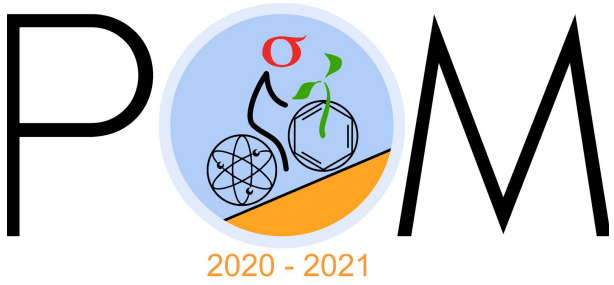
PROBLEM OF THE MONTH



December, 2020

2020 - 2021

MATHEMATICS

5 points: A circle is inscribed into a circular sector that has a central angle of 60 degrees. Find the ratio of the area of the inscribed circle to the area of the sector.

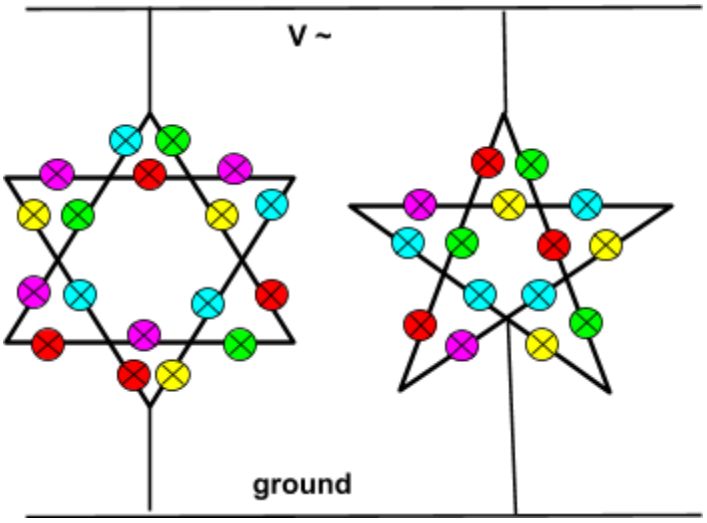
Hint: Use the theorem of the side of a right angle triangle opposite to the angle of 30 degrees.

10 points: In triangle ABC angle A is 120 degrees, point D lies on the bisector of angle A and AD = AB + AC. Find the angles of the triangle DBC.

Hint: Construct equilateral triangles ABB' and ACC' with points B' and C' lying on AD. Consider the triangles BAC, CDC', and DBB'.

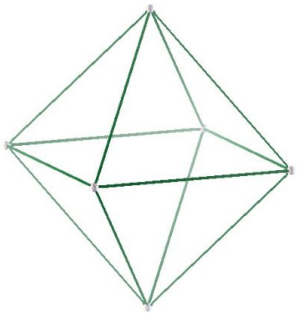
PHYSICS

5 points: Two neighbors have built their holiday decorations: one shaped like a 5-point star and the other like a 6-point one. Each edge contains one light bulb (see the figure), all bulbs have the same constant resistance. Which of the two will consume more power when plugged into the outlet with the same voltage? Find the ratio of their respective powers. Each line represents a wire with zero resistance.



Hint: For a given voltage V, power $P = V^2/R$, where R is the total resistance of the circuit.

10 points: An octahedron (see figure) is made out of wire. Each edge has resistance R, and each vertex is a perfect electric contact between 4 edges. Find the resistance of this circuit between two vertices that share the same edge.



Hint: Consider a plane of symmetry half way between the two edges, A and B. Each point where it intersects the octahedron, has the same voltage (which is the average of the voltages in A and B). If we now connect all those points together, this will not change the resistance of the circuit (since no current would flow between the points at equal voltages).

CHEMISTRY

This month, the topic is: **Redox reactions**

IMPORTANT! In this PoM season, we do an experiment: each month, an online lecture will be given. This lecture may be helpful for those who want to solve Chemistry PoMs, although it is not supposed to provide direct hints.

This month, the lecture will be on Dec 20 morning. At 11:00, a Zoom conference will start where October PoM solutions will be discussed. After that, approximately at 11:30, the lecture starts.

To join the Zoom conference, use this link:

https://us02web.zoom.us/j/4817690592?pwd=T2djSjRETEpDSHFZdWJpYlBTYzdjQT09

Meeting ID: 481 769 0592

Passcode: 879615

The recording is available here: https://youtu.be/RzE1UPZs_Rg

5 points:

Having lost, by the will of the elements, your backpack with food and matches on a hike to remote places, you remembered that an abandoned hut was indicated on the map nearby. Fortunately, it was easy to find. There were logs in the fireplace and cereals and salt in the cupboard, but there was no indication of any matches or lighters. During a search, an old first-aid kit was found, which contained some pills, jars of ointments, potassium permanganate, glycerol, bandages and cotton wool. In the shed, you found some rusty piece of iron, a bag of bleach and a bottle of battery acid. "Not bad, that may be a solution" - you concluded. In a few minutes you were already enjoying the warmth of the fire in the stove, on which a kettle was gurgling. How have you managed to kindle a fire?

Hint:

A fire may result if a strong solid oxidant comes into contact with reducing agents.

10 points:

The teacher decided to answer the question of an inquisitive student about the origin of the name of one of the elements with a small demonstration. He took a bright yellow liquid and added a colorless one to it. The resulting mixture immediately turned red-orange. Then the teacher dipped a roll of aluminum foil into the resulting solution. Gas bubbles began to rise from the metal surface, and the color of the solution began to change. This happened gradually, the color changed to greenish, and over time became sky blue. The student exclaimed: "Oh, now I understand it!"

What element was the student asking about? What chemical transformations took place during these color transitions? Write down their chemical equations.

Hint:

Some transition metals have different colors in different oxidation states.

BIOLOGY

5 points:

In one of the Harry Potter books there was a creature called basilisk (a fire-spitting animal with a dragon body, rooster head, and tail of a snake). According to the Care of Magical Creatures class textbook baby-basilisk comes out of an egg, which is produced by a rooster once in 100 years and hatched by a toad. There are no other ways for reproduction.

Envision a biologically-correct life cycle that fits this description and explains why rooster and toad stages are necessary for basilisk reproduction.

10 points:

There is a specific condition called "Compound V deficiency" in humans. People suffering from this condition were treated in one of the hospitals. The first cohort of patients with this deficiency was given Compound V-rich food. Some of the patients got better, but the conditions of others did not change. Doctors decided to give them Compound V as intravenous injections. This resulted again in the full recovery of some patients, but not all of them.

Could you suggest as many reasons as you can to explain this? Also, how can the rest of the uncured patients be treated?

LINGUISTICS

5 points:

The following sequence of sounds represents a phrase in Japanese language without spaces: *kakikuebakaneganarunari*. The part of the Japanese dictionary with all words that could be present in this phrase is given below.

aka	i	na
akane	iku	nari
aki	ikue	naru
an		naruna
ana	ka	narunari
ane	kaki	ne
ari	kan	
	kane	u
baka	kanega	un
bakan	ki	unari
bakane	kiku	ueba
	kikue	
gan	ku	e
ganar	kue	eba
ganaru	kueba	
ganaruna		ri

Determine how many ways are there to break this phrase into words using this dictionary and **justify your answer**.

10 points:

The following sentences are in a Caucasian language that uses a concept called **evidentiality**, where a statement is parsed differently (its grammar changes) depending on how much information is available to the speaker of that statement. Have a look at the several examples below. (ʕ and ɤ are two consonants with no direct analogues in English, while ə is a short vowel like the ‘a’ in *about* or ‘e’ in *water*.)

Muhamad Aslan dʕaydziɤit’	Muhamad thinks that Aslan came.
Aslan Fatima dchwadziɤun	Aslan thought that Fatima fell asleep.
Fatima Zarema aqəshw ljwadzəlɤit’	Fatima thinks that Zarema washed the window.
Aslan chay yzhwədzəlɤit’	She thinks that Aslan drank the tea.

Muhamad chay lzhwədziɕun Muhamad thought that she drank the tea.

Translate the following sentences into English and **explain your solution**. The last sentence (c) has two translations (try to find both of them):

a. Aslan Muhamad aqəshw yjwadziɕun

b. Muhamad dchwadzəlɕit’

c. Zarema dʕaydzəlɕit’

Also, translate these statements into the Caucasian language and **explain your solution**:

d. Aslan thought that Zarema drank the tea.

e. She thought that he fell asleep.

COMPUTER SCIENCE

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files with specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.
  No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!

Introduction:

This month the problem will deal with word shuffles. We will define a word shuffle as a combination of two words where the letters of the shuffle come from the original words in such a way that the relative order of the letters coming from the same word is maintained, however each letter of the shuffle can be drawn from either of the words. For example, a shuffle of TOURNAMENT and DINNER could be TDINOURNANMENTER. The shuffle would essentially weave two original words together utilizing all the letters.

5 points:

Write a program that given two original words and a shuffle would determine whether the shuffle is “legitimate” (i.e. produced correctly). The input of your program is a file **input.txt** , which contains 3 lines: two original words and the shuffle. The output of your program is a file **output.txt**, which should contain either CORRECT, if the shuffle is produced correctly from two original words, or INCORRECT otherwise.

10 points:

Write a program that given a shuffle would determine which two words from the list of two English words were used to produce it. The list of allowed words is supplied via words.txt file (you can download this file here: https://raw.githubusercontent.com/eneko/data-repository/master/data/words.txt).

Input is provided in **input.txt** file which contains a single word - the shuffle.

Output should be written to **output.txt** file and should contain two original words in alphabetical order, or, if the provided shuffle cannot be produced from any combination of the provided words, write IMPOSSIBLE. If there are multiple pairs of words that produce the input shuffle, then write any one of the pairs to the output.txt file.