PROBLEM OF THE MONTH

December, 2015

2015-2016

MATHEMATICS

5 points:

You have 100 beads, all different. How many different necklaces of length 42 can you make using these beads? The clasp on the necklace can be ignored, in the sense that 1-2-clasp-3-4 is the same as 1-clasp-2-3-4.

10 points:

Solve the following equation:

$$2015\ fractions\left\{\begin{array}{l}\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{\ddots\ -\dfrac{1}{1-\dfrac{1}{1-x}}}}}\end{array}\right. = x$$

Hint:

(i) Denote $f(x) = \frac{1}{1-x}$ and try to find a pattern using this notation.

(ii) Note, that an equation may, or may not have real solutions.

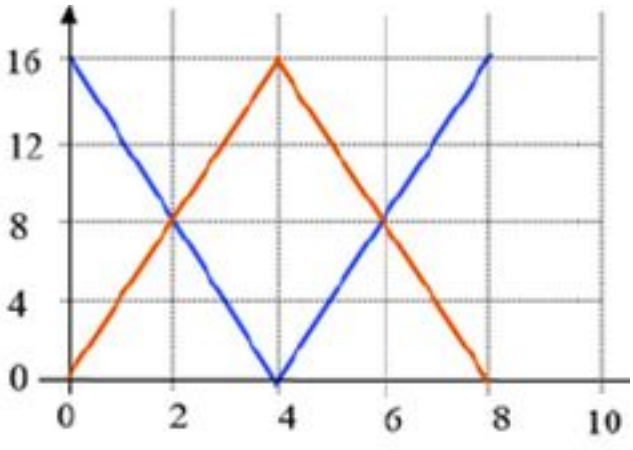
PHYSICS

5 points:

Two cars (blue and red) move in the same direction. The velocity-time graphs below represent their motion (time is given in minutes and the velocity in m/s). At initial moment of time, the red car is 1440 m ahead of blue car. Find the maximum distance between two cars.

There are two cars, one blue, one red, driving along the same straight road. The blue car starts 1440m behind the red car. The graph below indicates their velocities at each point in time, with the red line that starts at a velocity of 0m/s representing the red car, and a blue line starting at a velocity of 16m/s representing the blue car. Find the maximum distance between the two cars over the 8 minute time period.

Velocity, m/s



time, minutes

Hint: the distance traveled is given by the area under the velocity graph.

10 points:

A bus is moving along the straight road at a constant speed of 5 m/s. A boy, standing 30 meters away from the road, sees the bus when at the moment the bus is 50 meters away (from the boy). Is it possible for the boy to catch the bus, by reaching some point on the road before the bus reaches this point, if the boy can run at the maximum speed of 3 m/s? Explain.

A bus is moving along a straight road at a constant speed of 5 m/s. A boy, standing 30 meters away from the road, sees the bus when it is 50 meters away from him. Is it possible for the boy to catch the bus by reaching some point on the road as the bus reaches it if the boy can run at the maximum speed of 3 m/s? Explain and provide the point relative its distance from the truck at the start of the problem if the point exists.

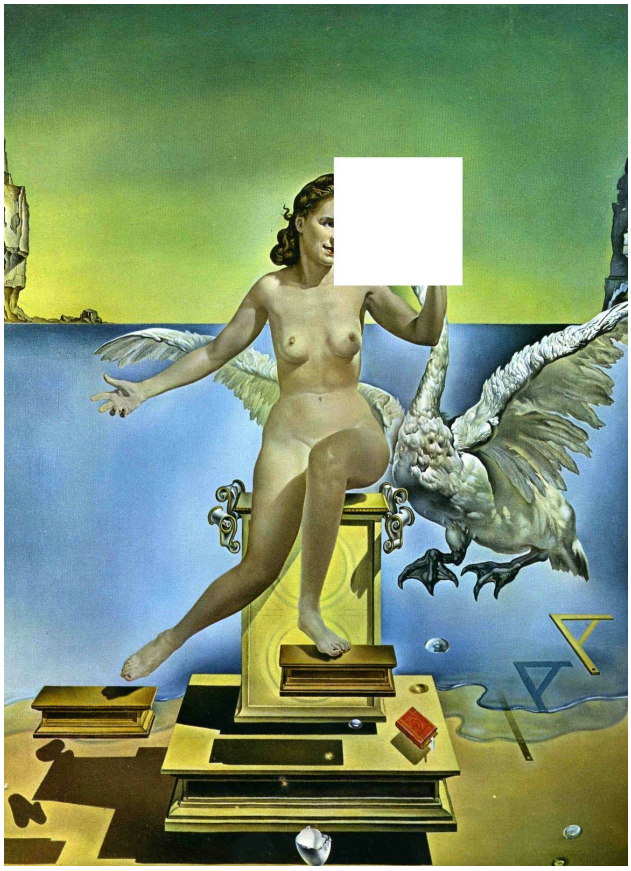
Hint: try to find the point x on the road which the child can reach simultaneously with the bus.

CHEMISTRY

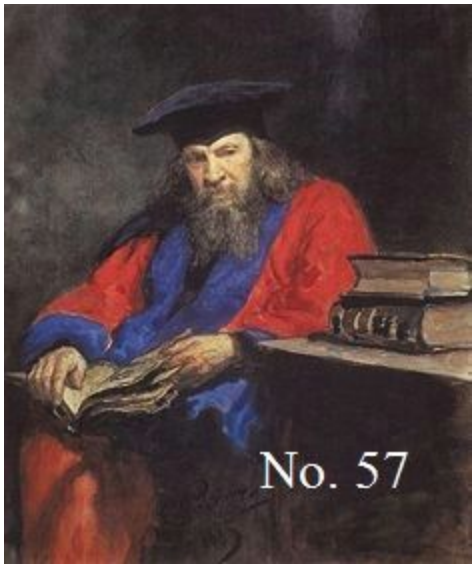
5 points:

Imagine you are playing the Escape the Room game. To open the lock, you need to know some secret word (probably, four letters long). You tried almost all hints that you found in the room, but your efforts were futile. The last three objects that seem potentially useful are the following:

1. A fragment of the photocopy of some painting:



2. A glass beaker with hot water. There is a test tube in the beaker, and there are few drops of some liquid metal in the tube. The tube is sealed. When you take the tube out of the beaker and let it cool down, the metal solidifies. When you put the tube into the hot water, the metal melts again.
3. A portrait where some odd message is added:

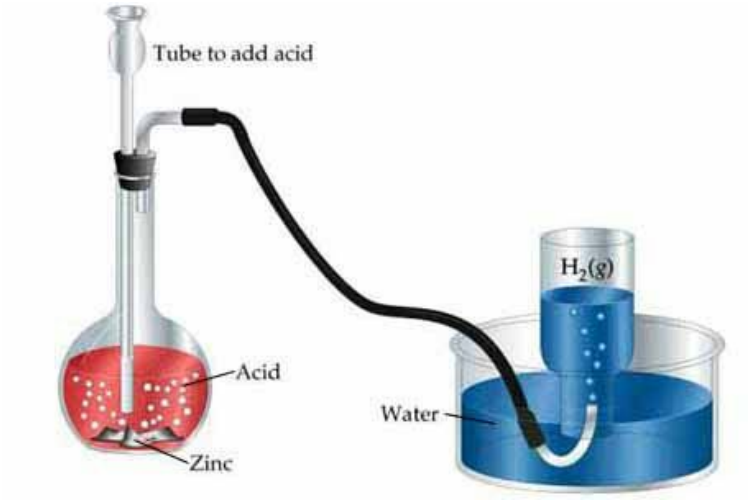


These three objects are your last hope, because you have no other idea on how to obtain the secret word.
Try to find that code.
PS. And, yes, we all live in the Internet era, so the whole power of Google is available to you (you have your smartphone with you, and the coverage is excellent).

Hint: Try to find what is the name the man on the second portrait, and what was his major contribution to science.

10 points:

When Bob, a Chemistry lab technician, came to Alice's lab, she was staring at some piece of a strange rock at her table. "What's that?" Bob asked. "Are you contemplating some new experiment?"
"Yes, Bob" Alice answered. "This rock is a fragment of *Chelyabinsk* meteorite. It exploded about a Russian city Chelyabinsk a couple years ago. My friend brought me a small fragment he picked up near the epicenter of the explosion, and I am going to take a part of this fragment to do its chemical analysis during the next class."
"Are you sure we will be able to do a comprehensive chemical analysis using our equipment?" - Bob asked.
"Of course, no. But we can measure the iron content in it. Wikipedia says there is about 10% of elementary iron in *Chelyabinsk*, and I think we can try to check is that is correct. To determine the percentage of iron, you Bob will take a small piece of this rock (about 50 grams) and grind it using our mortar and pestle. After that, weigh the amount of the powder, put it into the Cavendish's apparatus (shown on the picture below), and add hydrochloric acid



to the powder. Of course, in our case the meteorite powder plays the role of zinc: all hydrogen will be generated during the reaction of the meteorite iron with HCl."
"Ok, I understand, Alice", Bob says. "I add no zinc, and I use the meteorite material instead. What is my next step?"
"When the evolution of the hydrogen gas cease, measure the gas volume and calculate the iron content. Do you need more detailed instruction?"
"No, Alice, I understand. Alice, may I do this experiment with our students by myself? I believe I have enough experience, so your participation is not necessary."
"Yes, Bob, do that," Alice said. "Good luck. But don't forget to take excess of HCl, otherwise your results will be misleading.".

After the class, Bob came to Alice and said: "Alice, we took 53 grams of the meteorite material and 200 mL of 20% HCl. The volume of gas was 1.95 L."
"Good job, Bob. And what does that mean? What is the iron content in the meteorite? Is Wikipedia right or wrong?"
"Alice, Wikipedia is …."

What did Bob say?

Hint: Iron reacts with HCl according to the equation:

$Fe + 2\ HCl \rightarrow FeCl_2 + H_2$

That means one atom of iron produces exactly one molecule of the hydrogen gas.

BIOLOGY

5 points:

It is known that some animal species (wolves, geese, monkeys, etc) form communities with a very complex social structure, where a well developed hierarchy exists, and the members of the community establish strong personal relationships with each other. Obviously, to make that possible, the animals have to be capable of recognizing their congeners personally. Accordingly, in the communities of those animals who are incapable of recognizing each other personally (most fish species, majority of birds) no hierarchy or other forms of social structure exist (as a rule). Indeed, when the community is totally anonymous, it is almost impossible to propose the mechanism of formation of personal relationships. (*To demonstrate this idea, let's imagine some Internet community (for example, a chat) whose members are not allowed to have any nicknames, and who get a new random avatar every time they post a new message. Obviously, it would be very difficult for the members of such a chat to form any type of a more or less organized community.*)

Lizards, as well as other reptiles, are incapable of recognizing each other personally. However, in nature they form a community with a strong hierarchy: the strongest male occupies the best burrow in the most comfortable place, and it controls the greatest territory that it uses for hunting. The lizards establish the control over their territory by attacking invaders, so after a series of fights each member of the community gets their own hunting area, depending on their strength and fighting capabilities. Interestingly, the lizards form stable pairs, where the strongest male and the strongest female live together in their own burrow and use (and protect) the same territory. Despite that, even after a long period of living together the male and the female lizards do not recognize each other personally (a male lizard cannot recognize "his" female in the presence of other female lizards when they are placed in some artificial landscape).

Can you explain the mechanism of formation of stable couples between lizards despite the fact that they (i) do not recognize each other personally, and (ii) have to attack invaders to protect their territory?

10 points:

The development of reliable anti-AIDS therapy is a formidable task. One of the features that makes it extremely difficult is the fact that the enzyme responsible for copying HIV's genome (this enzyme is called "reverse transcriptase") works very unreliably and makes a lot of errors. That means, majority of viral particles are mutants that have numerous amino acid replacements in every HIV protein, including the reverse transcriptase itself. Normally, the non mutated virus

copies have an advantage over the mutants, but in a situation when HIV is subjected to some antiviral therapy, some mutant copies of HIV may have an advantage, so the new population of mutant virus forms that is not sensitive to this particular anti-HIV drug. In some cases, just a couple of weeks are needed for HIV to develop resistance to a particular drug.

HIV is a very primitive virus, and its genome encodes just few proteins. One of the key proteins is a reverse transcriptase itself, and this protein is the major target for anti-HIV therapy. Accordingly, most anti-HIV drugs (such as zidovudine) are mimics of nucleotides (which are the building blocks a reverse transcriptase uses to synthesize the viral DNA). Usually, HIV reverse transcriptase incorporates such a “pseudo-nucleotide” into the growing DNA chain, and that leads to termination of the viral DNA synthesis. Since the reverse transcriptase is a very imprecise enzyme, it utilizes the nucleotide analogs that are ignored by our own DNA synthesizing enzymes. That is why zidovudine and similar anti-HIV drugs are almost non-toxic to humans, but are capable of killing HIV.

However, since the reverse transcriptase makes a lot of errors, HIV, as well as the reverse transcriptase itself, evolve very rapidly, and new HIV mutants become more precise, and they ignore zidovudine and other first generation anti-HIV drugs. When the scientists faced this problem, they initially believed they are doomed to develop more and more novel anti-HIV drugs, each of which will be becoming obsolete rapidly.

However, about 10 years ago, the scientists developed a totally new strategy: they started to treat patients not with a single anti-HIV drug, but with a combination of four drugs: three of them were designed to suppress HIV reverse transcriptase, and the fourth one killed a second HIV enzyme. Surprisingly, the scientists found that HIV was incapable of developing resistance towards this type therapy, and no AIDS symptoms have been being developed in the patients who were taking this medication continuously and without interruptions.

Can you tell why that therapy does not lead to development of resistance?

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in C, C++, Java, or Python
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Submit the problem in a plain text file, such as .txt, .dat, etc. **No .pdf, .doc, .docx, etc!**

5 points:

One-Dimensional "XOR" Game of Life

The Game of Life (https://en.wikipedia.org/wiki/Conway%27s_Game_of_Life) is a game developed by the British mathematician John Horton Conway in 1970. In this two-dimensional game the state of the next generation of cells placed in a rectangular grid fully depends on the state of the neighbor cells in the previous generation. In this assignment, you will develop a simplified, one-dimensional Game of Life. As an input you will be a given a string that consists of 0s and 1s. This string represents a colony of cells, with each digit in this string representing a single cell. Cell state could be either 0 or 1. The state of the cell in the next generation will be logical XOR operation of its neighbors' states in the current generation. What this means is that if the neighbors of the cell are equal, 1 and 1 or 0 and 0, the cell will turn into 0 in the next generation. However, if we neighbors of the cell are different, 0 and 1 or 1 and 0, the cell will turn into 1. (Note: border cells, i.e. the leftmost one and the rightmost one, have one of the neighbors outside of the colony, and the state of those "outside neighbors is always 0). So, our colony of cells will change generation after generation. To make the output more appealing, you will print each generation of the colony on a separate line, you will use space instead of 0 and "x" instead of 1. For example, given the input of 1101010, first 6 generations of the colony will look like this:

```
xx x x
xx    x
xxx  x
x xxx x
  x x
 x   x
```

Write a program that takes a string of 1s and 0s on input and prints out 16 generations of the colony. You can use the following input :

```
000000000000000000000000000000000000000000000000000100000000000000000000000000000000000000000000000000
```

and see what comes out.

10 points:

You are given a rectangular area that consists of x's and spaces. You need to write a program that calculates the number of "strands" in this area. A strand is a sequence of x's that are either horizontally or vertically (but **not** diagonally!), connected. For example, here is a single strand:

```
xxxxxxxx
x      x
```

but here there are 3 strands:

```
xx
  xx
    xx
```

and here there are 3 stands as well:

```
xxxx xxxx


        x
```

Note that a single x not connected to anything else is counted as a strand.
Your program should take on input two numbers N and M, N defining the number of lines in the area, and M defining the length of each line, followed by N strings that should consist of x's and spaces. On the output print the number of strands detected.

You can play with the following sample inputs:

1)

```
4 9
xxxx xxxx
   xxx
x   x   x
xxxxxxxxx
```

2)

```
8 11
xx x xx x
```