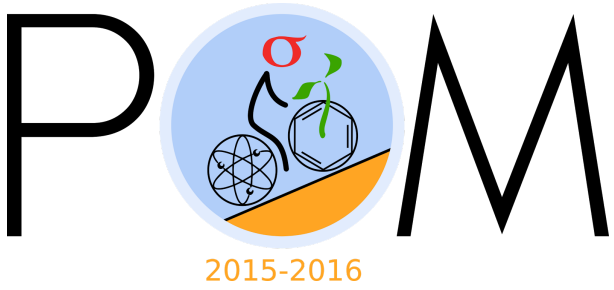
PROBLEM OF THE MONTH



January, 2016

MATHEMATICS

5 points:

Find the number of digits in the number 2^{100001}. You may use a basic calculator, but not the logarithmic function. Describe your procedure.

Hint: note that $2^{10} = 1024$

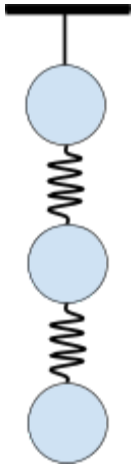
10 points:

Find the number of digits in the number $125^{1,000,000}$. You may use a basic calculator, but not the logarithmic function. Describe your procedure.

Hint: note that $125 = 1000/2^3$.

PHYSICS

5 points: Three identical balls are connected by identical weightless springs and suspended on the string (see figure). Find the accelerations of the balls at the moment the string is burnt.



Hint: The tensions of springs do not change immediately after the string is burnt.

10 points

Superman jumps from a bridge of height H, falls vertically and lands on a cart of mass M moving with velocity V. Find the maximal amount of heat released during the collision of Superman with the cart if the mass of Superman is m.

Hint: Use the conservation of the horizontal component of momentum and the conservation of energy.

CHEMISTRY

5 points

When Alice, a chemistry teacher, came to her lab, she saw Bob, her technician, staring at the glass flask with some liquid. “Look, Alice. Don’t you find it odd?” - Bob said.
“What exactly?” Alice replied.
“The flask. You asked me to prepare 3% hydrogen peroxide solution. I’ve done that. And it started to bubble. It started to bubble fifteen minutes ago, and I have no idea when it is going to stop.”
“What did you do with it, Bob?”, Alice asked.
“Nothing special, Alice. I just took 900 mL of distilled water, added 100 mL of 30% hydrogen peroxide solution to it, and stirred the liquid with a glass stick.”
“Bob, what happened with your finger? Why have you bandaged it?”
“That is just a minor cut, Alice. The glass stick had a sharp edge, and it cut my finger. The amount of blood was small, I even didn’t have to interrupt my work: I finished stirring, and stepped out to bandage my finger. By the way, that is why I didn’t see the moment when the bubbling started: when I came back, the solution has already been bubbling.”
“Now I see what is the reason, Bob. I am afraid, you have to prepare a fresh solution, and to clean the glassware carefully before that. The reason why the bubbles form is …”
Can you continue Alice’s explanations?

Hint: Bubbling (gas formation) means some reaction occurs. However, to initiate a reaction, Bob had to add some reactant (and, since the bubbling is intense, the amount of the added reactant should be significant), or to heat the flask, or to irradiate it with a bright light, or to do something of that kind. However, it seems he hasn’t done anything. Or he has? Frankly, it cannot be ruled out that he accidentally dropped something into the flask (just a tiny drop he haven’t noticed). What could it be, and why could it cause bubbling?

10 points:

You are again playing the “Escape the Room” game. You found a secret safe mounted to the wall. The safe is locked, and the lock seems to be controlled electronically. You found that turning off the light in the room deactivates the lock, so you can open the safe when the light is off. Unfortunately, the door of the safe seems to be connected to some switch that does not allow you to turn the light on when the door is open. In other words:

- when the light is on - the safe is locked;
- turning the light off unlocks the safe;
- opening the safe’s door blocks the light switch, so you cannot turn the light on until the safe’s door is closed;
- turning the light on locks the safe.

There is no windows in the room, and when you turn the light off the room is absolutely dark. When you examined the opened safe by touch you found nothing in it. Your hypothesis is there is probably some message written on the safe's internal surface. Unfortunately, you cannot check this idea, because, according to the rules of this game, all players are supposed to leave all electronic gadgets (smartphones, flashlights etc) outside.
The complete list of artefacts and materials in this room is as follows:

- The book "How to win friends and influence people" by Dale Carnegie.
- The magnifying glass;
- Two portraits of Sherlock Holmes;
- Three used smoking pipes;
- A box of Havana cigars;
- An empty box of matches;
- A flashlight with completely empty batteries;
- Pliers, a knife, and scissors;
- Porcelain dishes and cups;
- A bottle of mineral water;
- Two 1 L glass flasks with transparent and colourless liquids; the flasks are ⅓ full, the labels on the flasks say: "1% luminol solution", and "3% sodium hydroxide solution";
- A 200 mL bottle of standard 3% hydrogen peroxide solution;
- The book "Cat's cradle" by Kurt Vonnegut; the book is opened on the page 33;
- There is also the number "221b" on the wall.

Assuming your guess about the secret message was correct, how can you obtain the code hidden in the safe?

Hint: Since it is not possible to see anything in the absence of light, you have to find a way to produce it. Your friend (he is the guy with whom you are playing this game; he took Latin classes in school, so he knows Latin a little bit) told you that the name "luminol" resembles Latin "lux" (which means "light"). Maybe, this is a clue? Indeed, immediately upon having learned about that, you remembered that luminol, when oxidized by hydrogen peroxide in a basic media, produces light. But you also remember some catalyst is needed for that. What substance can serve as a catalyst? You cannot remember that. You are waving a flashlight with completely empty batteries (one of the things you found in the room), and mumbling: "Alice, Bob. Alice, Bob…" Why have these two names come to your mind? Why are you waving the flashlight? You yourself cannot explain that…..

BIOLOGY

5 points:

Question: Mammalians and birds are homeotherms, which means they maintain the temperature of their bodies constant. In contrast, the majority of other animals have no special mechanisms that allow them to maintain temperature constant. Nevertheless, some species, including insects, are exceptions to that rule. Which insects are capable of maintaining body temperature at the level that is considerably higher or lower than that the ambient temperature, and which mechanisms do they use?

10 points:

Question: As we know, many anticancer treatments, such as gamma irradiation, or some chemotherapy agents (cisplatin etc) may by themselves cause cancer in healthy humans. What is the reason, and does it mean every anticancer therapy should be potentially carcinogenic?

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in C, C++, Java, or Python
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Submit the problem in a plain text file, such as .txt, .dat, etc.
  No .pdf, .doc, .docx, etc!

5 points: Neat Words.

Some words in the English language are just "neater" than others: their letters are tidily arranged in alphabetical order. Take *chinos* as an example. A good example of how things should be organized in your room, isn't it? Then again, some words are clearly rebellious: they arrange their letters in reverse to the alphabet, e.g. *yolked*.

Your task this month is to write a program that checks the words for neatness. You will receive a list of words on the input, and on the output you should indicate for each word whether its letters are neat, messy or rebellious. For example, given the following input:

```
sponged fiddle begins yellow yolked biopsy
```

you should produce the following output:

```
sponged IS A REBEL
fiddle IS MESSY
begins IS NEAT
yellow IS MESSY
yolked IS A REBEL
biopsy IS NEAT
```

10 points: Garden Watering Optimizer

You have a rectangular garden where crops are arranged on a grid with integer coordinates. Normally there is enough usual precipitation to keep your garden healthy. but once a drought struck, and it is wreaking havoc to your garden. You lost a lot of crops, and you need to save as

many of the remaining ones by adding a sprinkler. Unfortunately, you can add only one as water during drought times is rationed. The sprinkler rotates 360° and has a fixed watering radius (integer) creating a circular watering area. Your task is to find an optimal location where to place the sprinkler so that the maximum number of surviving crops fall within its watering radius. The sprinkler can only be placed strictly on a grid, and if there is already a surviving plant at the point, that plant is killed.

Your program will take 3 integers as an input: two dimensions of the garden (N rows, M columns) and the watering radius R of the sprinkler. Then you are given a map of the garden, which is N strings, each of length M and consisting of either dots ('.'), representing no surviving crops, or x's, representing growing plants.
Additional rules:

- all the plants and the sprinkler have integer 0-based coordinates
- the sprinkler covers the plant if the distance from the sprinkler is less than or equal to the sprinklers radius. For example, the distance from (2, 2) to (3, 4) is SQRT(5), which is more than 2, and therefore (3,4) would not be covered by a sprinkler with R=2 placed on (2,2)
- if you place the sprinkler on a point with a crop, you destroy the crop, so handle accordingly
- in the event you find two or more placements that yield identical scores, pick any one of them

Example. You are given the following input:

```
6 7 2
..x...x
.xx..x.
...x..x
.x.....
..x..xx
x..x.x.
```

For this garden, the ideal location of the sprinkler is (2,2), which would cover 6 plants.

As a bonus, output the map showing location of the sprinkler and watered plants, like this:

```
..X...x
.XX..x.
..OX..x
.X.....
..X..xx
x..x.x.
```