



# SigmaCamp Qualification Quiz 2018

Dear prospective SigmaCamper,

To complete your application to SigmaCamp, you need to:

1. Fill out the registration form on our website: [sigmacamp.org/2018/apply](http://sigmacamp.org/2018/apply)
2. Submit your solutions to the Qualification Quiz, which you will find on the pages that follow. There are two problems from each of the five main disciplines at Sigma – math, physics, chemistry, biology and computer science. The second problem in each category was designed to be more challenging than the first problem. You are not expected to solve all the problems. We will evaluate your quiz submission based on your approach to the problems and the quality of reasoning. You can use the Internet, books and even help from someone else but **state precisely what sources you have used to solve each problem**. Also, please do not collaborate with other applicants. Your solutions can be handwritten or typed; if handwritten, please scan them as a PDF file. Save solutions for each subject as a separate file.

Note: if you were accepted to Sigma through PoM, you do not need to submit the QQ.

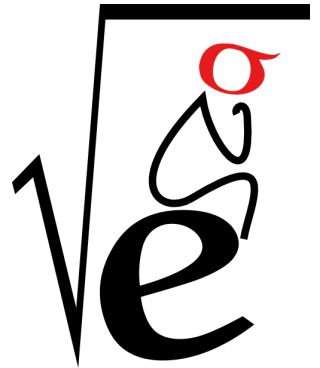
3. Write an brief essay telling us about yourself and your math/science interests.
4. Obtain two letters of recommendation: one from a Mathematics or Science Teacher and one from an adult who knows you personally. If you are a returning camper, you are not required to submit letters of recommendation – we already know who you are! Recommendation letters can be submitted online via our website or returned to you in a sealed envelope. Detailed guidelines for recommendations are on the website.

We will email you a confirmation upon receiving your application.

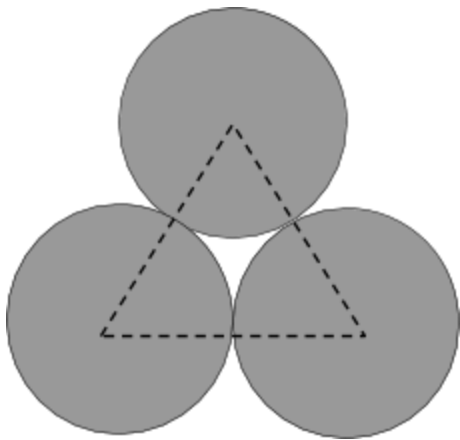
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**Good luck with your application!**

## Math



### Problem 1.



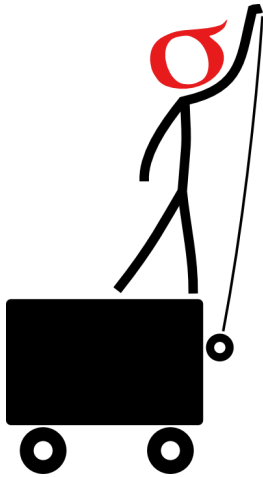
Three circles are centered at the vertices of an equilateral triangle with side 1, and are touching each other.

Find the area of the triangle not covered by these circles.

### Problem 2.

There are five students in the class, each has a favorite candy bar: Bob likes Butterfingers, Maggie likes Milky Way, Harry likes Hershey's, Charles like Crunch Bars, and Samantha likes Snickers. The teacher had one of each type of candy above, but handed the candies out randomly. Only one of the students got their favorite kind. What is the probability of such an outcome?

# Physics



## Problem 1.

During a rainstorm, an empty bucket moving up with vertical speed  $V$  gets filled in 2 minutes. The same bucket moving down with the same vertical speed  $V$  gets filled in 8 minutes. How long will it take to fill a motionless bucket?

## Problem 2.

Two blocks with masses  $M$  and  $m$  ( $M > m$ ) are attached to the ends of a relaxed massless spring and are moving with equal constant velocity,  $v$ , towards a wall.

- Find the velocity of the center of mass after the first block of mass  $M$  collides elastically with the wall.
- Find the the maximum distance  $x$  between the two blocks during the motion.

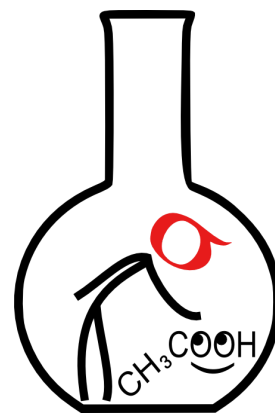
The initial length of the relaxed spring is  $l$  and the spring constant is sufficiently high (that is, it does not squeeze to zero length) and equal to  $k$ . The first block undergoes only one collision with the wall. All processes are elastic in the sense that there is no energy loss in the course of the collision, or motion.



# Chemistry

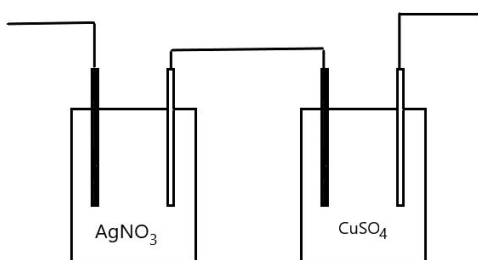
## Problem 1.

Karlsson-on-the-Roof is a character from a book by the Swedish author Astrid Lindgren. He is a very plump, short, charming and overconfident man who lives in a small house on the roof of an apartment building in Stockholm (if you never read this book, or didn't see a cartoon, we recommend it). When Karlsson pushes a button on his belly, it starts a clever little motor with a propeller on his back, allowing him to fly. Karlsson prefers to eat various jams and cakes, and this food is the only power source for his motor. Assuming that the motor's fuel efficiency is 70% (Karlsson's metabolism is similar to metabolism of other humans), and that in flight Karlsson consumes the same amount energy as a conventional helicopter would if it weighed the same as Karlsson, calculate how much jam (which contains 30% sugar by weight) he has to eat daily to be capable of flying for 2 hours each day.



## Problem 2.

Two glass beakers contain 100 mL of 1.7% solutions of silver nitrate ( $\text{AgNO}_3$ ) and copper (II) sulfate ( $\text{CuSO}_4$ ), respectively. A silver (white) and iron (black) electrodes are inserted into the left beaker, and a copper (white) and iron (black) electrodes are inserted into the right beaker as shown in the figure below.



The electrodes are connected by wires as shown in the figure. The ends of the wires are connected to a power source. The power source was turned on, and direct electric current (low voltage, around 10 V) was passing through this system for some period of time. When the power was turned off, the copper electrode was rinsed, dried and weighed. Its mass increased by 500 mg. How did the mass of the silver electrode change?

# Biology

## Problem 1.

In mesozoic ecosystems, large herbivore dinosaurs (triceratopses, titanosaurs, etc) played the role of modern rhinoceros and elephants, but they were reptiles who laid eggs. The eggs cannot be too big, otherwise they will be fragile (eggshell cannot be too thick, because it would be impermeable for air, which would make embryo's aspiration impossible).

These effects limit the maximal size of dinosaurs egg and the size of newborn dinosaurs.

It has been proposed recently that due to a relatively small size of dinosaurs eggs, the relationship between mature animals and their offsprings, and the ecosystem of mesozoic savanna as whole were significantly different from what we observe in modern savanna ecosystems. What could these differences have been?



## Problem 2.

Imagine we create a robot (named Valya 2.0) with artificial intelligence run by a computer (extrapolating from current, state-of-the-art technology). We can further imagine that this robot Valya 2.0, might require "sleep" to perform the same essential functions that the mammalian brain requires of sleep. Which of the computer's specific processes in the maintenance of optimal functioning would correspond to each stage of sleep, as well as dreaming?

# Computer Science

- You can write and compile your code here:  
<http://www.tutorialspoint.com/codingground.htm>
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easy gui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on  
<http://www.tutorialspoint.com/codingground.htm> before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc.  
**No .pdf, .doc, .docx, etc!**



The goal of the CS Qualification Quiz problems this year is to build a basic AI (Artificial Intelligence): We will teach a computer to play checkers! There are two parts to this. Problem 1: Teach the computer the rules of the game; and Problem 2: teach computer to strategize.

Rules of standard checkers can be found here:

[https://www.itsyourturn.com/t\\_helptopic2030.html](https://www.itsyourturn.com/t_helptopic2030.html). **Note:** To simplify things, we introduce significant modification to the rules:

- 1) There are no *kings*: This means that once a piece makes it to the opposite end of the board and can no longer move forward, it cannot move at all.
- 2) The game ends when a player cannot make a move on their turn. Then player with greatest number of pieces then wins.
- 3) Only one “jump” is allowed per turn: you can take at most one of opponent’s pieces per turn.

Problem setup: The state of the game is defined by an 8x8 array of integers, where 1 represents a square occupied by a white piece, 2 represents a square occupied by a black piece, and 0 represents an empty square. The initial state is:

```
board = 0 1 0 1 0 1 0 1
        1 0 1 0 1 0 1 0
        0 1 0 1 0 1 0 1
        0 0 0 0 0 0 0 0
        0 0 0 0 0 0 0 0
        2 0 2 0 2 0 2 0
        0 2 0 2 0 2 0 2
        2 0 2 0 2 0 2 0
```

Additionally, we need a boolean variable to remember whose turn it is:

```
isWhiteTurn = True
If False then it is black's turn.
```

## Problem 1.

Write a program that takes as input the state of the game (the 8x8 `board` array and boolean variable `isWhiteTurn`), as well as a coordinate on the board, and produces the following result:

- If the square at the given coordinate is empty, print “empty”.
- Else, print the possible moves that the piece at the current square can make. It is up to you what format to return it in.

**Note:** In order to make your life easier, we do not require that the input to the program be from the command line: You can just define `board` and `isWhiteTurn` at the beginning of the code, and we will modify it directly in your code when testing it.

## Problem 2.

Now that we taught the computer the rules of checkers, let's teach it to strategize. The ideal way to do this in theory is to have the computer calculate all possible game scenarios, and at every step make the move that maximizes the number of scenarios that result in it winning the game



(assuming opponent is strategizing as well). But computing the whole game of checkers is extremely difficult computationally ([it took 13 years to brute-force through checkers](#)).

The alternative is to compute not the whole game, but just several moves ahead. **Your goal is to write a program that, given a state of the board, determines the best next move by computing just two moves ahead:** Your next move, your opponent's next move, and then your next move again. Assume that your opponent is not strategic at all, and just wants to maximize the number of pieces they take that move. You, on the other hand, are a bit more strategic: **You want to maximize the number of pieces you take after two moves.**

*This problem is somewhat open-ended, and there is no one correct solution. We encourage you to submit partial solutions to this problem, and to explain your reasoning throughout your code.*

**This is the end of SigmaCamp 2018 Qualifying Quiz.**

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