Problem of the Month. Solutions: October 2012.

MATHEMATICS

One fine day in SigmaCamp Mark came to the chemistry lab where he held semilab for the campers. He was unhappily surprised by a frog jumping out at him from the test tube box. Now all the test tubes were contaminated and unfit for the experiment he had in mind. Unless someone washes them really well…

So Mark decided to appoint this task to the guilty party. He knew that only the members of his semilab (α-camper, γ-camper, τ-camper, ε-camper) and the helping counselor had access to the lab. When questioned, each of them made three allegations:

Counselor:
- ***I didn't do it.***
- ***I have never touched a frog in my life.***
- ***τ-camper did it.***

α-camper:
- ***I didn't put a frog in the test tube box.***
- ***I am 12 and my frog-playing days are long behind me now.***
- ***ε-camper knows who did it.***

γ-camper:
- ***I didn't do it.***
- ***I didn't know ε-camper before coming to the SigmaCamp.***
- ***τ-camper did it.***

τ-camper:
- ***I am not guilty.***
- ***ε-camper did it.***
- ***γ-camper is lying when he says I put the frog into the test tube box.***

ε-camper:
- ***I didn't put the frog in the box.***
- ***α-camper is guilty.***
- ***γ-camper can vouch for me because we take the same classes in SchoolNova.***

Later, each suspect said that two of his/her allegations were true and one was false. Assuming they didn't lie this time, which of them put the frog in the box and will now spend his evening washing all the test tubes?

Solution

τ-camper is innocent because he says so twice. Then the third statement from γ-camper is a lie, which makes his second statement true and thus he really didn't know ε-camper before coming to the SigmaCamp. Then ε-camper's last statement is a lie, which makes his second statement true. So α-camper did it.

PHYSICS

In deep outer space, where gravity can be ignored, Hegemon − the rebel spacecraft, moves towards Peace station with a constant velocity of 500 m/s. When the distance to Hegemon is 14.5 km Peace opens fire with small nuclear balls at a rate of 1 ball per second. The muzzle speed of each ball is 1500 m/s. Hegemon turns-on its protective shield, and the balls elastically bounce off. Assume that the bounced nuclear balls do not interact with the incoming ones, instead they pass through and go directly back to Peace.
i) How long will it take the first ball to reach Peace, counting from the moment when the first nuclear ball was fired?
ii) Starting from the moment when the first ball returns and hits Peace station, how long will it take until the second ball hits Peace?

Solution

(i)

The speed of the incoming ball relative to the Hegemon is $500+1500=2000\text{m/s}$

Hence, the first ball will hit Hegemon in

$$t_1 = \frac{14500}{2000} = 7.25sec$$

Then it will bounce back with a speed 0f *2000* ***relative to Hegemon***, i.e. the speed relative to the Peace will be *(2000+500) m/s*. Thus we can find the time it takes the first ball to reach the Peace counting from the moment it bounces back from Hegemon.

$$t_2 = \frac{14500 - 7.25 \cdot 500}{500 + 1500 + 500} = 4.35sec$$

So the total time is:

$$t = t_1 + t_2 = 7.25 + 4.35 = 11.6sec$$

(ii)

The second ball will reach Hegemon in

$$t_1 = 1 + \frac{14000}{2000} = 8sec$$

(because the distance to Hegemon at the moment the second ball is fired is 14000).

Similarly to the first part of the problem we can get the time it takes the second ball come back to Peace from the moment it bounces off of the Hegemon:

$$t_2 = \frac{14500 - 8 \cdot 500}{500 + 1500 + 500} = 4.2sec$$

Hence:

$$t = t_1 + t_2 = 8 + 4.2 = 12.2sec.$$

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Therefore the interval between the balls hitting Peace is

$$\delta t = 12.2 - 11.6 = 0.6sec$$

Remark. Note that while originally the time interval between the balls fired by Peace was 1.0 second, the time interval between the incoming balls hitting the Peace is less: 0.6 seconds. This phenomenon is similar to the so called Doppler Effect when the frequency of the sound (or light) measured by the observer moving towards the source turns out to be higher than the frequency of the sound emitted by the source.

CHEMISTRY

You are given two metal cubes of equal size. One cube is made of gold, while the other is made of aluminum.

1) Which of the two cubes contains a greater number of atoms?
2) How many times greater?
3) Can you comment on your answer?

Solution

Density of aluminium is 2.70 g/cm3; atomic weight of aluminium is 26.98;
Density of gold is 19.32g/cm3, its atomic weight is 196.97.

Obviously, since gold is more dense, a golden cube weighs 19.32/2.70=7.16 times more then the aluminium cube of the same size.
However, an atom of gold is 196.97/26.98=7.30 times heavier then the atom of aluminium. In other words, for the golden cube to have the same amount of atoms as the aluminium one, the former must be 7.30 times heavier then the latter. However, as we found, the golden cube is just 7.16 times heavier. That means the aluminium cube will contain 7.30/7.16=1.02 times more atoms then the golden cube.
However, 1.02 is very close to 1, so we can conclude that aluminium and golden cubes of the same size contain almost the same amount of atoms. That is very interesting and unusual, because that means much heavier atom of gold has almost the same volume as the atom of aluminium.

BIOLOGY

There are spring-blooming and fall-blooming plants (the former bloom only in the Spring and the latter bloom only in the Fall). Why don't the spring-blooming plants usually bloom again in the fall when the temperature outside is the same as it was in the spring? How would you explain the fact that sometimes plants do make a mistake and bloom? (We are not talking about indoor plants).

Solution

Why and how do plants flower?

The flowering plants include trees, grasses, shrubs, and, of course, flowers. When the plant matures and is ready to reproduce, it develops flowers.
Why and how do plants flower is something that has puzzled botanists for centuries. Even now there is still quite a bit about the process that we don't understand.
Plant growth and flowering are influenced by number of external and internal factors.
The external factors affecting plant's growth are: light, temperature, oxygen, carbon dioxide, soil water, soil nutrients, etc.
The internal factors are growth regulators, carbon-to-nitrogen ratio, and genetic factors.

In some species, the timing of flowering is primarily influenced by environmental (external) factors. These factors include photoperiod (i.e. day length), light quality (spectral composition), light quantity (photon flux density), vernalization (exposure to a long period of cold) and nutrient and water availability.
Other species are less sensitive to environmental variables and appear to flower in response to internal cues such as plant size or number of vegetative nodes.
Flowering can also be an important response to the **stresses** such as nutrient deficiency, drought, and overcrowding. This response enables the plant to produce seeds, which are much more likely to survive the stress than is the plant itself.

Most important external factors are:
Temperature: Many plants will not flower unless they have had a period of prolonged cold. Vernalization is a period of prolonged cold that some plants require before they can flower. This ensures that they only produce flowers after the damaging cold of winter has passed. This mechanism was uncovered only in 2011 and the results were published in leading scientific journals in 2012 – Nature and Science).
Read more:
http://phys.org/news/2012-07-species-programme-climates.html
http://www.sciencemag.org/content/337/6094/584.abstract
http://www.nature.com/nature/journal/v484/n7393/full/nature10928.html

Length of the day (Photoperiodism): Some plants are obligatory dependent on photoperiod - they will not flower until the dark period reaches a certain length. Warmer weather earlier in the year has no effect on their flowering time. Garner and Allard conclusively demonstrated the role of photoperiod in flowering in the 1920s in their classic experiments with the Maryland Mammoth mutant of tobacco.
Read more:
http://www.ag.auburn.edu/hort/landscape/lightduration.html
http://en.wikipedia.org/wiki/Photoperiodism
http://www.newyorker.com/online/blogs/culture/2012/05/early-blooming-plants.html#ixzz29NAVeNaP

How is a plant "tricked" into thinking it's Spring?

It is unusual but not rare to see some plants flower out of season. It mostly happen with plants which form their flowers for next spring in the late spring and early summer of the previous year.

Blooming out of season is a response of plants that have been stressed.

The type of plant determines what sort of stress will cause re-blooming. Plant blooming is a survival mechanism. It results in pollination thus increasing chances of plant's survival.

1. Many spring flowering plants like lilac, forsythia, dogwood, apple, and pear produce quite a few flowers in the fall.
This usually occurs when cooler temperatures with a rain follow a hot dry or otherwise stressful summer. The plants are "tricked' into reacting like it's spring and put out a few blossoms. They confused the long, dry summer for their normally dormant winter months.

Read more:
http://www.plantcell.org/content/10/12/1973.full
http://www.newyorker.com/online/blogs/culture/2012/05/early-blooming-plants.html#ixzz29OEceDSr

2. When the summer was exceptionally cold in July and August, some bulbs reached the threshold of cold needed to bloom earlier than normal.
The cool summer was followed by a very warm autumn and this triggered early flowering.

3. Big storms and hurricanes leave behind interesting phenomenon - many of the affected trees are blooming only on the windward side. This is same side that they lost most of the their leaves on during the hurricane. Because storms remove many leaves when the day length is still long and temperature is warm, flower buds are allowed to swell and open. During the normal development of these trees, the leaves keep the flower buds from developing. The loss of leaves triggers a hormone response that promotes flowering.

4. Strong drying wind and plant's diseases may also trigger out of season blooming.

Do plants really make this mistake?

September 23, 2011, United Kingdom: "An unusual series of weather events is causing widespread fall flowering of perennials and shrubs. A severe drought was followed by heavy rains and a summer cool period, followed by an unprecedented hot spell with temps in the high 80s".
This series of extreme weather changes "fooled" plants into believing that they had experienced a winter and that spring had arrived.
Read more:
http://www.guardian.co.uk/environment/2011/nov/13/warm-autumn-wildlife-oddities
http://www.dailymail.co.uk/news/article-2061590/Spring-November-Butterflies-wing-apple-trees-blossom--Has-nature-topsy-turvy.html

September 26, 2012, Kansas City: "After a brutally dry summer followed by cooler temperatures and a touch of rain, some of Kansas City's flowering plants might be under the impression that it's already spring.
Magnolia trees have bloomed over the last few weeks along with crab apple and pear trees and lilacs that were starting to blossom a good seven or eight months before they typically do".
Read more:
http://www.kansascity.com/2012/09/24/3832284/summer-drought-has-tricked-plants.html#storylink=cpy
http://www.weather.com/news/drought-tricks-plants-into-bloom-20120926

September 2011, East Coast: Re-blooming after Hurricane Irene.
A lot of Bradford pears damaged by Hurricane Irene were blooming in the end of September.

September 2008, Texas: Hurricane Ike: Some redbuds, Bradford pear trees and other Central Texas plants are blooming out of season.
Read more:
http://aggie-horticulture.tamu.edu/galveston/recovery/Plants%20Produce%20Striking%20Out-of-Season%20Blooms%20After%20Ike%20by%20WilliamJohnson.html
http://en.wikipedia.org/wiki/Plant_physiology

September 2004, Texas: Caribbean, Texas: Hurricane Ivan: Tulip trees along with red maples normally bloom in February. Hurricane Ivan has shocked several varieties of spring-blooming trees into a second bloom.