PROBLEM OF THE MONTH

December, 2019

2019 - 2020

MATHEMATICS

5 points:

With a pen, Alex drew a rectangle on the lines in his quad ruled notebook. Using a pencil, he successfully divided the rectangle into 3x1 rectangles. Once again with a pen, he colored in the middle square of each 3x1 rectangle. He then erased all the penciled lines. Is it always possible to recover the initial division into 3x1 rectangles?

Hint: Let's assume it's not possible to recover the initial division into 3x1 rectangles. This means that there must be at least one colored square that represents a horizontal 3x1 rectangle in one possible division and a vertical rectangle in another. Let's take the highest such colored square. What does this mean for the (non-colored) square directly above it?

10 points:

In triangle ABC angle A is 60 degrees. Let BB_1 and CC_1 be two angle bisectors of this triangle.

a) (2 points) Prove that the quadrilateral AB_1IC_1, where I is the intersection of the angle bisectors, can be inscribed in a circle.

b) (8 points) Show that the point symmetric to A relative to the line B_1C_1 lies on the side BC.

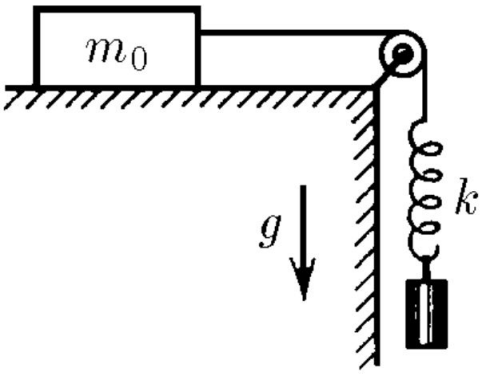
Hint: Google theorems about inscribed angles in a circle.

PHYSICS

5 points: The scales are based on a spring with spring constant *k=10000N/m* and are calibrated to show the weight in kilograms. Imagine now that you drop a rigid block of mass $M = 1kg$ from height $h = 1$ meters, right on those scales. What will be their maximum reading (in kg)?

Hint: Use conservation of energy to find the maximum spring deformation.

10 points: The system consisting of two blocks, rope, spring, and pulley is shown in the Figure. The friction coefficient between the block of mass m_0 and the horizontal surface of the table is μ. The block attached to the spring is released while the spring is unstretched. What should be the minimum mass M of that block so that the block on the surface is displaced from its original position? Assume that the rope, the spring, and the pulley are weightless, there is no friction in the pulley and the rope is unstretchable.



Hint: See hint to 5 pt. Problem

CHEMISTRY

5 points:

Brass is an alloy of copper and zinc. Zinc content varies in different types of brass. To measure a composition of some brass sample, 10 grams of brass shavings were placed into a narrow neck flask, 100 mL of 20% HCl was added to it, and a rubber balloon was attached to the neck. When the chemical reaction has ceased, the volume of the balloon was measured. 2.24 L of gas formed in this process. Using this information, calculate the composition of the brass sample.

Hint:

Keep in mind that zinc reacts with HCl to form a hydrogen gas and a water soluble salt (zinc chloride), whereas copper does not.

10 points:

Each year in SigmaCamp, Mark is having the same problem: how to organize the experimental part of the tournament, keeping in mind that (i) the tournament's participants are not supposed to operate with hazardous materials, and (ii) the majority of interesting chemical experiments use some dangerous reactants. Imagine that during one SigmaCamp, Mark managed to collect a set of pretty harmless chemicals that included:

1. Sodium hydrosulfate;
2. Potassium bicarbonate;
3. Magnesium shavings;
4. Calcium chloride;
5. Copper wire;
6. Alkaline batteries;
7. Sodium carbonate;
8. Quicklime (calcium oxide);
9. Aluminium foil;

   and some glassware (test tubes, flasks, beakers, etc), scales, other simple chemistry lab equipment, as well as various stuff, such as ropes, balloons, scotch tape etc..

Propose an experimental problem for the tournament that uses only chemicals from the above list.

Hint:

No single correct answer exist. At least three different experiments can be proposed. Also, keep in mind that some compound from this list is a moderately strong acid, whereas another one is a source of a relatively strong base.

BIOLOGY

5 points:

In living cells, DNA is being copied by an enzyme called DNA polymerase. This enzyme adds, step by step, nucleotides to a growing chain of DNA according to well-known rules: A is added opposite T, G opposite C, etc. Once a nucleotide has been added, the DNA polymerase molecule shifts forward, and the next nucleotide is added. As a result, DNA polymerase is crawling along the template DNA strand with an approximate speed of 1000 nucleotides per second. How long a division of a single procaryotic and/or mammalian cell would take if only one DNA polymerase molecule were present in the cell?

Hint:

Your answer will be somewhat counter-intuitive, but, nevertheless, it might be correct, because DNA copying in a living cell never occurs in the way described in this problem. In a real living cell, a large number of DNA polymerase molecules are involved in DNA replication, and they are doing that concurrently.

10 points:

The Nobel Prize in Physics for 2018 was awarded to Arthur Ashkin for his Optical Trap (OT).

Why did this Physics discovery became most important for the development of Molecular Biology?

What is it about the biological molecules that we can see with Optical Trap that we would not be able to see otherwise?

The Optical Trap is able to apply stretching forces to the individual single folded biological molecules (proteins or nucleic acids, i.e. DNA and RNA), and also is able to simultaneously measure the extension between two attachment points of the molecule. Using the conventional laws of mechanics, what can you learn about particular biological molecule? Give at least one example.

Hint 1: The conventional biochemical methods observe the average behavior of the huge number of molecules. However, biological molecules often have alternative conformations that perform specific functions. This is especially true of the molecular machines: molecules that walk, carry cargo, polymerize or unwind double stranded DNA, etc. What can the Optical Tweezers (version of Optical Trap used in molecular biology) do to tell us more about each molecule? Why is this instrument called a Tweezers? Please, describe and explain one good example of the biological molecule that people learned a lot about using Optical Tweezers.

Hint 2: The free energy of the particular folded structure of the biological molecule can be estimated as a product of the measured transition force and extension for a particular force-induced transition. The examples may include unfolding of RNA or protein structures, force-induced melting of the double stranded DNA, unwrapping the DNA from nucleosome, etc.

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc. **No .pdf, .doc, .docx, etc!**

5 points:

For a given finite sequence of N integer numbers a_1, a_2, … a_N, let's define a reverse-weighted sum (RWS) as

RWS = $\sum_{i=1}^{N}(N+1-i)*a_i$

Write a program that receives on input:

- a list of integers a_1, a_2, … a_N

- a target integer T

The program needs to find an integer x that will when inserted at some position in the list will result in RWS of the list being equal to T. Print the list with x inserted at the desired location, or, if such an operation is impossible, indicate so.

Example:

Initial list is 1, 2, 3 and T=15. Output: x=1, updated list: 1, 2, 1, 3.

Hint:

If you have RWS for numbers a_1, a_2, … a_N, think how would it change if you insert b in k-th position.

10 points:

Alice and Bob are playing a collaborative video game. They are given a target number **T** (integer). Each of them can press the UP or DOWN arrow, and upon each press their current Collaborative Number goes up (or down) by **a** for Alice's presses and **b** for Bob's. The game starts with the Collaborative Number being 0. Write a program to help Alice and Bob reach their target number T. Your program should take integers a, b and T on input, and it should print the number of up or down presses for each of Alice and Bob to win the game. If the target can't ever be reached, it should print so.

Example:
a=2 b=3 T=10
Output: Alice presses UP 2 times, Bob presses UP 2 times.

Hint:

Try to express the task in mathematical terms. Then look up algorithms for solving it.

LINGUISTICS

5 points:

Once upon a time a large and friendly family from one Northern-European country gathered for a holiday dinner. Almost everyone from the family could make it, save for three people. In total, 16 people gathered at the table:

1) The head of the household Algirdas;
2) his wife Irma;
3) Irma's brother, Jonas;
4) Irma's sister, Iolanta;
5) Algirdas's sister, Lada;
6) Lada's husband, Giedrius;
7) Lada's and Giedrius's son, Juozas;
8) Lada's and Giedrius's daughter, Anna;
9) elder daughter of Algirdas and Irma, Grazina;
10) middle daughter of Algirdas and Irma, Rasa;
11) husband of the yourngest daughter of Algirdas and Irma, Rimas;
12) Rimas's brother, Edgaras
13) Rimas's sister, Elena;
14) Elena's husband, Aidas;
15) Rasa's daughter, Maria;
16) Rimas's daughter, Elzbieta.

The last names of the participants of the dinner are given below in a random order:
Jurenas, Šeštokas, Balsiene, Matulite, Balsyte, Matulis, Šeštokas, Ambraziene, Adomaytite, Jurenaite, Matulis, Adomaitis, Jureniene, Matuliene, Ambrazas, Šeštokaite

Question 1: Determine the last names of the following people and explain your reasoning:

a. *Algirdas*
b. *Jonas*
c. *Elena*
d. *Elzbieta*

Question 2: Determine the last names of the three people who did not participate in a holiday dinner and explain your reasoning

a. *Rasa's husband*
b. *Rimas's wife*
c. *Elena's and Aidas's daughter.*

10 points:

Roman numerals are written in the following way:

1. 1, 2, 3, 4, 5, 6, 7, 8, 9 are I, II, III, IV, V, VI, VII, VIII, IX
2. 10, 20, 30, 40, 50, 60, 70, 80, 90 are X, XX, XXX, XL, L, LX, LXX, LXXX, XC
3. 100, 200, 300, 400, 500, 600, 700, 800, 900 are C, CC, CCC, CD, D, DC, DCC, DCCC, CM
4. 1000 is M

A number containing several decimal places is represented, as in the Arabic system, by writing its power-of-ten parts – thousands, hundreds, tens and units – in sequence, from left to right, in descending order of value. For example:

39 = 30 + 9 = XXX + IX = XXXIX.
246 = 200 + 40 + 6 = CC + XL + VI = CCXLVI.
789 = 700 + 80 + 9 = DCC + LXXX + IX = DCCLXXXIX.
2,421 = 2000 + 400 + 20 + 1 = MM + CD + XX + I = MMCDXXI.

Any missing place (represented by a zero in the Arabic equivalent) is omitted, as in Latin (and English) speech:

160 = 100 + 60 = C + LX = CLX
207 = 200 + 7 = CC + VII = CCVII
1,009 = 1,000 + 9 = M + IX = MIX

Question: provide a formal algorithm that allows one to add two Roman numerals (from 1 to 2,000) without converting them to decimal system. You can use any operations (substitution, order change, rewriting, etc), and anyone should be able to execute them, without knowing anything about Roman numerals.