SigmaCamp Qualification Quiz 2017

Dear prospective SigmaCamper,

To complete your application to SigmaCamp, you need to:

1. Fill out **the registration form** on our website: sigmacamp.org/2017/apply
2. Submit your **solutions to the Qualification Quiz**, which you will find on the pages that follow. There are two problems from each of the five main disciplines at Sigma – math, physics, chemistry, biology and computer science. The second problem in each category was designed to be more challenging than the first problem. You are not expected to solve all the problems. We will evaluate your quiz submission based on your approach to the problems and the quality of reasoning. You can use the Internet, books and even help from someone else but **state precisely what sources you have used to solve each problem**. Also, please do not collaborate with other applicants. Your solutions can be handwritten and scanned, or typed. Note: if you were accepted to Sigma through PoM, you do not need to submit the QQ.
3. Submit an essay describing why you are interested in SigmaCamp, as well as tell us a bit about your science/math interests.
4. Obtain **two letters of recommendation**: one from a Mathematics or Science teacher and one from an adult who knows you personally. If you are a returning camper, you are not required to submit letters of recommendation – we already know who you are! Recommendation letters can be submitted online via our website or returned to you in a sealed envelope. Detailed guidelines for recommendations are on the website.

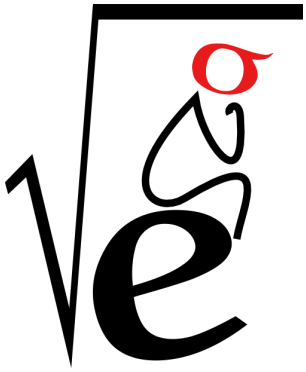
You can see the complete application instructions at: sigmacamp.org/2017/apply

We will email you a confirmation upon receiving your application.

The application deadline is April 18, 2017.

Math

Problem 1.

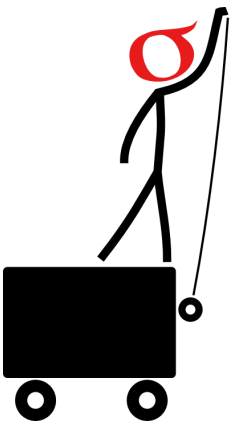


In a math club each boy is friends with exactly two girls and each girl is friends with exactly three boys. It is known that in a classroom where the club meets there are exactly 19 desks (each desk fits not more than 2 people) and that 31 of the math club members are high school seniors. How many students are in the math club?

Problem 2.

There are 13 elephants aligned in a row on a circus arena. The weight of each elephant (in kilograms) is an integer number. It is known that if one adds the weight of any elephant (except for the heaviest elephant standing at the very right) to half of the weight of its neighbor to the right one obtains 6000 kilograms. What is the weight of the lightest of all elephants?

Physics



Problem 1.

During his 3-mile ride, a bicyclist is turning pedals at a constant rate. However, due to a changing terrain, he had to change the gear ratio (i.e. the ratio of number of teeth of the front chainring to those of the rear sprocket). He rode the 1st mile in exactly 3 minutes, by setting the gear ratio to 3/1. He rode the 2nd and the 3rd miles by setting the gear ratio to 2.5/1 and 5/1, respectively. Find the total time of his 3-mile ride.

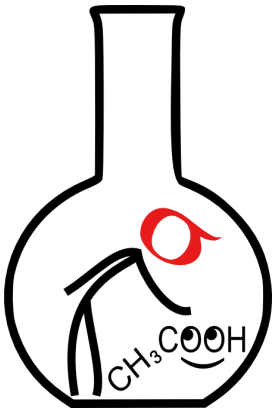
Problem 2.

To ride a bike at constant speed of v=5 m/s on a flat road, a girl has to generate P=100 Watt of power.

a) How much power does she have to generate to ride the bike at the same speed uphill on a 5% slope (which means that the road rises by 50 meters per every kilometer of the horizontal displacement)? The combined mass of the girl with the bike is 60 kg.
b) The girl now bikes downhill at the same speed, without any braking or pedaling at all. Find the slope of the road (i. e. how much the road descends per every km of the ride).

Chemistry

Problem 1.



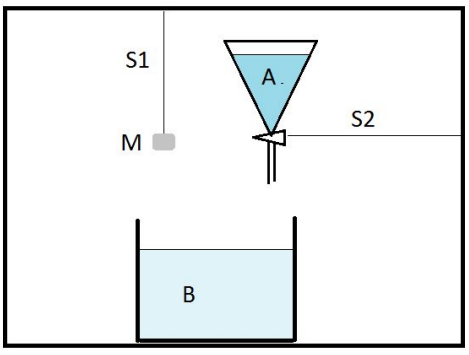
During the last several years, astronomy has made enormous progress. The discovery of exoplanets (the planets orbiting stars other than our Sun) is arguably one of the most impressive examples of that. Now we know that many stars have their own planets, and these planets' sizes, compositions, climates, and landscapes are very diverse. It is quite possible that in a reasonably close future automatic spaceships will be sent to some of these planets.

Imagine you are living in the 22nd century, and you are an engineer designing a robot for exploration of an exoplanet, Hades. This planet, which is orbiting one of the stars in the Orion nebula, has not been discovered yet, but it can be discovered in the next century. This planet has a silicate crust (like our Earth), but its atmosphere is composed of nitrogen, water vapors and hydrogen chloride. The average temperature on this planet is about 100° C, the pressure is 50 bar. In addition, volcanic activity is extremely high on this planet, so there are a lot of lakes there filled with hot liquid sulfur. The weather is windy on Hades, and, due to constant and fast erosion of Hadean rocks, the atmosphere contains a considerable amount of fine quartz particles.

Propose a material for the robot's body that will survive in these severe conditions for at least 2 years. Explain your answer.

Problem 2.

Consider the following experiment, which takes place in a sealed, thermally insulated box. The box contains a thermometer to monitor temperature change. Inside the box, a one gram piece of calcium metal (**M**) is hanging over a beaker (**B**) on the string (**S1**). The beaker **B** contains 100 mL of water. A *dropping funnel* (**A**) is filled with 20 mL of 10% hydrochloric acid solution. A string (**S2**) is attached to the



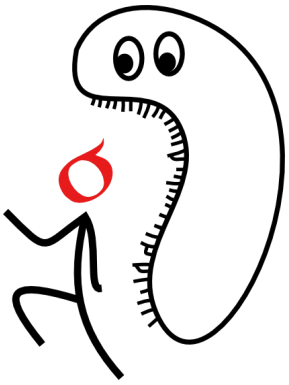
funnel's stopcock, so that pulling the string **S2** opens the stopcock, releasing the acid into the beaker.

A researcher releases the string **S1**, letting the calcium drop into the beaker. After 10 minutes, they pull the string **S2**, letting the acid pour into the beaker. When the system comes to an equilibrium (about 10 minutes), the researcher records the final temperature.

What if, instead, the researcher makes the same experiment, but the order of operations is reverse: **S2** is pulled first, and **S1** is released second? In which case would the final temperature be higher? Explain your answer.

Biology

Problem 1.



Mr. Oak was appointed a director of the Big Hole National Park. Since the park has a long history of having a unique population of ungulates, the new director treated them very tenderly. In response to even minor changes in an overall number of animals Mr. Oak took a swift action: when their population increased, superfluous ungulates were being immediately caught and released outside the park, whereas after the population decreased, the lack was compensated by buying new ungulates and releasing them at the Big Hole. When Mr. Oak was fired from his job as a result of this activity and got an advice to learn some biology at SigmaCamp, he was very indignant.

Explain why such a practice lead to adverse consequences for the park's animal population.

Problem 2.

One of the most common techniques in modern biotechnology is called "recombinant protein production". In the course of this procedure, a researcher forces some cells (usually *Escherichia coli*) to produce a specific protein of interest that these cells don't normally make on their own. This is achieved by introducing specifically engineered DNA (a plasmid) into the cell. The cell treats this DNA as its own and is able to read genetic information from it. As a result, the desired protein is produced, or, as biotechnologists say, "expressed". At the end of the procedure, the cells are mechanically crashed and the protein of interest is released and then separated from the mixture of broken cells. This technology is widely used for both research and industrial protein production. For example, insulin, which is a vital medication for millions of people suffering from diabetes, is currently produced using this method.

At the SigmaCamp semilab "From Gene to Protein in 5 Days", we express a Green Fluorescent Protein (GFP) in *E. coli* and purify it using a His-tag purification technique. This technique relies on the addition of a six-histidine peptide segment to the end of the GFP gene. Expression of this gene leads to the production of a modified GFP bearing 6 histidine residues (a "His-tag") at one terminus. The tag does not affect the properties of GFP, but it allows the resulting protein to specifically bind to a modified surface of a highly porous material, thereby dramatically facilitating its purification.

Imagine, this year we decided to try another tag, the Strep-tag. This tag is an eight-amino acid segment (Trp-Ser-His-Pro-Gln-Phe-Glu-Lys) that interacts with a molecule called Strep-Tactin with very high specificity. This new tag makes GFP purification even easier, and allows us to

obtain a very pure GFP suitable for various interesting experiments. We have eight GFP plasmids, each having a different DNA sequence.

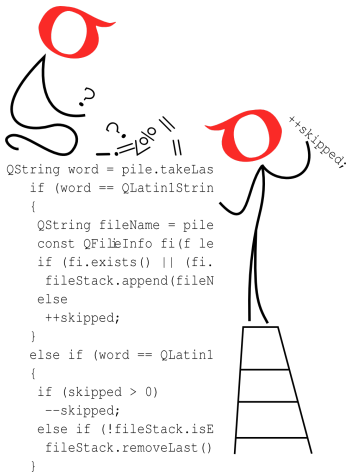
Shown below are the short segments of these plasmids' sequences that are supposed to encode for the Strep-tag. Please look at these sequences and, assuming that the reading frame starts at the first nucleotide, tell:

1) which sequences correspond to the fully functional Strep-tag,
2) which plasmids will be capable of expressing a protein of interest,
3) which plasmids will produce the protein suitable for affinity tag purification (not necessarily via the Strep-tag approach).

Sequence 1: TGGTCTCATCCTCAATTTGAAAAG - GFP_protein_sequence
Sequence 2: TGATCTCATCCTCAATTTGAAAAG - GFP_protein_sequence
Sequence 3: TGGTCCCATCCTCAATTTGAAAAG - GFP_protein_sequence
Sequence 4: TGGTCTCTCCTCAATTTGAAAAAG - GFP_protein_sequence
Sequence 5: TGGTCTCATCCTCAATTTGAAGAG - GFP_protein_sequence
Sequence 6: CATCATCACCATCACCACCATCAC - GFP_protein_sequence
Sequence 7: TGGTCGCATCCACAATTTGAAAAA - GFP_protein_sequence
Sequence 8: TGGTCTCATCCTCAATTTGAAAAGTGA - GFP_protein_sequence

Computer Science

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easy gui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc. **No .pdf, .doc, .docx, etc!**



Problem 1.

The problem: You are given 4 digits. If you can make from them (by changing their order) the last 4 digits of your credit card number, you win a prize, otherwise, you lose. Write a program that determines if you win or lose.

Input: 4 digits and also a 4-digit credit card number.

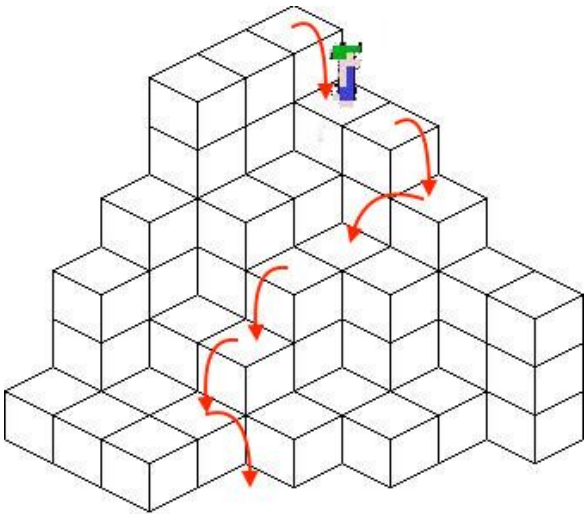
Output: WIN or LOSE

Example:

Input: you are given 0, 4, 6, 8, and your credit card number is 6440. Output: LOSE

Input: you are given 0, 4, 6, 8, and your credit card number is 6480. Output: WIN

Problem 2.



The problem: A lonely Lemming is stuck on top of a stack of boxes. He can walk on flat surfaces, hop off boxes, but cannot climb upwards. Furthermore, he cannot withstand a fall greater than the height of one box, or he will hurt himself. Can you write a program that determines if there is a path for him to get down to the ground level?

Technical details: The information about the boxes is in a 6x6 array of numbers, where each number signifies the height at that coordinate. The Lemming starts at coordinate (0,0). Every Lemming's step can be in one of the 8 directions (north, east, west, south, northwest, southwest, northeast, southeast). Every step can be associated with a change of elevation 0 or -1. It can not be greater than 0 (Lemming can't climb), and cannot be less than -1 (Lemming will hurt himself). The Lemming has found the way out if there is a path to a square with number 0 -- ground level.

Input: Your program should take as input the 6x6 array of box heights (it is acceptable to hard-code it in). Make sure to check that there are no negative heights, and that there is at least one 0, corresponding to the ground level.

Output: Your program should output YES (Lemming found way out) or NO (Lemming is stuck), with an optional list of path coordinates the Lemming took if YES.

[Note: We encourage you to submit partial solutions, but they must be accompanied with an explanation -- how far you got, what challenges were you not able to overcome, etc. If you are able to solve the problem fully, you don't need to include an explanation of your code -- if the code runs, it will speak for itself.]

The application deadline is April 18, 2017.

We will notify applicants regarding acceptance decision no later than **May 1st**.

Good luck with your application!