

**PROBLEM OF THE
MONTH**



December, 2017

MATHEMATICS

5 points:

A large experimental scientific facility began operation in 2007, collecting 1 Terabyte (TB) of data that year. The amount of data newly collected each following year was double the amount of all the data accumulated prior to that year. How much data will the facility have accumulated by the end of this year, 2017? How much data will there be by the end of 2027?

Hint: At the end of 2008, there were 2 TB new data collected, and 1TB from the previous year, i.e. 3 TB total. How much will there be at the end of 2009? Try to figure out a general rule.

10 points:

Find all solutions of the following equation:

$$f(f(f(\dots f(x)))) = x$$

2017 times

Where the function which is successively applied 2017 times is $f(x) = \sqrt{4x^2 - 1}$

Hint: Try to solve equation $f(x) = x$. After that, you can demonstrate that (i) you found a solution for the original problem, and (ii) there are no other solutions. Alternatively, you can find the function $(f(f(\dots f(x))))$

PHYSICS

This month Physics problems are on the momentum and energy conservation. You might find the following links useful.

<http://hyperphysics.phy-astr.gsu.edu/hbase/ke.html>

<http://hyperphysics.phy-astr.gsu.edu/hbase/pegrav.html>

<http://hyperphysics.phy-astr.gsu.edu/hbase/mom.html>

<http://hyperphysics.phy-astr.gsu.edu/hbase/conser.html#conmom>

<http://hyperphysics.phy-astr.gsu.edu/hbase/elacol.html>

5 points

A block of mass m is moving with the velocity V on a flat surface without a friction, and collides with another block of mass $M = 2m$, standing on the same surface and attached to the rigid horizontal wall through a massless spring with the spring constant k . Find the the maximum compression of the spring after absolutely inelastic collision* of the blocks.

* The phrase “absolutely inelastic collision” essentially means that blocks stick together after the collision.

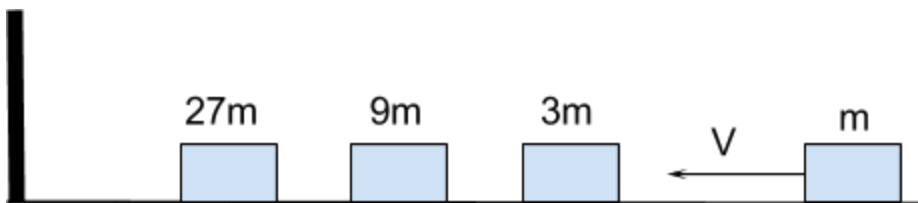


Hint: Use conservation of momentum for the collision and then conservation of energy.

10 points

A block of mass m is moving with the velocity V on a flat frictionless surface towards three blocks of masses $3m$, $9m$, $27m$ arranged along the straight line in the order shown in the Figure. Find the velocities of all blocks after all collisions between blocks and the wall (see the Figure) take place. Assume that all collisions are absolutely elastic*.

* The phrase “absolutely elastic collision” means that mechanical energy is conserved during the collision.



Hint: Find the result of the collision of mass m and $3m$ using both conservation of momentum and energy.

CHEMISTRY

Introduction to both problems.

Karel Čapek (1890–1938), a Czech playwright and novelist, is best known as the author of the very successful satirical plays R.U.R. (Rossum's Universal Robots) (1920) which gave the word 'robot' to the languages of the world. Besides that, he authored many novels that can formally be considered science fiction novels, although they are not just SciFi.

For example, in his novel *Krakatit*, which, for some reason is considered merely a satires on misused science he tells an amazing and fascinating story of a talented inventor named Prokop, who is desperately trying to find his only love and at the same time to save the world from the devastating weapons ("*Krakatit*") that he created and that fell into hands of irresponsible criminals. In the beginning of the novel, Prokop, who fell ill as a result of his experiments, tells his friend Tomasz, in delirium, about his research:

"...Look, I've been working for twelve years now."

"What does that give you?" - Tomasz objected sharply.

"Well, something happens. I sold explosive dextrin this year."

"How much?"

"Ten thousand. But it is, trifles. Just some lousy explosive for mines. If I wanted to ..."

"Are you better now?"

"Wonderful! I found such methods ... You know, my friend, just cerium nitrate - strong, a bastard! And chlorine, chlorine, tetraphase of nitrogen chloride explodes with light. You light a light bulb - and trrrah! But that's nothing but a trifling. Listen", - he suddenly blurted out, thrusting out from beneath the blanket a thin, terribly mutilated hand. "I have to take something in my hands, and I ... feel the movement of atoms. Like goosebumps. And every substance tickles in its own way, you know?"

"No."

"It's power, you know? The power contained in matter. Matter has a monstrous power. I ... I feel to my touch that everything is in it and it's crawling ... And all this is being harnessed by an incredible effort. It is worth to shake from the inside - and bummm! - decay. Everything is an explosion. Any thought is an explosion in the brain. You give me your hand, and I feel something exploding in you. I have such a fine touch, brother. And hearing. Everything is noisy like soda in the water. These are all tiny explosions... Oh, how my head buzzes! Ta-ta-tata - like a machine gun."

"All right," said Tomacz. "Now take aspirin."

"Good. Blows ... explosive aspirin. Perchlorinated acetylsalicylic acid. Nonsense. But I, you know, discovered exothermic explosives. Actually, any body is an explosive. Water ... water is an explosive. The Earth ... and the air are also explosives. The feather, the fluff in the featherbed is explosive. Well, as long as it has only theoretical value. And I discovered atomic explosions. I ... I invented an alpha explosion. Everything brrreak ... breaks up into positive particles. Thermochemistry does not exist. De... destruction. Destructive chemistry, that's what. This is a grandiose thing, Tomasz, from a purely scientific point of view. I have such tables at home ... Oh, if I had any equipment! But I have only eyes ... and hands ... "

5 points:

Krakatit, an explosive invented by Prokop possesses an immense explosive power: just one gram of this material was sufficient to destroy a large building. The detailed description of this compound is missing, but it seems to be some substance prepared by chemical means, and it is composed of argon, lead and some other elements. Is it possible to prepare a chemical explosive that fits Čapek's description, and how do you estimate the upper limit of the explosive energy a chemical explosive can produce?

Hint:

The total amount of energy that evolved during a chemical reaction (including explosions) is the difference between the energy of all chemical bonds that break and all chemical bonds that form.

10 points:

Carefully read the above conversation between Prokop and Tomasz and tell if you see anything wrong or inaccurate in what Prokop says? Namely,

- which Prokop's statements are correct, and which are wrong?
- which substances mentioned by Prokop can be prepared in a chemistry lab?
- is there any reason to expect these substances are explosive, or Prokop's intuition was wrong?
- explain your answer.

Hint:

Explosions are redox reactions, which means some atoms in the explosive molecule are being reduced, and other atoms are being oxidized (these atoms may belong to the same molecule, if the explosive is a pure compound, or to different molecules, if the explosive is a mixture). During reduction, oxidation state of an atom decreases, whereas during oxidation it increases. Some substance can be explosive when at least one electronegative atom in it has a very high oxidation state, and at least one atom has a low oxidation state.

BIOLOGY

5 points:

As alternative to using chemical control for agricultural pests, a biological control could be used. Biological control is an organism which would dramatically reduce population of the pest. It could be a predator that attacks the pest or a parasite that uses the pest's organism as a host. Imagine that you are a minister of agriculture, and you have a problem: the main agricultural plant in your country is being massively attacked of some beetle. Two different research groups proposed their solutions to this problem: one group is working on a novel chemical pesticide that efficiently kills the beetle, whereas another group proposes to introduce a new species of a wasp that was recently identified in South America: this wasp lays eggs on the beetle, so wasp's larvae develop inside the beetle's body and kill it before the beetle produces its own offsprings. You have to decide which project should be funded (you can fund only one project). Please, explain which factors have to be taken into account for making an optimal decision. In which case the first project should be selected, and which considerations can make the second project more competitive?

10 points:

We know that mutation rate during a replication of many viruses (especially, HIV) is much greater than the rate of mutation in host cells (for example, human cells). This, as well as a rapid (exponential) growth of the population of viruses in an infected organism allows viruses to quickly develop resistance towards many antiviral drugs. As a result, first generation anti-HIV drugs (for example, zidovudine), which were quite efficient 20 years ago, are currently almost inefficient, and next generation drugs are needed to suppress HIV.

We also know that cancer cells have increased mutation rate too, and they also grow exponentially, so it is not a surprise that the drugs that efficiently suppress tumor's growth at the early stage of cancer development may become inefficient at later stage of the disease. However, in contrast to AIDS, anti-cancer drugs do not lose their efficiency in new patients, so the drug that efficiently suppressed cancer growth, e.g. 20 years ago, works with the same efficiency for new patients.

Can you explain why many antiviral drugs lose their efficiency in several years (or decades) after they became popular, but anticancer drugs do not?

COMPUTER SCIENCE

- You can write and compile your code here:
<http://www.tutorialspoint.com/codingground.htm>
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on
<http://www.tutorialspoint.com/codingground.htm> before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc.
No .pdf, .doc, .docx, etc!

5 points:

You know that numbers can be expressed in different base number systems. Every day we operate with numbers in base 10, while computers like base 2 and base 16. But what if we are given a number and not told what base is it in. For example, 13. We understand what it is in base 10. In base 8 it is same as 11 in base 10. However, it cannot be a number in base 2!

Your program should enter a number from input. The number may contain digits 0-9 as well as a-f. For this number your program should calculate what is the smallest base this number could be written in, and then transform and print its value in base 10. For our example above, given 13 as an input, the smallest base would be 4, and 13 base 4 is 7 base 10.

10 points:

You may know a game called Reversi (<https://en.wikipedia.org/wiki/Reversi>). In this game, played on a 8x8 board, players take turns placing disks on the board. Each disk is white on one side and black on the other. For a player playing white a valid move is to place a disk white side up on an empty square so that there is another white disk present in a straight vertical, horizontal or diagonal line, following a number of black disks. This would cause all the "surrounded" black disks flipped (or reversed - hence, the name Reversi) to their white side.

In modern Reversi, the initial position (let's say, X is a black disk, and O is a white disk) is:

```
-----  
-----  
-----  
---OX---  
---XO---  
-----  
-----  
-----
```

In this position valid moves for X are (denoted by *):

```
-----  
-----  
---*-----  
--*OX---  
---XO*--  
-----*-----  
-----  
-----
```

Your program should enter some Reversi position from input, figure out whose move it is (let's presume that X always starts the game), and print the board marking all possible next moves, the way we did above.