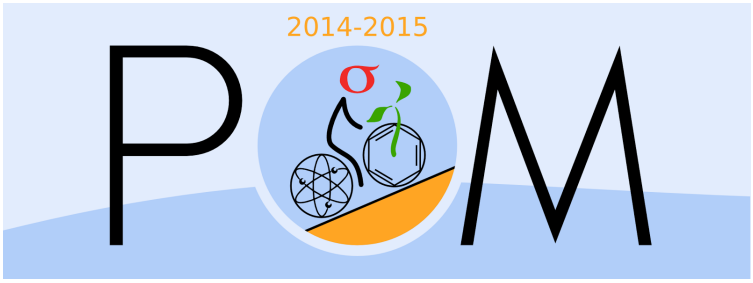
PROBLEM OF THE MONTH



DECEMBER, 2014

MATHEMATICS

5 points:

Three friends are deciding who is going to run to the store to get bagels. How could they choose one of them fairly by flipping a coin?

Solution:

There are many solutions to this problem. For example, the following algorithm might be used.

1. the friends flip the coin twice
2. Then depending on the results they do the following
    a. HH the first friend runs to the store
    b. HT the second friend runs to the store
    c. TH the third friend runs to the store
    d. TT the results of coin flipping are annulled and friends go to the step 1., i.e flip the coin twice again and ...

It is clearly that the algorithm is fair. Of course there is a theoretical possibility that the friends will continue to flip the coin until the store is closed getting TT all the time. However, the probability of TT is ¼ and the probability of having TT ten times in a row is (¼) to the power 10 which is less than one part in a million.

10 points:

It is known that some family has two children. It is also known that one of them is a boy born on Tuesday. What is the probability that the other child is a boy as well?

Note: This is a math problem. Therefore, assume that the probability of a single child being a boy is 50% and that having birth on any day of the week is equally probable.

Solution:

Answer: 13/27

There are 14 equally probable possibilities for a one child: a boy born on Monday, a boy born on Tuesday, …, a girl born on Monday, …, a girl born on Sunday.

It is known that one of two children is a boy born on Tuesday. However, we do not know whether this was the first child or the second. Let us consider both cases. If the first child is a boy born on Tuesday, there are 14 possibilities for the other child and out of them 7 possibilities correspond to the second child being the boy. If the second child is a boy born on Tuesday there are 14 possibilities for the first child and out of them 7 possibilities correspond to the first child being the boy. In this counting we counted the case when both children are boys born on Tuesday two times. Therefore, we have 14+14-1=27 equally probable possibilities to have one of the children to be boy born on Tuesday (we subtracted the case we overcounted). Out of them we have 7+7-1=13 equally probable possibilities to have both children to be boys (again we subtracted the case we overcounted). As a result we have the conditional probability that the other child is a boy to be 13/27.

PHYSICS

5 points:

An egg is dropped from the top of a very high building. As the speed of the egg increases, the force of air resistance changes proportionally to this speed squared: $F_{air} = kv^2$ (here v is the speed and k is certain unknown constant). As a result, the egg first accelerates but eventually reaches a constant speed $v_0 = 40 m/s$ (called terminal velocity).
Find the acceleration of the egg at the moment when its speed was $v = 30 m/s$.

Solution:

When speed is $v_0 = 40 m/s$, air resistance force is equal and opposite to the gravitational force acting at the egg, mg. Therefore, when $v = 30 m/s$, that force is $F_{air} = -mg(30/40)^2 = -9/16\, mg$. The total force acting at the egg is $F = mg - \frac{9}{16}mg = \frac{7}{16}mg$. The acceleration can be found from the 2nd Newtons Law: $a = F/m = \frac{7}{16}g = 4.3 m/s^2$

10 points:

A pendulum is made of a string and a mass *m* attached to it. Initially, the mass is held so that the string is horizontal. The mass is released with no initial speed, and the pendulum starts to oscillate. Find the maximum tension in the string during these oscillations. Neglect air resistance or any other energy loss.

Solution:

Let L be a length of the string. The tension is maximum when the pendulum is at its lowest point. At that moment, the speed of the mass is the largest, and so is the centripetal acceleration, v^2/L. This acceleration is caused by the tension T minus gravity force mg:

$mv^2/L = T - mg$. Therefore, $T = mg + mv^2/L$.

We now just need to find the speed by using the energy conservation: $mv^2/2 = mgL$. Hence, $T = 3mg$.

CHEMISTRY

5 points:

LZ 126 *Hindenburg* was a commercial passenger airship (dirigible). It was used on regular transatlantic flights connecting Germany and the US (New Jersey) from 1936 till 1937. *Hindenburg* made 63 flights, and it was destroyed in a catastrophic fire during landing. 62 passengers and crew perished during this tragic incident.

What was the reason of the fire, and was it possible to change *Hindenburg's* design to eliminate a possibility of fire? Was it possible to make a dirigible with exactly the same size and payload? If the answer is yes, prove it by making necessary computations, if the answer is no, calculate the maximal payload of the modified *Hindenburg.*

Solution

The *Hindenburg* catastrophe was a result of the overconfidence of German engineers, who decided it was safe to fill *Hindenburg's* balloons with hydrogen. Since hydrogen is an extremely flammable gas, a small leakage in the stern balloon, followed by a small spark turned the airship into a giant torch. This tragic incident could be avoided if another, non-flammable gas was used in *Hindenburg*. Which gas could it be? As we all know, equal amount of molecules of any gas occupy the same volume at the same

temperature and pressure. We know the weight of one mole of hydrogen (i.e. the weight of $6 \cdot 10^{23}$ molecules) is 2 grams, and the weight of one "mole" of air is approximately 29 grams (this weight is calculated assuming that the air is composed of 80% of nitrogen and 20% of oxygen will masses 32 and 28, accordingly). That means, the buoyancy of one mole of hydrogen is 29 (the molar mass of air) minus 2 (the mass of hydrogen itself), or 27. In other words, one mole of hydrogen can lift a 27 gram load at normal pressure and temperature. Obviously, only the gas with the same (or lower) molecular mass could serve as a totally adequate replacement for hydrogen. Unfortunately, no such gas exists, and the gas with the second lowest molecular mass is helium. Its mass is 4, which means one mole of helium can lift a 29-4=25 gram load. In other words, if *Hindenburg* had been filled with helium, its load would be $25/27 \cdot 100\% = 93\%$ of the load hydrogen filled *Hindenburg* could carry. Therefore, it would be impossible to make a safe dirigible with exactly the same size and payload. To lift the same load, *Hindenburg's* volume had to be about 9% bigger.

Other gases with a density lower than the air density are carbon monooxide (CO, molecular mass 12+16 = 28), nitrogen (N_2, 14*2 = 28), methane (CH_4, 12+4 = 16). However, all of them, except nitrogen, are combustible, and their density is much higher than the density of hydrogen and helium.

It is necessary to note that German engineers did have plans to replace hydrogen with helium in *Hindenburg*, and changed its design accordingly. Unfortunately, the only country that produced helium on industrial scale in 1930s was the US, and the US government did not allow selling helium to foreign companies.

10 points:

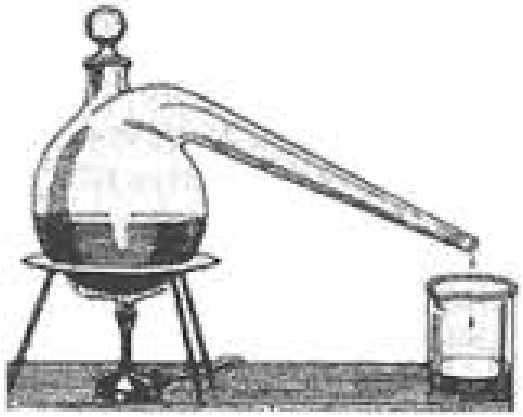
Al kohl (the 'kohl') is an Arabic alchemical term that was initially used to describe a tiny and volatile powder obtained after heating some minerals. Later, the alchemists, both in the East and in Europe, used this term to describe any volatile substances obtained after heating liquids or solids followed by condensation of the vapors formed. In English, this term later transformed into 'alcohol', and it is used now mostly for ethanol (ethyl alcohol). Ancient alchemists believed by heating wine, and by collecting and condensing the vapors, they obtain a 'spirit' of vine, or its 'essence'. Accordingly, the Medieval Latin name of ethyl alcohol was *spiritus vini*, or 'the spirit of vine'. By the way this name, was preserved in many European languages (German *spitirus*, Spanish *espiritu*, Russian *спирт* (pronounced *'speert'*), Italian *spirito,* etc).

Besides *spiritus vini,* Medieval alchemists obtained other *spirits*. They did that by heating various substances, or mixtures of substances, and by collecting the vapors formed during that process. Please, name as many these *spirits* as possible, and give their

modern names. It would be good if you provided the equations of the corresponding chemical reactions.
Happy googling!

Solution

Two other *spirits* were obtained by heating two salts, table salt, sodium chloride, and saltpeter, potassium nitrate, with sulfuric acid in the apparatus similar to that shown on the figure.



In both cases, the exchange (double replacement) reactions between sulfuric acid and the salts yielded volatile acids, hydrogen chloride and nitric acid, accordingly:

$NaCl + H_2SO_4 \rightarrow Na_2SO_4 + HCl\uparrow$

$KNO_3 + H_2SO_4 \rightarrow K_2SO_4 + HNO_3$

The arrow ($\uparrow$) means the product is in a gas state at normal conditions. The first product, hydrogen chloride, was gas, however, it is extremely easily soluble in water, so it combined with water vapors, and condensed into the concentrated water solution of hydrogen chloride, which was known to alchemists as the ***spirit of salt***.
The nitric acid, HNO_3, which forms in the second reaction, boils at 83° C and upon condensation yields a liquid known to alchemists as the ***spirit of saltpeter (niter)***.
The non-volatile byproducts of these two reactions were sodium sulfate and potassium sulfate, accordingly.
The third *spirit* was obtained by heating ammonium chloride (*sal ammoniac*) with some strong base, for example, with sodium hydroxide:
$NH_4Cl + NaOH \rightarrow NaCl + H_2O + NH_3\uparrow$
Ammonia NH_3, like hydrogen chloride, is a gas, and it is also extremely soluble in water. The solution of ammonia in water has a very specific smell, it is basic, and it was known as a ***spirit of ammonia.***

Other *spirits* were obtained by distillation of naturally occurring sulfuric acid (“vitriol”) or its salts, through dry distillation of wood (by heating wood in a closed container in the absence of air), or by heating some salts of acetic acid. These products were occasionally called “spirit of vitriol”, “wood spirit”, and “pyroacetic spirit”, accordingly. However, these terms were used very rarely.

BIOLOGY

5 points:

Around 300 BC a physician from Alexandria named Herophilus was studying human anatomy and physiology. He concluded that the brain is the organ which performs thinking. What observations could lead you to the same conclusions?

Solution

Let' us first review who was Herophilus and what he did. Herophilus lived in ~300BC. With his younger colleague, Erasistratus, he did the first ever scientific dissections of human bodies, using cadavers. Dramatically, this kind of medical work has been done only for a short period of 30-40 years, and then human dissections were forbidden for the next 1800 years, until Renaissance. Herophilus is called the Father of Anatomy. He wrote 13 manuscripts, but none of them survived until our time; all we know about Herophilus comes from writing of others.

Herophilus realized that the network of nerves originates in the brain, and then spreads throughout the body. He concluded that the brain was the controlling organ in a human. This discovery went against common belief at the time that the heart was the source of human intellect and reason. He described many individual nerves (such as optic and auditory nerves), and observed difference between sensory nerves (those carrying sensation signals to the brain) and motor nerves (those carrying command from the brain to muscles). He discovered that damage to the motor nerves caused paralysis. He stated that the brain initiated voluntary motion.

Herophilus made anatomical discoveries in other organs systems as well, but we will not discuss them here.

Now, let's see what other observation could have been made to conclude that the brain performs thinking.

- People surviving brain trauma to certain brain areas would lose some of the functions. For example, damage to motor cortex will cause loss of the motor control - e.g. control of the muscle movement. Damage to sensory areas of the cortex will cause loss of corresponding senses - for example, damage to the visual cortex on the back of the brain will cause loss of vision even if eyes are not damaged.

- Damage to peripheral organs do not cause impairment of thinking - arguing that these organs (liver, heart, etc) are not responsible for central control functions. This is not very strong argument though, for the following reasons: (1) brain damage does not necessarily causes "thinking" impairment; for example, damage to visual cortex will cause blindness, but will not impair "thinking". (2) damage to peripheral organs may cause "thinking" impairment; for example, failing liver will cause confusion and disorientation due to toxicity.

- Modern technology allows us to directly see brain activity during thinking process. Functional magnetic resonance imaging (fMRI) allows to detect increased usage of oxygen and glucose or increased blood flow is particular brain areas as small as 1 cubic millimeter. When person is "thinking", or experiencing emotions, or imagining playing sports, or controls muscles, etc., the neurons responsible for these activities need more oxygen and glucose. This causes increase in blood flow in capillaries. All these effects can be directly observed in people, proving that brain is the organ performing thinking and central control functions.

References:

http://www.historyinanhour.com/2012/12/17/herophilus-and-erasistratus/#sthash.WVhDIsrp.dpuf

http://www.britannica.com/EBchecked/topic/263634/Herophilus

http://www.ncbi.nlm.nih.gov/pubmed/9762750

10 points:

Unicellular organisms are extremely successful in their environments and, under the right conditions, can reproduce rapidly. Why do you think multicellular life has developed and became so widespread?

Solution

As for many biological questions, there is no definitive answer. We can only speculate about evolutionary processes and trends that led to the emergence and development of

multicellular life forms. . Interestingly, multicellularity has been independently acquired multiple times during the evolution of eukaryotes, suggesting that multicellularity is beneficial for the survival. The resulting organisms developed into more than 20 different lineages including animals, plants, fungi, slime molds, green and brown algae, and several other eukaryotes.

There are two major strategies responsible for this process: (1) clonal division of a single cell or (2) aggregation of different cells. The first mechanism must be predominant since intra-organismal competition in the aggregation scenario is expected to be evolutionarily unstable. Thus, eukaryotic lineages that attained the most complex multicellular lifestyles (i.e., plants and metazoans) arose through clonal cell division. The second mechanism, aggregative cell behavior, typically represents a transient life cycle stage. This type of multicellularity arose within several eukaryotic clades, including the famous social amoebae (dictyostelids) and others.

There are several more or less obvious benefits of multicellular organization:
1. Cells can be specialized for different functions. Such specialization allows cells to devote more resources for single function, omitting other functions, which make the cell more efficient. For example, muscle cells perform contraction and glucose absorption, but don't need to maintain complex biochemistry to break down toxins, which is a liver cell's function.
2. Different cells (cell types) can communicate with each other, performing more complex functions than a single cell can do.
3. Multicellular organisms are usually larger. This means that more complex processes of signals/nutrients exchange and communication with the environment have been developed.
4. Better homeostasis and specialization/compartmentalization of biochemical pathways resulted in significantly longer lifespan.
5. Multicellular organisms are much better adapted to environmental changes. The principal difference here is that there is no need to reroute the whole metabolism – changes in some organs and tissues may suffice.
6. Multicellular organisms can use resources better. The typical example here is a tree that can use both inorganic (through its roots) and photosynthetic (through its leaves) sources. The trunk consists of specialized cells forming channels for water and mineral delivery.
7. Multicellularity has allowed multiple new ways of communication, movement, and social organization.

As always in evolution, all advantages come with a price tag.

1. Because of their extreme specialization, cells of multicellular organisms lose their ability to live independently.
2. Almost all multicellular organisms have different sexes. This significantly slows down their reproduction rate.
3. Evolution is slower, making it more problematic to adapt to dramatically changing conditions.

As for the unicellular life forms, they:
1. Are usually more tolerant to changing environmental conditions. While mammals, for example, cannot withstand changes of about ±6 – 10°C of their normal body temperature, some unicellular organisms can survive in the temperature range of ±60°C.
2. As a rule, have a wider range of available and usable food sources. They often can use a limited set of available organic compounds for all their biochemical and physiological needs.
3. Have significantly shortened reproduction rate (doubling time).
4. Are very abundant and wide spread.
5. Often can live as symbionts or parasites with other organisms.

COMPUTER SCIENCE

Solutions must be typed and submitted in one of following formats:
.txt .c .cpp .java .py
Solutions written in Java, C, C++, Python and pseudo-code are accepted.
Pseudo-code guidelines are at
http://users.csc.calpoly.edu/~jdalbey/SWE/pdl_std.html

5 points:

Given a 4-digit integer print whether all the digits are different or not.

Solution (Python):

```
n = input('Enter your 4-digit interger: ')
print 'It is ' , 10**len(set(str(n)))>n , ' that your number has no repeating digits'
```

10 points:

You are given a chess-board with various pieces at specified locations. You have a knight at square (a, b). Determine if it is possible to move the knight to square (i, j) without moving any other pieces. The knight can make any number of moves. 1-2 points will be awarded for finding the smallest number of moves needed.

Hint: First, write the algorithm to see if you can do it in just a few moves, say, at most 2.

Technical details: You can define variables that define your chess board however you like, but make sure to write a few sentences in the comments of your code that explain your setup. One way to store the data about pieces on the board is to define a 2D array where 0 corresponds to empty square, and 1 corresponds to “occupied by a piece that is not the knight” (all pieces except for the knight are there just to take up space).

Solution:

We will use the concept of the Breadth First Search (BFS) algorithm to solve this problem.

http://en.wikipedia.org/wiki/Breadth-first_search

In a nutshell, this process works by making our knight try all possible paths simultaneously -- every time there are 2 possible squares to go to, it "makes of copy of itself" and goes to both of them.

---

board = two dimensional, 8x8 array with *board*[i][j] = 0 meaning the square in row i and column j is empty (or has the knight), and *board*[i][j] = 1 meaning the square is occupied (and does not contain the knight).

x_init = the initial row coordinate of the knight.

y_init = the initial column coordinate of the knight.

x_destination = the destination row of the knight

y_destination = the destination column of the knight

num_moves = *0*; we will keep track of the number of moves the knight needs with this variable.

knight_visits = an 8x8 array like *board,* but starts with all elements equal to 0. We will use this array to track the knights moves: *knight_visits*[i][j] = 0 means the knight has not visited the square (i,j); value of 1 means the knight has just visited the square, and value of 2 is assigned if the knight visited the square earlier, and already moved onto the next square.

knight_next_move = 8x8 temporary array, all values initialized to 0.

Accept user input to determine the values of *board, x_init, y_init*.

Record where the knight starts: knight_visits[x_init][y_init] = 1

Until there are no squares in *knight_visits* with value 1
{
Initialize all squares in *knight_next_move* to 0.

add 1 to *num_moves*

For each square *knight_visits[i][j]* with value 1
{

    Move the knight 2 squares to the right, one up:
    set i' = i + 2
    set j' = j + 1

    if i' and j' are between 0 and 7, inclusive (otherwise we're off the board)
    {
        if *knight_visits*[i'][j'] is equal to 0 (that is, the square is not occupied and the knight has not visited there yet)
        {
            set *knight_next_move*[i'][j'] to 1

```
        }
    }
```

Repeat the orange section with the other 7 possible moves (2 left 1 up, 2 right 1 down, 2 down 1 left, etc…)

}

We want to make a distinction between the squares the knight JUST visited and the ones it visited earlier. To all squares *knight_visits[i][j]* with value *1* we assign 2.

We want to add to *knight_visits* all squares that the knight just visited. That is, for each *knight_next_move[i][j]* that has value 1, we set *knight_visits[i][j]* to 1.

Now, we check if the knight got to the square it needed to, by seeing if *knight_visits[x_destination][y_destination] == 1.* If so, return *num_moves.* Otherwise, repeat the cycle.

}

If we got to this point, it means that the knight went to all the squares it could, but has NOT gotten to its destination. Thus, return "false”