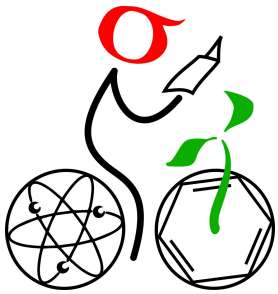


SigmaCamp Qualification Quiz 2020

Dear prospective SigmaCamper,

To complete your application to SigmaCamp, you need to:

1. Fill out the registration form on our website: http://sigmacamp.org/2020/apply. This also creates an account for you (if you didn't have one) so that you can login later to view the status of your application and check which application materials have been received.

2. Submit your solutions to the Qualification Quiz, which you will find on the pages that follow. There are two problems from each of the six main disciplines at Sigma – math, physics, chemistry, biology, computer science, and linguistics. The second problem in each category was designed to be more challenging than the first problem. **You are not expected to solve all the problems**. We will evaluate your quiz submission based on your approach to the problems and the quality of reasoning. You can use the Internet, books and even help from someone, but **state precisely what sources you have used to solve each problem**. Note that you **cannot** repost the problems to any Internet or other public forums and solicit help that way. Also, please do not collaborate with other applicants.

3. Your solutions should be submitted either as PDF, plain text (.txt) or Word document files (.doc or .docx), with the exception of Computer Science, where it should be .java or .py files (see Computer Science section below for more details).

4. Your solutions can be handwritten or typed. Handwritten solutions must be scanned as PDF (not as JPG or PNG) files.

5. Files containing solutions should be named as follows: *Subject.ProblemNumber.Your name.extension*, where Subject is Math, Physics, Biology, Chemistry, CS or Linguistics. ProblemNumber can be 1 or 2, corresponding to the problem number in the corresponding subject. For example, here are valid file names: Chemistry.1.John Smith.pdf, Biology.2.Jane.Miller.doc, CS.1.SamJohnson.java.

6. Each solution should be presented in a separate file. Please don't combine the solutions for problems 1 and 2 of the same subject into one file, and also don't combine the solutions across subjects.

7. If you were accepted to Sigma through the Problem of the Month contest (PoM), you do not need to submit the QQ. If you participated in PoM, but were not admitted among the top scorers, your effort will not be forgotten: 30% of your cumulative PoM score will be added to your QQ score.

8. Write a brief essay telling us about yourself and your math/science interests.

9. Obtain two letters of recommendation: one from a Mathematics or Science Teacher and one from an adult who knows you personally. If you are a returning camper, you are not required to submit letters of recommendation – we already know who you are! Recommendation letters can be submitted online via our website or returned to you in a sealed envelope. Detailed guidelines for recommendations are on the website.

The application deadline is April 15, 2020. We will notify applicants regarding an acceptance decision no later than **May 1st**.

Good luck with your application!

Math

Problem 1.

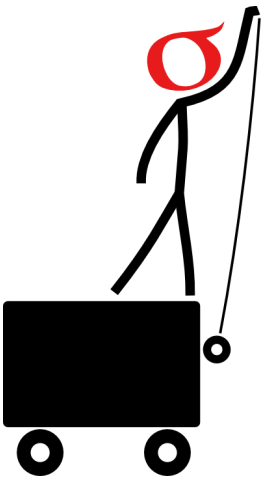
A hexagon ABCDEF has all equal sides of length a. Angles A, C, and E are right angles, while B, D, and F are obtuse. Find the area of ABCDEF.

Problem 2.

There is a secret society where every member is of either V, I, or P status. At the moment, there are 11 V members, 13 I members, and 18 P members. Whenever two members of different letter statuses meet in private, their status changes to the other letter. For example, if a V member and a P member meet, both become I members. After some number of such meetings, is it possible for every member of the society to have the same letter status?

(For full credit, you must either give an example for how this might occur, or a proof that it can't happen.)

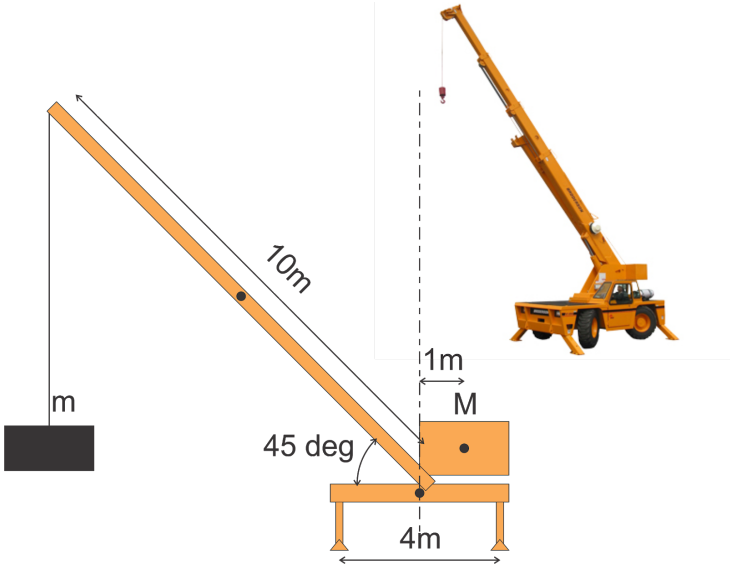
Physics



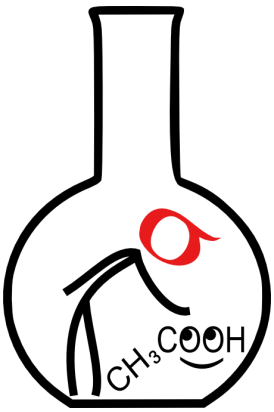
Problem 1.

A mobile construction crane has a $l = 10$ m long boom, which weighs $m_b = 1$ metric ton, with the center of mass in the middle. The boom is cantilevered by a $M = 10$ ton massive block whose center of mass is 1 m away from the center of the crane's lower platform, as shown in the picture. The weight of the lower platform is $m_p = 2$ tons and its center of mass is in the middle. In operation, supports of the lower platform are positioned symmetrically, with 4m spacing between them (see picture). With the boom at 45 degrees, what is the maximum payload, m, which this crane can lift? Assume no limitation on the crane's motor power that provides the pull of the hoist rope.

Problem 2. A construction crane has a 10 m long boom, which is positioned at 45 degrees (see picture). What is the minimum time required to lift a weight equal to 90% of the maximum crane payload, up the entire height of the boom's vertical dimension? Assume no limitation on the crane's motor power that provides the pull of the hoist rope.



Chemistry



Problem 1.

You have 3 glass beakers, water, salt, copper electrical wires, and 9 metal rods made of zinc, iron, nickel, lead, silver, tungsten, aluminium, magnesium and tin. Using these materials and equipment, what should you do to produce maximal voltage that can be measured using a standard multimeter? Breaking the rods is not allowed.

Problem 2.

A steel container was filled with a mixture of propane and air (5 : 95 by volume) at standard temperature and pressure (20 degrees Celsius and 1 atm, accordingly). The container was sealed, and the propane-air mixture was ignited using a built-in sparkling plug. Calculate the pressure in the container when the temperature comes back to 20 degrees.

Biology

Problem 1.

A 2019-20 human coronavirus, a.k.a. COVID-19, which caused the currently unfolding pandemia, has a genome composed of a single RNA strand, and in that sense it is similar to HIV. To stop HIV replication, various nucleotide analogs are used. They block an enzyme called "reverse transcriptase" that is responsible for the key step of HIV's RNA copying, and that allows virtually complete suppression of HIV propagation. That strategy capitalizes on the fact that human cells do not need to copy their own RNA, so a drug that blocks RNA copying may be harmless to a human organism. Taking into account that COVID-19 is an RNA virus too, it might be reasonable to try to use some anti-HIV nucleotide analogs against COVID-19. However, that approach is not working. Please explain why.

While solving this problem, you may enjoy this music (which is relevant to the subject): https://www.youtube.com/watch?v=DsNyWE0aSrM. It may make the problem solving process more relaxing :-)

Problem 2.

'Twas brillig, and the slithy toves
Did gyre and gimble in the wabe;
All mimsy were the borogoves,
And the mome raths outgrabe.

In his "Through the Looking-Glass", Lewis Carroll forgot to mention that, after the White Knight defeated the Red Knight and rescued Alice, he invited her to his castle. To Alice's big surprise, he appeared to be a passionate zoologist, and his castle was full of nice and spacious cages where the White Knight was keeping various animals that inhabited his country, including toves, borogoves, raths, and others.

"Look, Alice", the Knight said, "All animals in this cage are descendants of just one couple of toves: a female was black, and a male was light gray. Each litter of that couple was black, but and all subsequent generations, on average, have ¾ of black animals and only ¼ of gray ones.

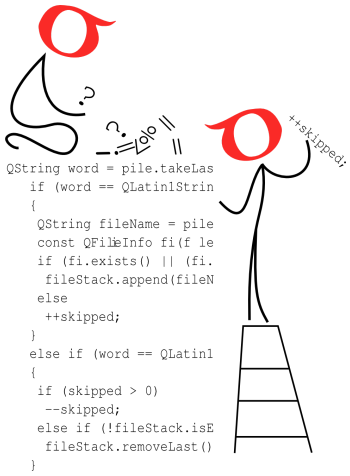
This situation reproduced every time, and I call it ‘White Knight’s law’. I was very proud of being a discoverer of that law. Then, Tweedledum caught a couple of nice borogoves, and brought them to me. They were a male and a female, both with nice, shiny, silverish fur. To my big surprise, in the first litter, only nine animals out of 12 were silverish like their parents, whereas three others were brown. The same situation reproduces in every next generation: although color distribution may be different in each litter, on average, three quarters of all animals are silverish, and one quarter is brown. I started to realize that the White Knight’s law is not as universal as I thought.”

“However, that was not the most terrible disappointment. Recently, Tweedledee gave me a couple of two raths, a female was beautiful: she was long haired, and her fur had a nice solid golden color. The black short haired male was not attractive at all. Every animal in their litter looked pretty similar: all of them were short haired, but a color was golden. However, the next generation of raths was a total mess: about 9/16 short haired golden animals, 3/16 black short haired (like their grandfather), 3/16 looked like their grandmother, and about 1/16 had a nice, long and soft fur, but … they were black.”

“Alice, I am totally confused. It looks like ‘The White Knight’s Law’ is not working at all”

Is the White Knight right, and does it mean his law is not working? Explain White Knight’s observations.

Computer Science



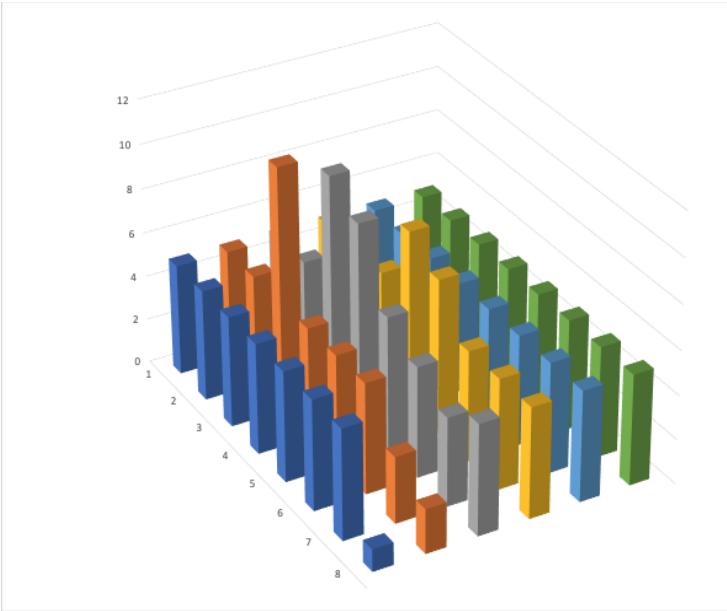
- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files with specified in the problem statement names
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py. **No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!**

You are given a map of elevations of a mountainous region, provided as a matrix of NxM positive integers. Your starting point is the bottom left corner of the map. From each area (matrix cell) you can move to the adjacent north, east, south or west area by ascending or descending by no more than 1 point.

Your program should read input file **inputQQ.txt**, which consists of N rows of M space-separated integers. Example input file:

```
5 5 5 5 5 5
5 5 5 5 5 5
5 11 10 5 5 5
5 5 9 8 5 5
5 5 6 7 5 5
5 5 5 5 5 5
5 3 4 5 5 5
1 2 5 5 5 5
```

visualized at right:



Problem 1 (5 pts)

Program needs to find a difference in elevation between the starting point and the highest peak in the region (defined as an area with the highest elevation). Results should be printed to the file named **outputQQ.txt**.

Problem 2 (10 pts)

The task is to conquer the highest peak in this region (defined as an area with the highest elevation).
The program needs to find a path to the highest peak and print it into the output file **outputQQ.txt**, step by step, coordinates of each area per line. Coordinates of the bottom left corner are 0, 0, with the first coordinate being the column number, and the second coordinate - row number. If such a path does not exist, print NO PATH to the file.

Example output corresponding to the input file above:

```
0, 0
0, 1
1, 1
2, 1
2, 2
2, 3
3, 3
3, 4
2, 4
2, 5
1, 5
```

Linguistics

Problem 1.

Consider the following sentences in some language spoken in Siberia:

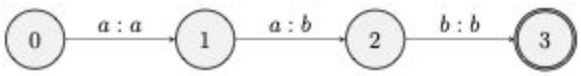
1. Atyrkar surureten	Old women left.
2. Atyrkan ŋinakinin suruden	The dog of the old woman will leave.
3. Nginakin kalarva eden ičere	The dog will not see boilers
4. Oronmi nginakinmatyn eren iktere	My reindeer didn't hit their dog
5. Akin oronin kalanmatyn ičeren	A reindeer of the brother saw the boiler of sisters.

Problem: Translate into this language:

a. Brothers will not leave
b. My sister saw dogs
c. His brother will see the boiler

Problem 2.

The following problem relates to finite state transducers (FST) - a computational devices or "machines" that are used a lot in computational linguistics. An example is shown below:



The machine has a finite set of states (hence the name "finite state") designated here with circles with numbers in them. There is a specified initial state; we will always note that state with the number "0". And there are one or more final states, which are conventionally notated with a double instead of single circle. And there may be states that are neither initial nor final. So in the machine above, there are four states, one initial state (0) and one final state (3). Between the

states there are arcs, and these arcs are labeled with pairs of symbols. The symbol on the left of the ":" on the arc is the input symbol, and the one on the right is the output symbol.

The machine is called a transducer because it transduces strings of symbols — e.g. a word — into other strings of symbols. How that works is as follows. Let us say you have a string *aab*, and imagine we have a pointer that points to where we are currently in that string; initially it will point to the beginning letter. You start in the initial state of the machine, and you ask: given the letter where the pointer is pointing, is there any arc that matches that first letter on its input label? In this case the answer is "yes": there is an arc labeled *"a:a"* that leaves that state, and goes to state (1). So I do three things:

- Output — from the arc's output label — the symbol *"a"*
- Move the machine from state 0 to state 1
- Move the pointer to the second letter of the string (so it now points at the second *a*).

We then continue the process from state (1) and the second position of the string. In this case there is also an arc, labeled *"a"* on the input and *"b"* on the output, and our little input pointer is pointing at an *"a"*, so we:

- Output — from the arc's output label — the symbol *"b"*
- Move the machine from state 1 to state 2
- Move our little pointer to the third letter of the string *"b"*

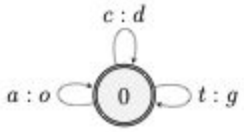
Now we are looking at the *"b"* and ask the same questions and by this point you should be able to see that we can:

- Output — from the arc's output label — the symbol "b"
- Move the machine from state 2 to state 3
- Move my little pointer to the next position — which is the end of the string.

Now, notice that we have reached the end of the string and are in state (3), which is a final state. If, and only if, both conditions hold — we've used up our input and we are in a final state — then the machine has successfully read the input and successfully output a string. In this case it read *aab* and output *abb*.

If we had given it a string such as *abb* the machine above would fail to match the input, and hence would fail to give any output.

Here is another example: the following two machines will change *"cat"* into *"dog"*, but the second one will also change the nonsense word "*tac"* into "*god"* (the first one will not accept *"tac"* at all).



The Problem: the following algorithm was often used in the past to abbreviate people's names. Its main advantage is that if the name is slightly misspelled the representation of it will still be correct.

1. Keep the first letter of the name, and drop all occurrences of non-initial *a, e, h, i, o, u, w, y*
2. Replace the remaining letters with the following numbers:
   b, f, p, v → 1
   c, g, j, k, q, s, x, z → 2
   d, t → 3
   l → 4
   m, n → 5
   r → 6
3. Replace any sequences of identical numbers , only if they derive from two or more letters that were adjacent in the original name, with a single number (i.e., 555 → 5).
4. Convert to the form *Letter Digit Digit Digit* by dropping digits past the third (if necessary) or padding with trailing zeros (if necessary).

Write an FST to implement this algorithm. For the purposes of this problem, assume that every last name contains the symbol # at the end (i.e. Smith#, Jones#, etc.)

The application deadline is April 15, 2020. We will notify applicants regarding acceptance decisions no later than **May 1st**.

Good luck with your application!