PROBLEM OF THE MONTH

OCTOBER, 2014

Dear Problem of the Month participants!
Welcome to 2014/2015 contest.

Each month you will be offered a set of 10 problems, two in each of the following subjects: Mathematics, Physics, Chemistry, Biology, and Computer Science. In each subject, there will be one advanced problem, worth 10 points, and one easier problem, worth 5 points. You may submit **any combination of at most FIVE problems** (you can submit more than one problem in the same subject, but not more than five problems in total).

Good luck!

MATHEMATICS

5 points:

Find all real solutions of the equation $\frac{x^{11}-1}{x^7-1} = \frac{x^7-1}{x^3-1}$.

10 points:

Suppose that $F(x) = x^2 + 10x + 20.$ Find all real solutions of the equation $F^{2014}(x) = 0$. Here we denoted $F^k(x)$ the function F applied to x exactly k times. For example, $F^3(x) = F(F(F(x)))$.

PHYSICS

5 points:

In order to find the distance to the epicenter of an earthquake, seismologists use the fact that different waves travel at different speed. For instance, so-called P-waves (P stands for primary) travel at speed about 5000 m/s. The Secondary, so called S-waves, move slower, about 3000 m/s. Find the distance D (in km) from the epicenter of an earthquake to the seismological station where S-waves were detected 20 seconds later than P-waves.

10 points:

The boat moves relative to the water at the speed which is 2 times smaller than the speed of the river. At what angle to the direction of the flow should the boat aim to cross the river so that it is taken down the river the least?

CHEMISTRY

5 points:

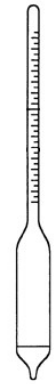


Fig. 1

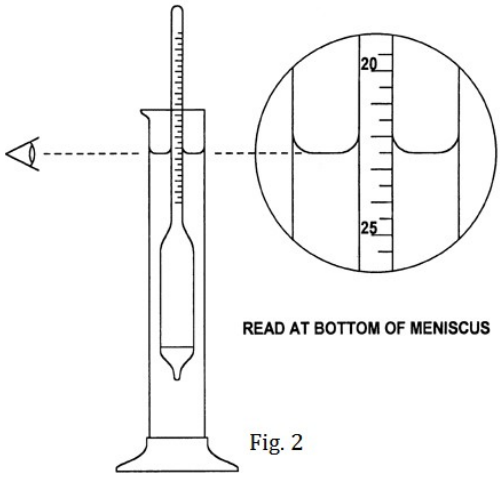


Fig. 2

Introduction I.

An areometer is an instrument used to measure the specific gravity (relative density) of liquids. An areometer (a.k.a. hydrometer) is a glass cylindrical stem and a bulb weighted with lead shot to make it float upright (Fig.1). To measure the density of a liquid, you pour it into a tall container, and gently lower the areometer into the liquid until it floats freely (Fig.2). The point at which the surface of the liquid touches the stem of the hydrometer corresponds to the liquid's density. Usually, areometers contain a scale inside the stem, so you simply read the number off the scale, and this number equals to the density of your liquid (Fig. 2).

Introduction II.

As we know, the density of water at 25ºC is 0.9971 grams per cubic centimeter (it is 1 g/cm^3 at 0ºC). The density of acetic acid is slightly higher, 1.044 g/cm^3. Interestingly, 69% aqueous acetic acid and 85% acetic acid have the same density, 1.0635 g/cm^3 (69% means that 100 grams of the acetic acid – water mixture contain 69 grams of acetic acid per 100 grams of the mixture. Chemists usually say the concentration of this solution is 69%).

You have a solution of acetic acid and the areometer. You measured the density of the solution and found it equals 1.0635 g/cm^3. Can you tell whether the concentration of this solution is 69% or 85%, if the only chemical you have is water, and the only equipment you have is the areometer and a tall glass container?

10 points:

You have 10 grams of the mixture of zinc and tin shavings, and 1 L of 5% solution of cupric sulfate (a.k.a. copper (II) sulfate). Can you determine a composition of the zinc-tin mixture, if the only materials and equipment available to you are filter paper, flasks, beakers, filtration funnel, and scales?

BIOLOGY

5 points:

Some plants have seeds in form of burs/prickles. Such seeds can stick to animals' fur, and then the animals spread seeds to new areas.
But how do these seeds fall off and get into the soil, if they are stuck in the fur? Suggest as many explanations as possible.
What are other ways that animals can spread seeds?

10 points:

In contrast to their negative reputation as disease causing agents, many viruses play very important and often overlooked biological roles (for example, in evolution and ecosystem maintenance). Overall, viruses helped shaping the world we live in today.

1. Please give at least 4 examples of viruses' beneficial roles in particular biological systems, and explain how that serves the system overall.
2. Please give at least 2 examples of viral infections in a) plants, b) animals, c) bacteria, and explain if those viruses are beneficial or harmful.
3. In what conditions, and how, can viruses be used by humans elsewhere? Please explain which virus features will be most important in your proposed application(s).

COMPUTER SCIENCE

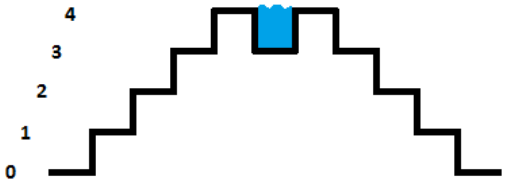
Solutions must be typed and submitted in one of following formats:
.txt .c .cpp .java .py
Solutions written in Java, C, C++, Python and pseudo-code are accepted.
Pseudo-code guidelines are at
http://users.csc.calpoly.edu/~jdalbey/SWE/pdl_std.html

5 points:

Write a program that would survey the popularity of 2 apps - SnapChat and Instagram - among 20 students:

1. Input the results (1: Instagram or 2: SnapChat).
2. Count the number of results of each type.
3. Display a summary of the results : number of students who prefer Instagram and number of students who like SnapChat more.
4. If more than 10 students prefer a particular app, print a message about a winning app.

10 points:



You have a 10-step staircase that is 10 ft from railing-to-railing with tall walls on either side of it, parallel to the railings. Each step is 1 ft wide. It is a special staircase, in that the steps don't just go up -- any step can either be one foot higher or one foot lower than the previous step. You are given the heights of all steps in the staircase as an array of non-negative integers, where the lowest step is at height 0. For instance, normal, ascending stairs would be represented by {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, descending stairs by
{9, 8, 7, 6, 5, 4, 3, 2, 1, 0}, and stairs that go down and then back up would be represented by {4, 3, 2, 1, 0, 1, 2, 3, 4, 5}.

Question: One day, it rains really hard. Given the array on the input, determine how much rain is trapped in the staircase. You may assume the input is valid (all steps are separated by one foot in height, all elements in the array are integers, array has size 10, etc.).

For example, on the input {0, 1, 2, 3, 4, 3, 4, 3, 2, 1}, the output should be 10. This is because the only step that traps water is the step between in the section 4, 3, 4 (see diagram), and the volume of that is 1 ft high by 1 ft wide by 10 ft long = 10 cubic feet.