PROBLEM OF THE MONTH

POM

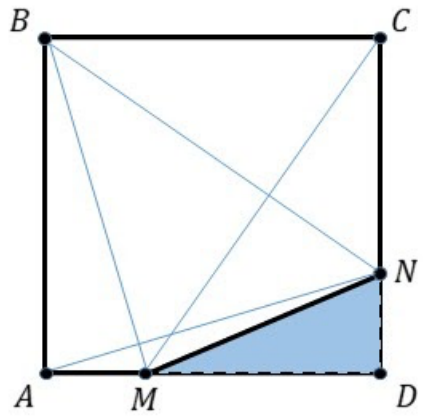
2021 - 2022

December, 2021

MATHEMATICS

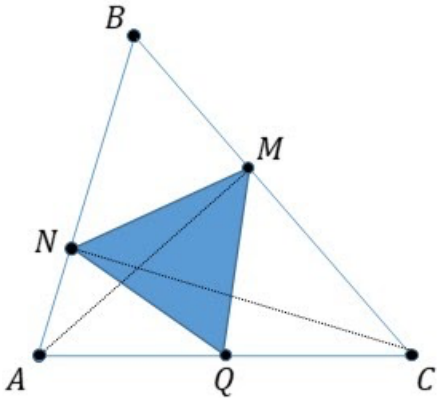
5 points:

Take a square ABCD and cut off the right triangle MDN so that |MD|+|ND| is equal to the side of the square. Show that the sum of angles MAN, MBN, and MCN is 90 degrees.



10 points:

In the acute triangle ABC, the angle B is 60 degrees. Point Q is the middle of the side AC, whereas AM and CN are altitudes of the triangle ABC. Show that triangle MNQ is equilateral. (Need a picture).



PHYSICS

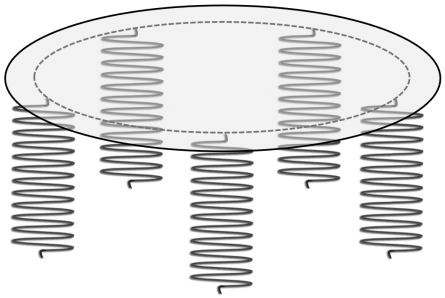
5 points:

A massless plank rests on two springs standing distance *L* apart. The two springs have equal natural lengths but different spring constants, k_1 and k_2. Where on the plank should one place a weight so that the plank is horizontal?

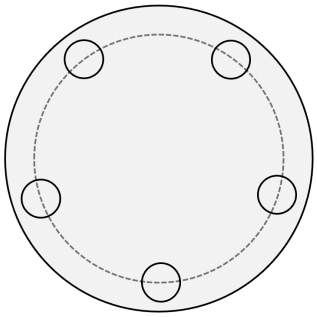
10 points:

N springs are arranged evenly in a circle, with a platform placed over the springs (see below for diagram). All of the springs have the same natural length, but they have different spring constants. Is there always a place on the platform where someone can place a single weight so that all of the springs are compressed to the same length? If so, show how one can find this place. If not, prove why not.

Diagram for $N = 5$



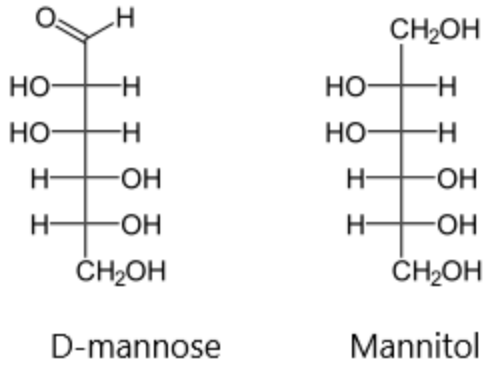
Top view:



CHEMISTRY

5 points:

A sugar *mannose*, as well as all other sugars, has an aldehyde group, which can be reduced to an alcohol, as shown at the figure below. The product is called “mannitol” with a formula $C_6H_{14}O_6$.. The same reaction can be performed with other sugars.
It is easy to conclude that eight six-carbon D-sugars can exist. Their names are “allose, altrose, glucose, mannose, galactose, talose, gulose, and idose. How many different compounds with the formula $C_6H_{14}O_6$ can be prepared from them by reducing their aldehyde group?



D-mannose Mannitol

10 points:

Legend has it that in the mid XIX century, one young French scientist, who would later become famous for many discoveries in biology and medicine was observing crystals of a salt of some organic acid. To his surprise, he noticed that each crystal had a small facet on one of its edges, but in some crystals it was oriented to the right, whereas in some other crystals it was oriented to the left. He could separate them using a tweezer. The crystals of these two types had exactly the same chemical composition and the same chemical and physical properties (such as solubility, melting point, hardness). Later, chemists established that the acid that formed this salt is a dicarbonic acid (i.e. it had two carboxylic groups), the molecule is composed of four carbon atoms, and its shape is linear. Besides carboxyl groups, this acid contains two hydroxyl groups (similar to ethanol), so its empirical formula is $C_4H_6O_6$. Draw Fischer formulas of these two acids.

Besides these two isomeric acids, chemists found one more isomer with the same empirical formula. This compound was a linear molecule too, but, in contrast to the first two isomers, its physical properties, such as a melting point or solubility were *different* from the properties of other two isomers. Draw the Fischer formula of this third isomer.

If you don’t know what the Fischer formula is, the Wikipedia article contains correct information about it (we checked it, and that article has no errors).

BIOLOGY

5 points:

Usually, Darwinean evolutionary theory is summarized as “survival of the fittest”. However, some traits, for example, a beautiful tail of birds-of-paradise, are not easy to explain from that point of view. Clearly, a male with a long and fluffy tail is more vulnerable and more likely to be caught by predators, which by no means is an evolutionary advantage. The same is true for its offspring.

Is this phenomenon consistent with Darwinian theory? Explain your answer.

10 points:

One of the reasons why religious people rejected Darwin was their interpretation of his theory as “survival of the fittest”, which they interpreted as justification of an utter selfishness and immortality. Furthermore, a Prussian philosopher Immanuil Kant concluded that the emergence of human morality and altruism cannot be explained by natural causes, and, therefore, that human altruism may serve as the major proof of the existence of God.

Is it true that modern science cannot adequately explain the origin of altruism and morality using the Darwinian paradigm, and, therefore, altruism is not a biological trait of the species *Homo sapiens*?

Explain your answer.

LINGUISTICS

5 points:

Linguistics is not only limited to spoken or written language - there are many forms of communication, such as sign language, that can also be treated as full-scale language forms. In some parts of the world, long-distance communication can be accomplished by whistling, such as Silbo Gomero in the Canary Islands.

Silbo Gomero roughly divides native Spanish pronunciation as follows (with some extra sounds)

	Low vowels			High vowels		
(lowest)	u	o	a	::	e	i (highest)

Low consonants											High consonants										
stops:	k	p	f	glides:	g	b	v	m	h	j	::	stops:	t	s	tʃ	ts	glides:	n	(ŋ)	l	r	d	dʒ	z

(Note: ‘j’ represents the sound ‘y’ in yes; tʃ is English ‘ch’ and dʒ is ‘j’ in ‘jam’; ŋ is English ‘ng’)

Now, imagine that this whistling system was used in English. Below are some audio samples where English words are converted into Silbo Gomero. Match each sample with one of the words listed below, and describe what helped you match the sound samples with the English words. Since English vowels are different from Spanish vowels, the vowel pronunciation was up to the whistler’s discretion.

Words: **1. camper** **2. chicken** **3. goodbye** **4. hello** **5. language**
6. music **7. never** **8. party** **9. physics** **10. sigma**

Link to sound files: **https://tinyurl.com/pomling21dec**

A: ___ B: ___ C: ___ D: ___ E: ___ F: ___ G: ___ H: ___ I: ___ J: ___

10 points:

Etymology refers to the history of a particular word and how its pronunciation or meaning changed as it transitioned from language to language. Studying etymology can serve as a crucial comparative tool to determine similarities between languages, classify them, and determine their age in relation to one another. However, it can also indicate whether a language borrows a **cognate** from another language and incorporates that word within its own vocabulary.

The diagram below represents the etymology of a word which was able, surprisingly, to appear in most languages commonly used today through cognate borrowing. Each tree node contains a rough phonetic transcription of the word, along with its language of origin.

Diagram A:

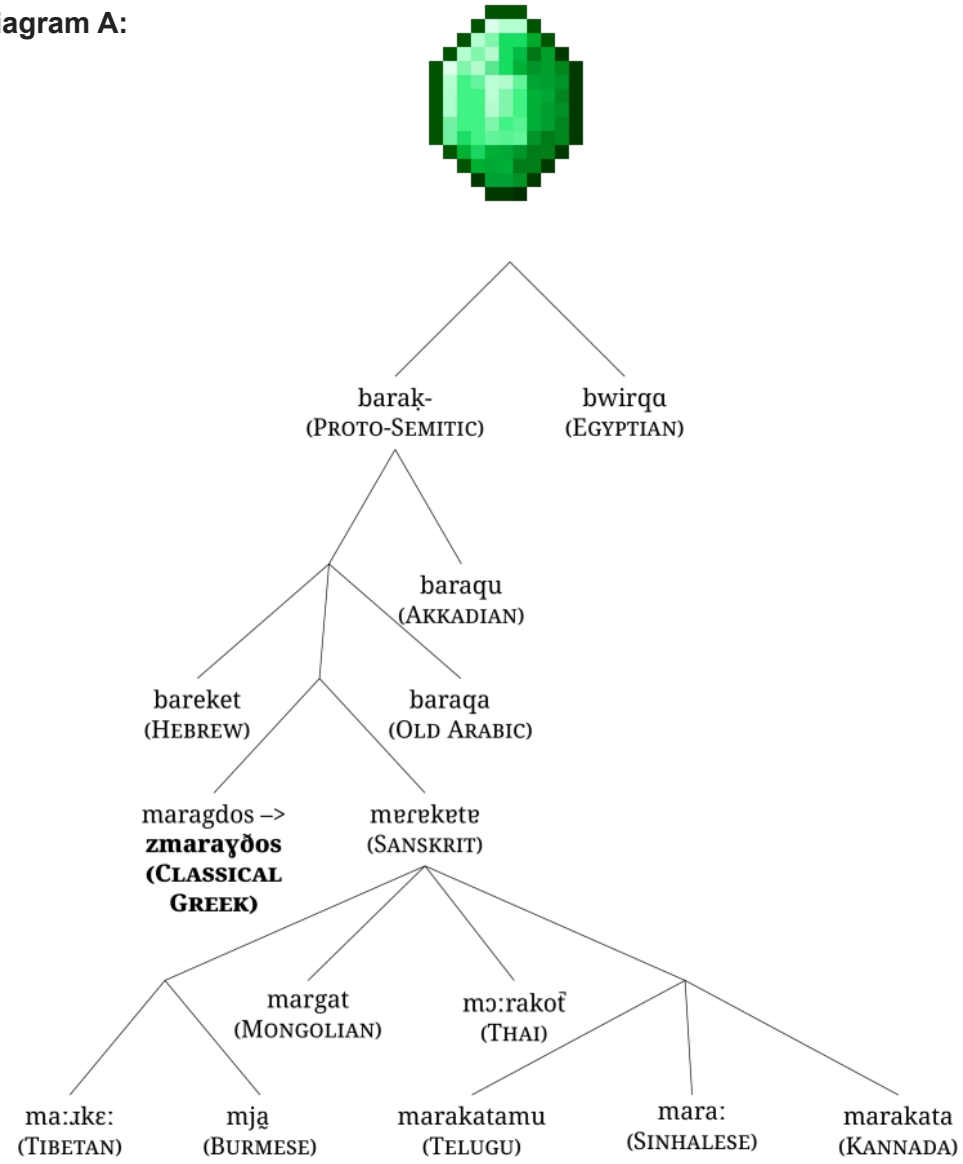

ɓaraḳ-
(Proto-Semitic)
ɓwirqɑ
(Egyptian)
baraqu
(Akkadian)
bareket
(Hebrew)
baraqa
(Old Arabic)
maragdos –>
zmaraɣðos
(Classical Greek)
mɐrɐkɐtɐ
(Sanskrit)
margat
(Mongolian)
mɔ:rakot̃
(Thai)
ma:ɹkɛ:
(Tibetan)
mjąˍ
(Burmese)
marakatamu
(Telugu)
mara:
(Sinhalese)
marakata
(Kannada)

Diagram B:

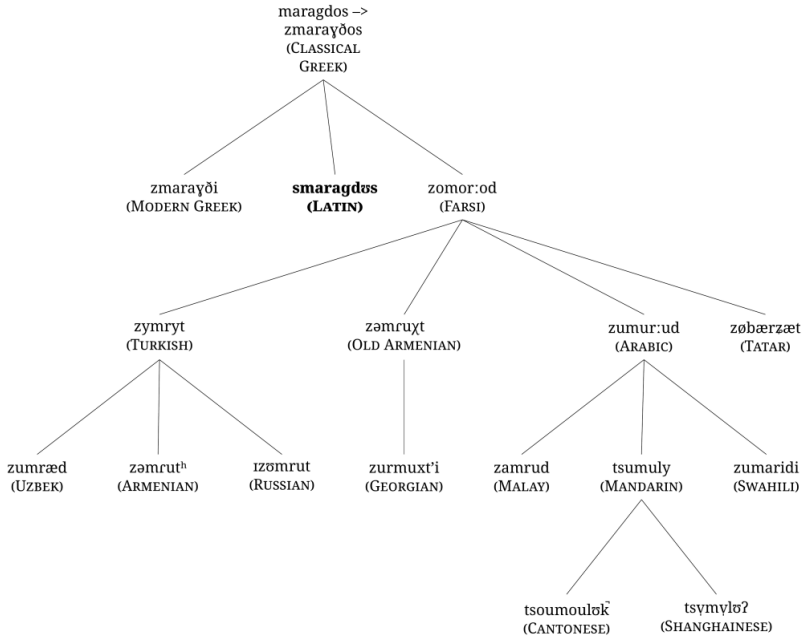
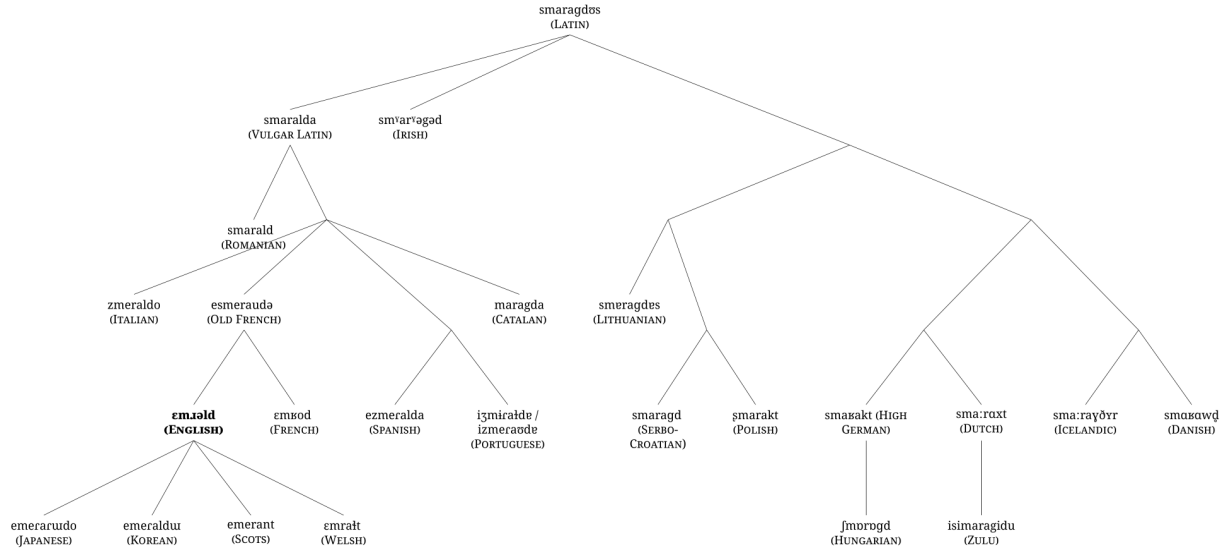


Diagram C:



Part A: Choose **5 different words** (at least one from each section A, B, C of the diagram) and discuss how the words changed phonetically from their predecessors. You must (1) present and (2) justify at least one reason for why each word you choose evolved the way it did. *Feel free to research the languages whose words you choose to explain.* Example:

The Lithuanian word ‘smɛragdɛs’ preserves its ‘ɛs’ syllable unlike other languages that loaned the Latin word for ‘emerald’ because all of its male-gender nouns use ‘ɛs’ endings.

Part B: As you may have noticed, while many languages loaned their terms for ‘emerald’ from closely related languages, some loans occurred between representatives of different language families. **Explain how these sorts of word exchanges take place.**

COMPUTER SCIENCE

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easygui in Python
- All the input and output should be via files named as specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py
- **No .pdf, .doc, .docx, etc! Programs submitted in incorrect format will not receive any points!**

To Make a COW out of a FLY

You may have played this game: you are given two words of the same length. The task is to transform the first word into the second by changing one letter at a time so that at every step a valid word is formed. For example:

FLY - FRY - CRY - COY - COW

Input: a pair of space-separated words in **input.txt** file. In the example above, the file would contain:

FLY COW

Output: for both problems below the output should be written to **output.txt** file.

Dictionary: a set of valid words can be loaded from **words.txt** file**,** one word per line. The dictionary can be downloaded from *https://raw.githubusercontent.com/dwyl/english-words/master/words_alpha.txt* (copy-paste the contents into words.txt file for your testing. We will be using the same file for our verification).

If the transformation is impossible write IMPOSSIBLE to the output file.

5 points:

Given two words from the input file, your program should determine if the first one can be transformed into the second one with exactly one intermediate step. For example, for a word pair BIRD BARN the intermediate word would be BARD.

The intermediate word should be written to the **output.txt** file. If transformation is impossible write IMPOSSIBLE to the output file.

10 points:

Given two words from the input file, your program should determine if the first one can be transformed into the second one, and, if possible, write a chain of transformations into the output.txt file, one word per line (only include the intermediate words, not the words given in the input). If there are alternative chains, any of them is acceptable.

For example, for the input:

FLY COW

the output should contain:

FRY

CRY

COY

If the transformation is impossible write IMPOSSIBLE to the output file.