Problem of the Month

2022-2023

December, 2022

# MATHEMATICS

**5 points:**

You are invited on a game show in which a host displays an infinite list of prizes to you, numbered consecutively starting from 1. You can assign each prize the color red, or the color blue. Then, the host will make a list of all of the distances between every pair of prizes of the same color, and any prize whose number is in the list of distances is colored red, and any prize whose number is not in the list of distances is colored blue. The host repeats this procedure until the colors of the prizes don't change, and you win all of the prizes colored blue. Is it possible for you to win a prize? If yes, provide an example of how you could win one, and if not, provide a proof of why it is impossible.

**Hint: Try some examples, what patterns do you observe?**

**10 points:**

You are then invited on a different game show, where you are displayed a finite list of positive real numbers, representing amounts of money in dollars. The host gives you two options: the Regular option or the Iterative option. Choosing the Regular option means you win the geometric mean of the values in dollars. However, if you choose the Iterative option, the list of values will be shuffled and you will win the iterative geometric mean* of the values in the list. Which option should you choose in order to win the most money on average?

*An iterative geometric mean of a list of numbers is defined as taking the geometric mean of the first two numbers, then taking the geometric mean of that result with the next number, and so on. For example, the iterative geometric mean of {1,2,3} would be $((1*2)^{½}3)^{½} \approx 2.06$.

Information about the geometric mean can be found here:
https://en.wikipedia.org/wiki/Geometric_mean

**Hint:** Section 1.3 of that wikipedia article might help you…

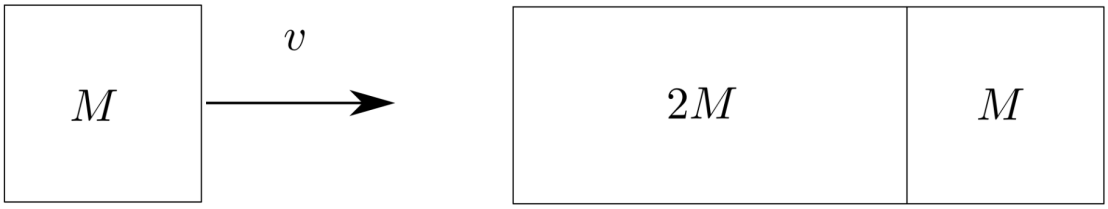
## PHYSICS

**5 points:** Captain John J. Roberts (Bob to his friends) finds himself stranded on a lifeboat after a kerfuffle with his crew. Bob is of mass $m$, and the lifeboat and its contents are of mass $M$. Feeling hungry, Bob scrambles to the other end of the lifeboat to his pack of rations travelling a distance $L$ in total. What distance across the water does the boat move in the process?

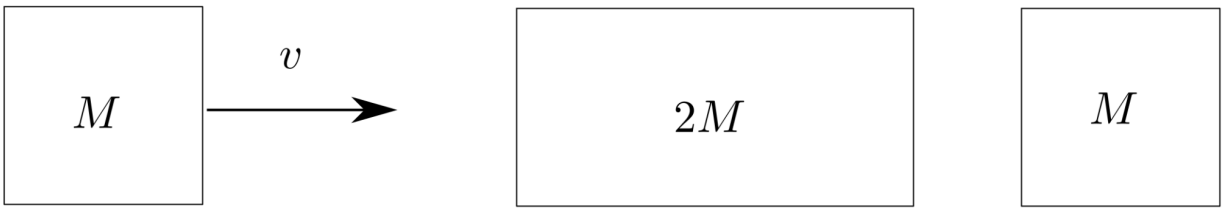
Neglect friction between the boat and the water.

**10 points:** A block of mass $M$ is fired in the direction of a mass $2M$block and a second mass $M$ block. In the first case, the latter two blocks are touching in the initial configuration. In the second case, they are not touching. What are the final velocities of all the blocks in both cases? Are they the same, or different? Explain your findings - why should they be the same or different? You may find this video demonstration helpful.

Case 1:




Case 2:



# CHEMISTRY

**5 points:** The idea the Periodic table is based upon seems trivial: if you arrange all elements according to their atomic weights, the periodicity becomes obvious. However, it is necessary to keep in mind that by the time Dmitry Mendeleev proposed his first table, some elements had not been discovered yet, and atomic weights of many known elements were measured incorrectly, which made discovery of the Periodic table not as straightforward as it seems today. Even worse, if you look at the modern Periodic Table, you may see that not all elements are arranged according to their atomic weight: some elements weigh less than their neighbors on the left. How many times is the order of atomic weight violated in the Periodic Table, and how does modern atomic theory explain this?

**10 points:** Tritium labeled leucine has been widely used in kinetic study of protein synthesis and secretion. A researcher needs to prepare 10 mL of 10 nM leucine solution that contains 10% of tritium labeled leucine and 90% of “cold” (unlabeled) leucine. He purchased a solution of tritium labeled leucine from Perkin-Elmer (https://www.perkinelmer.com/product/leucine-l-4-5-3h-net1166001mc), it contains 1 mCi in 1 mL of solution. He also prepared a 1 mM solution of unlabelled leucine. Calculate the volumes of labeled and unlabelled leucine solutions that should be mixed together to get a 100 nM leucine solution that contains 10% of labeled leucine.
Tritium half-life is ~ 12 years, and one Curie (Ci) is $3.7 \times 10^{10}$ decays per second.

# BIOLOGY

**5 points:** Anglerfish have a characteristic mode of predation, in which a front luminescent worm-looking fin ray acts as a lure for other fish. Fish that come to the light are seized by the angler's large jaws and eaten whole. Therefore, the genetic trait "being attracted to the angler's light" may be considered as a disadvantage, so it would be natural to expect the elimination of this gene from the population. Why isn't this happening?

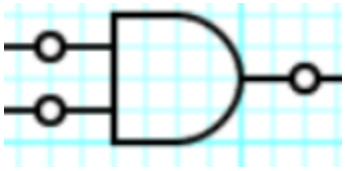
**10 points:** *Toxoplasma gondii* is a protozoan parasite that can infect any warm-blooded animal species. Its definitive hosts are cats (including big cats, e.g. pumas), whereas other animals (mice, humans, wolves etc.) are intermediate hosts. Toxoplasma changes behavior of its intermediate hosts in such a way that it becomes easier for cats to catch them. However, a study of the wolf population demonstrated that infected animals are much more likely to become the wolf pack leaders. How would you explain that?

# APPLIED SCIENCE

## 5 points:

Electrical circuits use switches called logic gates to determine the result of an input. Basic logic gates receive two or more inputs and determine a single output. Inputs and outputs are either 1 or 0, which can translate to on or off, or true or false. You can write the different combinations of inputs and outputs in a handy chart called a truth table. Here's an example:

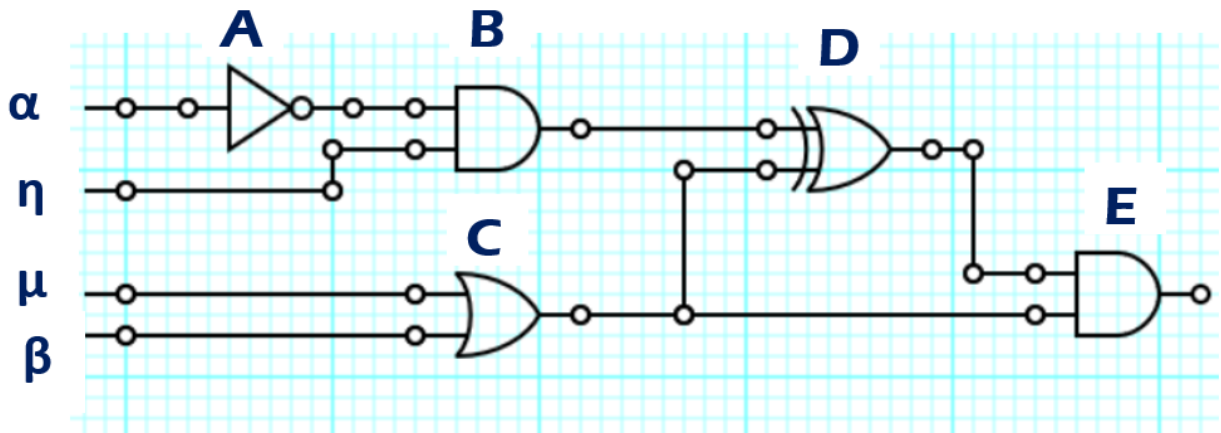
This symbol represents an AND gate with two inputs and one output.



The gate regulates the signals so that the output will only be "on" if both inputs are also "on."

| Input 1 | Input 2 | Output |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 1 |

When you put multiple gates together, the output of the first follows the wire and goes into the input of the second. For the logic gate circuit below, write out a truth table that includes the outputs of each of the intermediate gates, as well as the final output. You may use the template below or make your own.



| α | η | μ | β | A | B | C | D | E |
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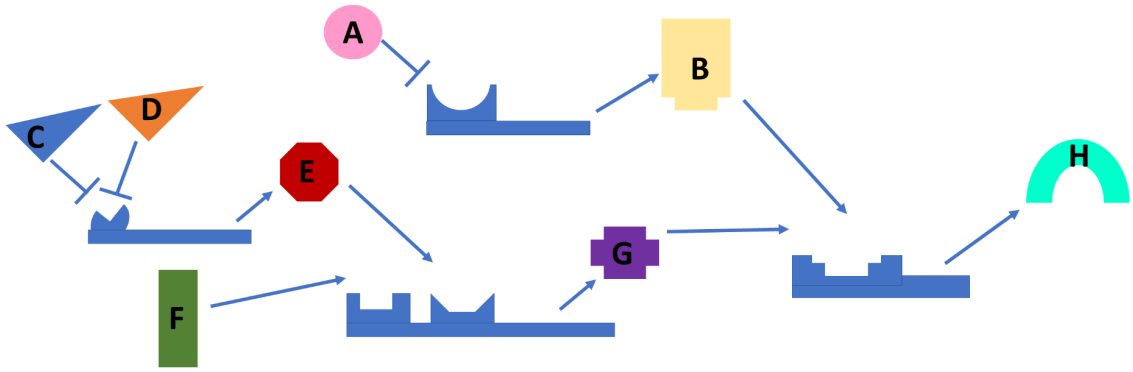
## 10 points:

Wildly enough, this is happening inside you right now! Gene regulation can be modeled using logic gates just like electrical circuits. These gene circuits have inputs in the form of signal molecules that must bind to the DNA in order to begin expression of the protein that the gene codes for. When a signal molecule is drawn with an arrow towards the binding site, that means the signal promotes gene expression. When a signal molecule is drawn with a bar facing the binding site, that means the signal inhibits gene expression. Look at the gene circuit below and draw the corresponding logic gate system. Additionally, if C and D were not present but A and F

were, then determine which molecules will be expressed.




**Hint:** Pay close attention to the shapes of the signals and their corresponding binding sites. What are the similarities and differences between the gates? Do some need one molecule or two? Can multiple molecules go in the same binding site?

# COMPUTER SCIENCE

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files named as specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.
  **No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!**

## 5 points:

Sigman is delivering presents to children around the world. He has a list of houses he needs to visit, indicated by how far each house is, in kilometers, from the previous one. For instance, if Sigman's list reads 540, 80, 90, 70, and 27, then the first house is 540 kilometers away from SigmaLand, the second house is 80 kilometers away from the first house, the third house is 90 kilometers away from the second, and so on. In order to make sure all presents arrive in prime condition, Sigman will take time to visit each building on his list individually and carefully place each present. Unfortunately, as it's no longer the 18th century and huge apartment buildings are everywhere, more than one family often lives in a single building. Each building's layout can be different, so Sigman may need to spend a different amount of time delivering presents to each family.

However, Sigman is limited by time: he has to deliver presents to as many families in the buildings in his list within a certain number of minutes! Luckily, Sigman has harnessed the power of physics to move insanely fast at a constant speed.

Write a program that receives the number of buildings Sigman is delivering to, the amount of time he has to deliver presents, Sigman's speed, and a list of the distances to each building, the number of families in each building, and the amount of time it will take to deliver presents per family in that building, and determines how many families Sigman can deliver presents to within the time limit. Assume that once Sigman stops at a building, he delivers presents to each family in the building before moving on to the next one.

Your program should receive the input file **input.txt**, which will consist of *four lines*:

- The first line contains three space-separated integers: the number of buildings on Sigman's list, the number of minutes he has to deliver presents, and Sigman's speed in kilometers per minute.
- The second line contains the distances in kilometers from each building to the next, formatted as space-separated integers. Note that the integers in this list may not necessarily be divisible by the speed of the sleigh.
- The third line contains the number of families living in each building, formatted as space-separated integers.

- The fourth line contains the number of minutes it will take to deliver presents per family in each building, formatted as space-separated integers.

Example input file:

```
6 60 200
540 80 90 70 27 200
5 40 1 3 80 1
2 1 3 3 2 2
```

Your program should produce the output file **output.txt**, which will contain the number of families that Sigman can visit within the time limit.

Example output file:

```
47
```

In the example above, Sigman has 6 buildings on his list, 60 minutes to deliver presents, and he travels 200 kilometers per minute. Within these time constraints, he is able to deliver presents to 47 families.

## 10 points:

The world's growing population has become a bit too much for Sigman to handle, so he's decided to outsource present delivery to a certain number of counselors, who can each only take up to a certain number of presents at a time. Each counselor will only take one trip to deliver presents.
Unfortunately, this year Sigman's has run into some supply chain issues, so each present will arrive at Sigman's warehouse to be delivered at a different time. Sigman's running on a very tight schedule, so he wants to minimize the amount of time a present waits at the warehouse to be delivered. This year, Sigman has reached the peak of optimization, and has found the best schedule possible.

Write a program that receives the amount of presents to be delivered, the amount of counselors, the maximum amount of presents an counselor can take at a time, and the arrival time of each present at the warehouse, and calculates the maximum amount of time a present will wait at the warehouse.

Your program should receive the input file **input.txt**, which will consist of two lines. The first line will contain the amount of presents to be delivered, the amount of counselors, and the maximum amount of presents each counselor can take at a time, separated by spaces. The second line

will contain the arrival time of each present at the warehouse, also separated by spaces. Note that the second line is not necessarily sorted!

Example input file:

```
7 4 2
1 2 5 4 8 13 10
```

Your program should produce the output file **output.txt**, which will consist of one line with the maximum amount of time a present will wait at the warehouse. If it isn't possible for the presents to be delivered, write "IMPOSSIBLE".

Example output file:

```
2
```

With 7 presents and 4 counselors, each of which can take 2 presents, the maximum amount of time a present will wait at the warehouse is 2 time units.