PROBLEM OF THE MONTH

December, 2018

2018 - 2019

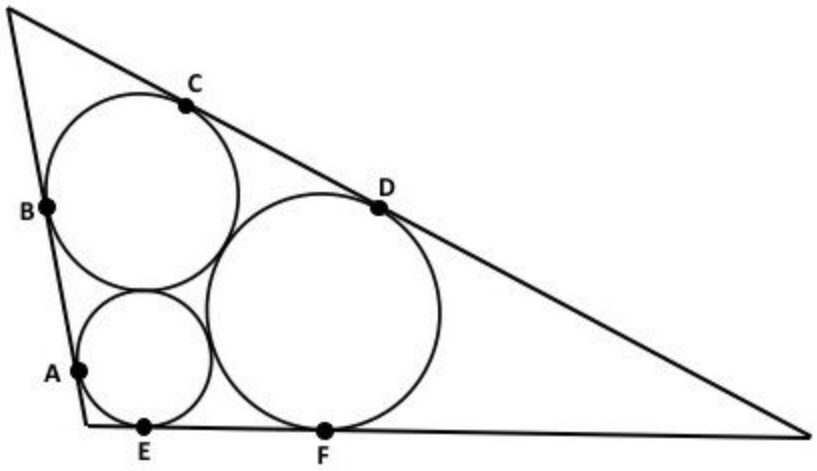
MATHEMATICS

5 points:

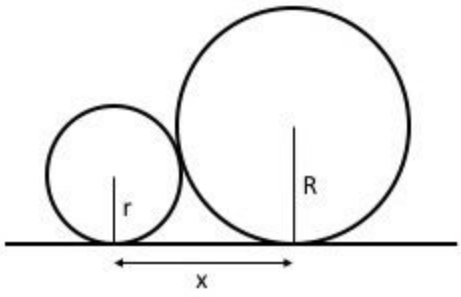
Three circles of radii R_1, R_2, and R_3 are pairwise externally tangent to each other, as in the diagram.
It is known that one can form a right triangle from the segments AB, CD, and EF.

Find R_1 given that $R_2 = 3$ and $R_3 = 4$.

(Note that there may be more than one correct answer.)



Hint:

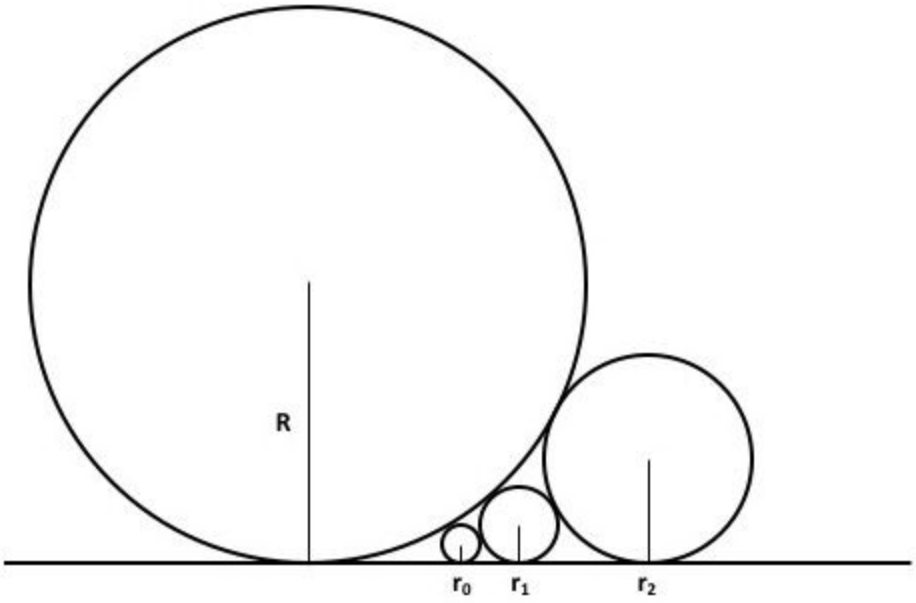


Find x in terms of R and r, and then apply it to the problem to write the pythagorean relation between AB, CD, and EF.

10 points:

A large circle with radius R is tangent to a line. Smaller circles with radii r_0, r_1, r_2, etc. are constructed such that each is externally tangent to the large circle, the line, and its neighboring smaller circles, as in the figure.
It is known that $r_0 = R / 2018$.

How many such smaller circles can be constructed?
What is the radius of the last one of these circles?



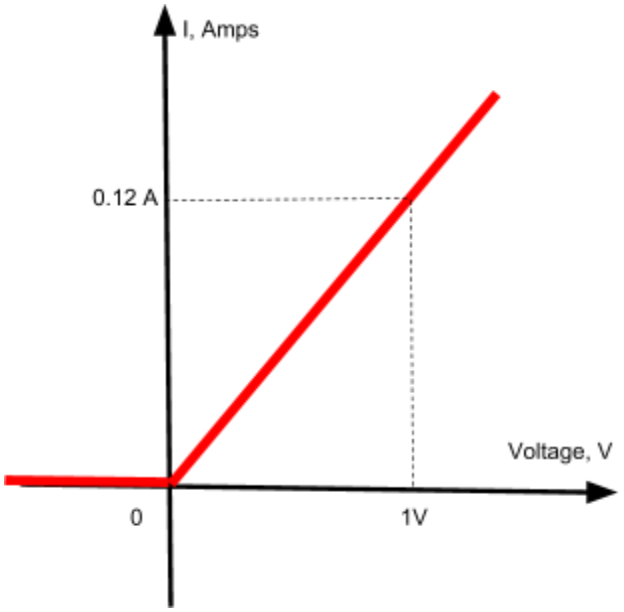
Hint:

Find the relation between r_k, r_{k+1}, and R.

PHYSICS

5 points:

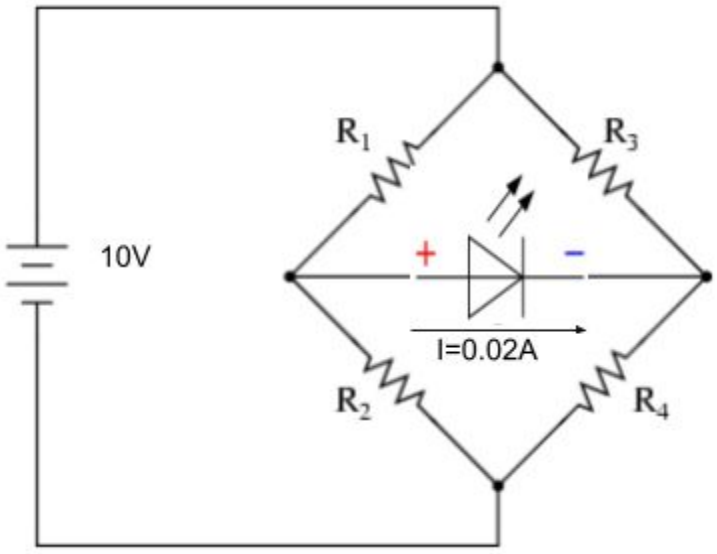
A Christmas tree is lighted by 55 light-emitting diodes (LEDs) connected in series and plugged directly into a to regular, 110V electric outlet. The outlet supplies an alternating current (AC): half of the time it flows in one direction and half of the time - in the opposite. The direction of the current changes back and forth 50 times per second. LEDs, however, conduct the current only in one direction, and shut down when the opposite voltage is applied. The figure shows schematically how the current depends on voltage for each LED. What is the overall power consumed by all the LEDs?



Hint: You can find AC voltage on each LED, and the current that corresponds to it. Note LEDs are off 50% of the time.

10 points:

A light-emitting diode (LED) is plugged into the circuit shown in the figure. The ideal battery has voltage 10V, and three of the resistors all have the same resistances: $R_1=R_2=R_3=100$ Ohm. For optimal operation, the voltage drop across this particular LED should be 2V, while electric current through it should be I=20mA=0.02A. Find the value of resistance R_4 needed to achieve this optimal regime.



Hint: Assume that the voltage drop across LED and the current flowing through it are optimal: 2V and 0.02A respectively. Let current through resistor R_1 be I_1. From here, find currents and voltages across other resistors.

CHEMISTRY

5 points:

"*We had two bags of Arabica coffee, seventy-five ounces of sodium hydroxide pellets, five kilograms of high purity acetic acid, a saltshaker half-full of zinc chloride, and a whole galaxy of multi-colored pH papers, rubber balloons, strings etc... Also, a quart of concentrated hydrochloric acid, a quart of acetone, a case of Poland Spring water, a pint of raw ether, and two dozen grams of some amyl alcohol. Not that we needed all that for our trip, but once you get locked into a serious chemicals collection*, *the tendency is to push it as far as you can. I even was not sure which amyl alcohol did we have: iso-amyl, sec-amyl, tert-amyl (a friend of mine, Lucas, explained me that he'd just invented a simple test to discriminate them), and I was absolutely confident we would be quite capable of figuring that out. The only thing that really worried me was the ether. There is nothing in the world more helpless and irresponsible and depraved than a man in the depths of an ether binge, and I knew we'd get into that rotten stuff pretty soon.*"

Am I right, and will I (using the chemicals from our list and with Lucas help) be capable of figuring out which of three alcohols do we have? If yes, describe the procedure.

Hint:

Actually, the hint is already there. We really can do that with Lucas help.

10 points:

Alice, a college faculty, came to her lab and found Bob, her technician, smiling meditatively. "*I know that look*", Alice said. "Were you binge-watching YouTube videos again?" *Yes!*", Bob said. "*People throw one-pound pieces of sodium into ponds, and it is amazing! We should do it right now. I found enough sodium in the lab; let's do it together – it is going to be awesome!*" "*Wait*", Alice said, "*Do you want to throw a pound of sodium into a pond in front of our building?*" "*Yes! YES!!*" "*But what about all the fish? Freshwater fish do not tolerate water when its pH is above 9. If throwing one pound of sodium into the pond will increase the pH of water up to 9 or higher, I would say you should not do that*", Alice said. "*I recall the average depth of the pond is 2 meters and its diameter is 40 meters. Can you please calculate the pH of water in the pond after throwing 1 pound of sodium there, and depending on the results, we will decide if we can do this experiment. In you calculations, assume the pH is exactly 7 now, and the water is fresh and free of other dissolved substances.* "

Make needed calculations and tell if Bob will be allowed to do this experiment.

Hint:

Just calculate the concentration of OH ions assuming that each sodium atom produces one OH^- upon reaction with water. A negative logarithm of concentration of OH^- is pOH, and pH+pOH always equal to 14.

BIOLOGY

5 points:

Based on what observations scientists might have assumed that the extinct species of animals was warm-blooded?

10 points:

In the late 1980th, Carolyn Napoli has studied flower coloration in petunias. She knew that the intensity of the purple color was determined by the chalcone synthase (CHS), a key enzyme in the biosynthesis of the purple pigment, anthocyanin. Carolyn hypothesized that in order to produce darker purple flowers, she would need to engineer plants with extra copies of the CHS gene. The rationale was straightforward: the more copies of the gene, the more mRNA is produced, so the level of CHS will go up, and the color of the flowers will be deeper. However, the result was utterly unexpected - instead of the dark purple, Dr. Napoli ended up with white flowers, and, after several years of extensive studies, a biologist discovered a totally new protein machinery that is present in almost all eukaryotic organisms. What this machinery is, how does it explain the white color of genetically modified petunias, and what is the primary function of this machinery in eukaryotes?

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc. **No .pdf, .doc, .docx, etc!**

Intro:

SigmaCamp always has a lot of returning campers and with that a lot of friends from previous years. Your program will receive on input a list of pairs of friends. For example:
Alice-Bob
Bob-Chris
Dan-Frank

5 points:

Given the list of friendships, your program needs to identify the disconnected friendship clusters. In the example above, Cluster 1: Alice-Bob-Chris, and Cluster 2: Dan-Frank. Clusters should be printed on output.

Hint:

Using *sets* makes the task much easier. Alternatively, you can keep a list of friends for each camper, and in the end combine the lists into clusters.

10 points:

Given the list of friendships, your program needs to identify a minimum set of new friendships that need to be established so that everyone in Sigma is at most 2 degrees of friendship separation from everyone else (that is everyone is either a friend or a friend of a friend or a friend of a friend of a friend). In the example above, either Bob-Dan or Bob-Frank are correct answers.
Friendships that need to be established should be printed on output. If no new friendships are required, your program should state so.

Hint:

A natural way to represent relationship between campers is via a graph. We could think of each camper as a graph vertex, and friendship between two campers as an edge between corresponding vertices.

Moreover, one of the most convenient ways to represent edges in a graph is via a so called adjacency matrix, which is a square matrix where 1 in a cell (i, j) denotes a presence of an edge between vertices i and j, while 0 represents an absence of such an edge. Now you can turn the task into a matrix manipulation one.

IMPORTANT NOTE: we are not asking your program to prove that the resulting set of friendships is an absolute minimum. Instead your program should attempt to minimize the number of additional connections. As we feel that the original problem statement was not clear on this, we will allow submissions of the solutions up to January 10 without deadline penalties.