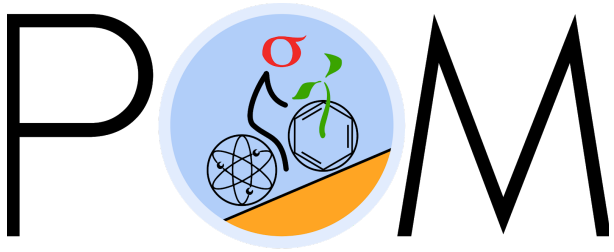
PROBLEM OF THE MONTH



January, 2019

2018 - 2019

MATHEMATICS

5 points:

There are 20 chairs in a room, each is either red or blue. Each chair is occupied by either a knight or a knave. Knights always tell the truth and knaves always lie. All 20 people declared that they are sitting on a blue chair. Then they switched seats in some way after which half say they are sitting on a blue chair and the other half say they are sitting on a red chair. After the switch, how many knights occupy a red chair?

Hint:

Notice that in the beginning, all knaves are on red chairs and all knights occupy blue chairs.

10 points:

The operation * has the following two properties:
a * a = 0
a * (b * c) = (a * b) + c

Here + is regular addition and parentheses indicate the order of operations.

Find 2019 * 2018.

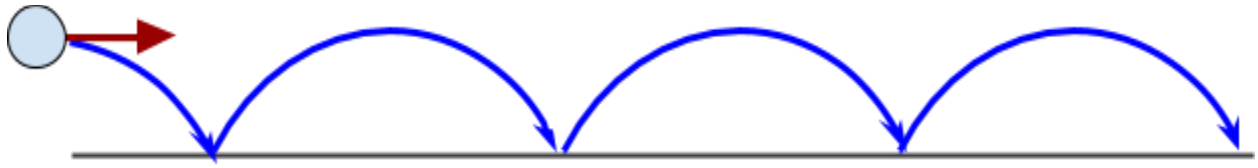
Hint:

Consider a*(a*a)

PHYSICS

5 points:

A broken robotic vacuum cleaner "Roomba" can only move with constant speed v=1m/s along a circle of radius R=2m, always turning to its right. When it collides with the wall, it bounces off elastically, i.e. at the same angle as it approached (see the figure). At the initial moment, the Roomba is released at distance R/2=1m from the wall, in the direction parallel to it. After some time, it collides with the wall, bounces back, and keeps doing this again and again. Find the average speed with which the robot moves along the wall. Assume its own size to be much smaller than R.



Hint:

Compare the length of the arc to the length of the chord knowing that the distance from the wall is R/2.

10 points:

An unknown mass is attached by an ideal spring to a point on a horizontal frictionless plane. The mass moves along a circular orbit centered at that point, with constant angular velocity ϖ. During this motion, the spring is stretched by 20% compared to its relaxed length. Based on this information, find the period of free oscillations of this mass on this spring.

You might find it useful to learn a background information on the following two topics:

- **Circular motion**

http://physicsnet.co.uk/a-level-physics-as-a2/further-mechanics/circular-motion/

- **Linear Oscillations**

http://physicsnet.co.uk/a-level-physics-as-a2/further-mechanics/simple-harmonic-motion-shm/

Hint:

Consider a uniform circular motion of the mass with angular velocity ϖ with radius $l = 1.2l_0$. Equate spring force to the centripetal force.

CHEMISTRY

5 points:

"*We had two bags of Arabica coffee, seventy-five ounces of sodium hydroxide pellets, five kilograms of high purity acetic acid, a saltshaker half-full of zinc chloride, and a whole galaxy of multi-colored pH papers, rubber balloons, strings etc... Also, a quart of concentrated hydrochloric acid, a quart of acetone, a case of Poland Spring water, a pint of raw ether, and two dozen grams of some amyl alcohol. Not that we needed all that for our trip, but once you get locked into a serious chemicals collection*, *the tendency is to push it as far as you can. I even was not sure which amyl alcohol did we have: iso-amyl, sec-amyl, tert-amyl (fortunately, Lucas, a friend of mine, already explained to me how to discriminate them, and I've been capable of figuring out that the correct name of the alcohol was isoamyl alcohol). The only thing that really worried me was the ether. There is nothing in the world more helpless and irresponsible and depraved than a man in the depths of an ether binge, and I knew we'd get into that rotten stuff pretty soon.*"

Using the stuff described above, will we be capable of obtaining at least 1 gram of reasonably pure caffeine? If yes, describe all steps of this procedure in as many details as possible.

Hint:

This is a final problem in the 2018-19 school year, and I think most of you correctly concluded that we eventually have to use *"that rotten stuff",* for, according to the Chekhov's gun principle, "If you say in the first chapter that there is a rifle hanging on the wall, in the second or third chapter it absolutely must go off. If it's not going to be fired, it shouldn't be hanging there."

10 points:

Alice, a college faculty, came to her lab and found Bob, her technician, assembling an apparatus composed of a round-bottom flask, a still head, Liebig's condenser, and a receiver. "Hi, Bob", - Alice said. "Are you going to distil something?" "Yes, Alice", - Bob replied, "You told me we need anhydrous ethanol for tomorrow classes, but I've just realized we have only reagent grade alcohol with concentration of 70%. That is why I decided to distill it: since ethanol's boiling point is much lower than 100 degrees, I expect it will boil first. I am going to collect the first fraction that boils at 78.4 degrees, and that will be a pure 100% alcohol". "Alas", - Alice said, "That will not

work. You apparatus will not allow you to do that. And even if you improve your apparatus, you will never obtain 100% alcohol, because ..."

Continue Alice's words and explain Bob’s mistakes.

Hint:

Do you know what does “azeotrope” mean? If you don’t, google it. That may be helpful.

BIOLOGY

5 points:

Three different species of animals are close to each other, and each of them has the same gene that plays similar role in their organisms. This gene contains a homologous segment with the following sequence:
Species 1: AAACGGCTTGCAGCTCTA
Species 2: AAAGGGCTTGCAGCTCTA
Species 3: ACATAGCAAGCGGTTCAA
Which of these species are more likely to belong to the same genus?

Hint:

Point mutations ("point mutation" means replacement of a single nucleotide) in genomes of most organisms occur with the same overall rate, so the closer two homologous sequences are in two organisms, the closer these two organisms are situated on the evolutionary three. The same hint works for the problem number 2.

10 points:

Five different species of animals are close to each other, and each of them has the same gene that plays similar role in their organisms. This gene contains a homologous segment with the following sequence:
Species 1: AAACAGTGACTTAGT
Species 2: AATCTGTGGGTAATT
Species 3: AAATTGTGCCTTATT
Species 4: AATCCGTGGCTAATT
Species 5: AAACTGTGCCTTATA
Assuming that mutation rate occurs with constant speed, tell which of those species had separated first during the evolution process, and which of them diverged just recently?

Hint:

See the hint to the 5pt problem.

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc.
  No .pdf, .doc, .docx, etc!

5 points:

Imagine yourself facing a choice of which lectures to go to at SigmaCamp. The complication is that lectures can start at different times and end at different times, unlike the 9:00am - 10:00am uniformity we know of. Assuming all lectures are equally good if we listen through from start to the very end, our problem becomes that of maximizing the number of lectures we attend and managing to avoid overlapping their scheduled times.

Write a program that takes a list of pairs of times in 24-hour notation as input (containing the times of beginning and end of scheduled lectures in [time_begin, time_end] format for each lecture) and outputs subset of this list containing the maximum number of lectures one is able to attend while avoiding any overlaps in schedule.

For example, an input of:

9:35, 16:31
10:45, 16:36
11:54, 14:30
13:59, 14:56
14:17, 16:34
11:20, 11:38
8:20, 9:19
14:10, 16:54
14:35, 15:59
8:16, 15:37

would produce an output of:

8:20, 9:19
11:20, 11:38

11:54, 14:30
14:35, 15:59

If any row (lecture) in the input contains beginning time of lecture that occurs later than its end time or if any of the inputs cannot be time in 24-hour notation, the program should state so and terminate.

Hint:

Picking a class that ends earliest, repeatedly, is the winning strategy.

10 points:

Imagine a new policy at SigmaCamp for determining the composition of teams. The policy is simple: no pair of classmates are allowed to be on the same team. Therefore, starting next season, SigmaCamp's teams need to be carefully composed to avoid having on the same team any pair of campers studying in the same class at their school: if campers A and B are classmates, they can be on teams, say 1 and 2, respectively, but never both on team 1 or any other team.
Write a program that takes a list of pairs of classmates (pair of camper names each) followed by a list of campers with no classmates present at SigmaCamp and outputs the composition of the minimum possible number of teams the campers can be divided into without any two classmates ending up on the same team.

For an example input with 9 campers:
A, C
B, D
B, F
D, F
E, G
H
I

The following output could be produced indicating that 3 is the minimum number of teams possible:
Team 1: A, B, E, H, I
Team 2: C, D
Team 3: G, F

Note that all pairs of campers that are classmates to each other are given as a comma-separated pair of names: as A, C or as B, D. Campers without any of their classmates present at Sigma are given as a single name on a line each: as H or as I.

If there is a problem in the input where campers A and B are classmates, B and C are classmates, but campers A and C are not classmates, the program should state the issue and terminate.

Hint:

The largest number of campers from one class determines the minimum number of teams.