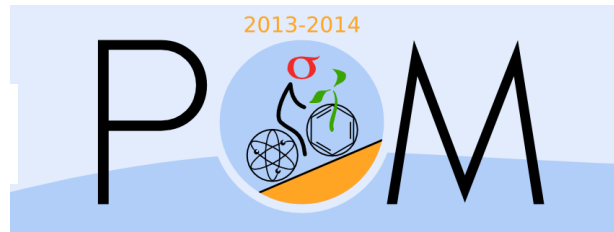


PROBLEM OF THE MONTH



November 2013

MATHEMATICS

Prove that $n^5 - 5n^3 + 4n$ is divisible by 120.

PHYSICS

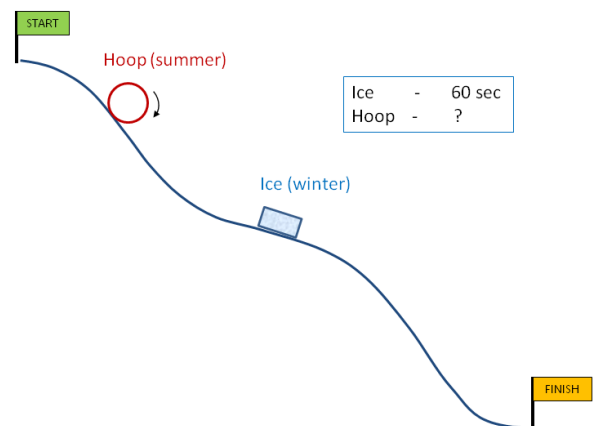
In winter, a flat piece of ice resting on the top of the mountain starts sliding down an icy track. The track has rather irregular shape, and its slope is not constant (see the picture). It takes $t=60$ seconds for the piece of ice to reach the bottom of the track.

In summer, an experiment is made with a hoop that rolls down the same track with zero initial speed without any sliding.

a) Will it take more, less or the same time for the hoop to complete the track? Substantiate your answer.

b) Can you find out, how much time (in seconds) it will take for the hoop to complete the track?

Note that kinetic energy of a rolling object is the sum of the kinetic energies of its translation and rotation. Neglect air resistance and all other losses of energy due to friction.



CHEMISTRY

On July 2, 1982, Larry Walters, a truck driver from Southern California, made his only daredevil historic solo flight. He fastened 42 helium filled weather balloons to a lawn chair and anchored it with a rope to a bumper of his jeep. He carried various supplies with him as well as a CB radio and a BB gun to shoot balloons one at a time to descend. Unfortunately, he dramatically underestimated the buoyancy of the balloons: when he cut the rope, he took off so quickly that in few minutes he reached altitudes of up to 16,000 feet, and for several hours he was floating in skies above L.A. According to an article in the New York Times, Walters was spotted by pilots from TWA and Delta Airlines, who radioed the tower that they passed a guy in a lawn chair with a gun! It was cold at 16,000 feet and he started shooting some of his balloons to descend, but dropped his BB gun and had to wait for his rig to come down on its own.

Suppose (theoretically) you are going to reproduce that flight, and you obtained needed amount of empty balloons, but you have no helium to fill them (your parents didn't approve your enterprise, and they did not allow you to buy helium). Therefore, you decided to use hydrogen instead of helium. You have an unlimited amount of battery acid and common materials. How many pounds of iron nails do you need to buy at a hardware store to take off sitting in a chair (assuming that chair's weight is 2 kg, and other parts of your "apparatus" have zero weight)?

To solve this problem, you may use information from the October PoM, and from the SchoolNova web site (http://schoolnova.org/nova/classinfo?class_id=chemistry101&sem_id=f2013), or to look up molecular masses and/or densities of various substances in the web.

BIOLOGY

Imagine yourself a biologist on the board of Tara*, a unique, specially equipped, 36-meter-long sailboat designed for the most advanced scientific experiments, in the middle of Indian Ocean. You study green microscopic algae. In the morning, you took a metal bucket and scooped a full bucket of ocean water and placed a bucket on the deck of the boat. You measured the amount of green microscopic algae in the bucket immediately and in the evening and noticed that the amount of green microscopic algae has increased significantly. Why do you think this has happened? Please, name as many factors that may influence the algae growth as possible. Try to select the most important factors (say 3) and propose experiments to test each of them.

*“Our seas and oceans are under constant threat. They are affected by many kinds of human activity, by coastal erosion, CO₂ and acidification, transport activities and climate change. But they are also an enormous source of wealth, much of which is still undiscovered”. (European Commission’s Research Directorate-General). To investigate all these issues, an international team of oceanographers, ecologists, biologists and physicists from prestigious laboratories set out on a three-year, round-the-world voyage (known as Tara Oceans Expedition), with the goal of analyzing plankton ecosystems in relation to physicochemical conditions throughout the world’s oceans, assessing their adaptation to a rapidly changing earth system. For more information check out the Tara Oceans Expedition website at <http://oceans.taraexpeditions.org/>

COMPUTER SCIENCE

1) 5 points

Write a Java program to compute the following

$$1/100 + 2/99 + 3/98 + \dots + 99/2 + 100/1$$

using a loop. Print out the result and try to make your program as short as possible.

2) 5 points

Write a Java program that defines and then uses a method (function) to return the volume of a sphere given its radius.

Use `java.lang.Math` for the value of PI.

Call your method from the main method 3 times with different arguments and print out the results.

Resources:

<http://csc.columbusstate.edu/woolbright/java/loops.html>

http://www.tutorialspoint.com/java/java_loop_control.htm

<http://math.hws.edu/javanotes/c2/s5.html>

http://www.tutorialspoint.com/java/java_methods.htm

<http://www.horsesnw.com/articles/JavaMethod.htm>