

# Sigma

## Qualification Quiz 2014

Dear prospective SigmaCamper,

To complete your application to SigmaCamp, you need to:

- 1) Fill out the registration form on our website: [sigmacamp.org/2014/apply](http://sigmacamp.org/2014/apply)
- 2) Submit your solutions to the Qualifying Quiz, which you will find on the pages that follow. There are two problems from each of the five main disciplines at Sigma – math, physics, chemistry, biology and computer science. The second problem in each category was designed to be more challenging than the first problem. You are not expected to solve all the problems. If you do not have a complete solution but have some ideas, send them too - we want to see your original thinking and approach, not just the final answer. You can use the internet, books and even help from someone else but **state precisely what sources you have used to solve each problem**. Also, please do not collaborate with other applicants. Your solutions can be handwritten or typed. Note: if you were accepted to Sigma through PoM, you do not need to submit the QQ.
- 3) Obtain two letters of recommendation: one from a Mathematics or Science Teacher and one from an adult who knows you personally. If you are a returning camper, you are not required to submit letters of recommendation – we already know who you are! Recommendation letters can be submitted online via our website or returned to you in a sealed envelope. Detailed guidelines for recommenders are on the website.

We will email you a confirmation upon receiving your application.

**The application deadline is April 20, 2014.**

We will notify applicants regarding acceptance decision no later than May 15.

## Math

**Problem 1.** Two trolls are dividing a stolen treasure, which consists of 9 gold pieces of varying sizes. A wizard tells them that they can easily split the treasure so that each troll gets half of the total weight of gold; all they have to do is to cut one of the pieces in two, so that each troll gets 5 pieces. Is the wizard right? That is, is it always possible to split a treasure in half by cutting only one piece and giving each troll 5 pieces? If not, give a counterexample; if so, explain how it should be done (your method should work for any combination of weights of the 9 pieces)

**Problem 2.** A square board is divided into  $n^2$  rectangles (not necessarily all squares!) by drawing on the board  $n-1$  vertical lines and  $n-1$  horizontal lines. These rectangles are painted black and white in the usual checkerboard pattern. It is known that the black rectangles forming one of the diagonals are all squares. Prove that the total area of the black cells is greater than or equal to the total area of white cells.

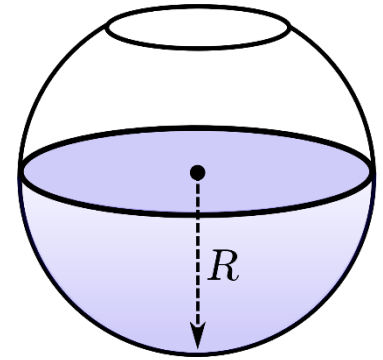
## Chemistry

**Problem 1.** In a high school workshop, students were preparing for a robotics contest, and generated a huge amount of waste: wood sawdust, small snips of copper wire, as well as polystyrene, iron, and aluminum shavings. They collected all the waste in one large container. Please, suggest a procedure to extract all the copper from this mixture. Can you give a procedure for separation of as many components as possible from this mixture? You may use any equipment and any chemicals you want.

**Problem 2.** In Chapter XV *The Mysterious Island* by Jules Verne, the colonists of Lincoln Island are engaged in production of iron from the iron ore. Please, re-read the description of what they did, and describe the chemistry behind the technology they used. The abridged English translation (<http://www.gutenberg.org/files/1268/1268-h/1268-h.htm#link2HCH0015>) contains some inaccuracies in the description of the chemistry of this process. Please identify these inaccuracies, and propose your own wording.

## Physics

**Problem 1.** An aquarium in the shape of a spherical shell of radius  $R$  is initially half-filled (see figure). Water evaporates from the surface at a constant rate, so that per unit time, the volume  $q$  of water evaporates from a unit area of the surface. How long will it take for all the water to evaporate from the aquarium?



**Problem 2.** A brick is sliding with initial velocity  $v$  up an icy 45-degree slope. After some time, it stops, and slides back down to its starting location. Find the friction coefficient between the brick and the slope if it took the brick 2 times longer to slide down than to slide up.

## Computer Science

**Problem 1.** Write a Java program that prompts the user for his/her height  $H$  in inches and outputs the ideal weight  $W$  in pounds. Use the following formula:

$$W = (H \times 2.5 - 100) \times 2.2$$

**Problem 2.** Write a Java program that given a string, returns the maximum number of repeating characters. For example:

`maxRepeats("Hello")` → 2

`maxRepeats("wallless")` → 3

`maxRepeats("ppffff33ZzZzZw")` → 4

`maxRepeats("")` → 0

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## Biology

**Problem 1.** Legends of wonderful creatures- dragons- have been around for centuries. They breathed out fire, battled with knights, ravaged towns, and guarded treasures. “Fairytale are a lie”, but the sheer amount of these legends could not have come from nothing. Here are drawings of the three most widespread depictions of dragons:



- a) The dragon, the way it is usually depicted in fights with Saint George
- b) Zmey Gorynych, from Russian fairytales
- c) A Chinese dragon

Propose which real-life animals could have served as a prototype for each of these drawings. Why do you think they could not exist in real life? Support your logic.

**Problem 2.** Human cells in the body live anywhere from a few days to a few years. Every minute, millions of our cells are dying. Unplanned cell death, called necrosis, can result from cell aging, a sudden traumatic injury, infection, or exposure to a toxic chemical. Imagine that scientists came up with a unique medicine completely halting cell death.

- Which physiological or biochemical changes in the organisms can be the result of this medicine?
- What are the potential side effects?

In the end of the last century, scientists discovered that cells are able to “commit suicide”, a process also known as programmed cell death or apoptosis.

Apoptosis occurs during the normal development of multicellular organisms and continues throughout adult life.

- What is the role of apoptosis and why is it important? Please, provide example of apoptosis in different organisms (e.g. animals, plants, fungi).