PROBLEM OF THE MONTH

POM

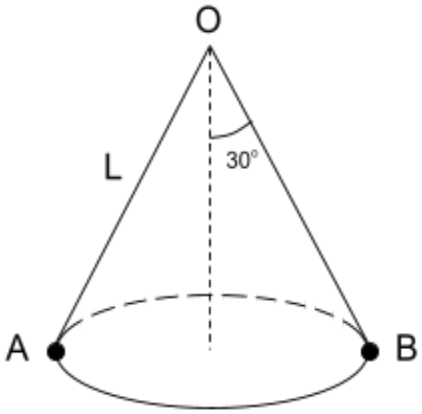
2020 - 2021

January, 2021

MATHEMATICS

5 points:

What is the length of the shortest path along the side of the cone connecting points A and B that lie opposite each other on the cone's base? The length of the side of the cone OA=OB=L and the aperture of the cone is 60 degrees as shown in Figure.



Hint:

Imagine it is a paper cone which can be cut with the scissors.

10 points:

Two straight roads OA and OB intersect at the crossroads O so that the angle AOB is 60 degrees. A runner can run along the road with velocity 20 km/h and off-road with velocity 10 km/h. What is the distance to the crossroad of the furthest point equidistant to both roads that the runner can reach in an hour?

Hint: Find all points on the plane which the runner can reach in an hour.

PHYSICS

5 points: Electric car Tesla 3 has a typical range of 260 miles, and its battery stores 0.7 $kW \cdot hr$ of electric energy per liter. An alternative technology being developed for powering electric vehicles is hydrogen-based. In that case, the energy would be stored in the form of compressed hydrogen. The chemical reaction between hydrogen and atmospheric oxygen in a so-called fuel cell can directly produce electricity (without the need of burning the fuel), and the only product of that reaction is water vapor. This way, 1 mole of hydrogen gas can be converted to approximately 280 kJ of electric energy. What would be the range of the fuel cell vehicle, if the volume of its hydrogen tank is the same as that of Tesla 3 battery, and the pressure of the gas is 700 atmospheres (70 MPa)? Assume normal temperature (300 K).

Hint: Note that at normal conditions 1 mole occupies volume of approximately 22 liters.

10 points: Flywheel is a mechanical device for energy storage. It is a heavy cylinder that may be rotated at very high angular velocity, thus accumulating substantial kinetic energy. This energy is limited since the outer rim of the flywheel experiences an outward pressure due to centrifugal forces. Find the maximum kinetic energy per unit volume that the flywheel can store, if the maximum pressure that material of the rim can sustain is P. How much more (or less) energy the flywheel would store, compared to hydrogen at the same pressure P? (use results of 5 pt problem).

Hint: Consider a small element of the flywheel at distance r from the center. Its centripetal acceleration is $r\varpi^2$, where ϖ is angular velocity of the flywheel. By using the 2nd Newtons law, you may find the pressure difference across that element.

CHEMISTRY

This month, the topic is: **Chemical reactions, heat and energy.**
IMPORTANT! In this PoM season, we do an experiment: each month, an online lecture will be given. This lecture may be helpful for those who want to solve Chemistry PoMs, although it is not supposed to provide direct hints.
This month, the lecture will be on Sunday morning, Jan 31st. At 11:00, a Zoom conference will start where December PoM solutions will be discussed. After that, approximately at 11:30, the lecture starts.
To join the Zoom conference, use this link:
https://us02web.zoom.us/j/4817690592?pwd=T2djSjRETEpDSHFZdWJpYlBTYzdjQT09
Meeting ID: 481 769 0592
Passcode: 879615

A recording is available here: https://www.youtube.com/watch?v=BEi8NQgxhEA

5 points:

One online textbook explains the concept of exothermic and endothermic reactions as follows:
"*When more stable substances are produced from less stable ones, such a reaction is usually exothermic; if less stable substances form from more stable ones, such a reaction is exothermic."*
What is wrong with that explanation?

Hint:

More stable state is always the state with lower energy

10 points:

Two compounds A and B are isomers. The compound A has a linear shape, like this:

Y-X-Y

so the angle between two Y-X bonds is 180º. In the compound B the angle between X-Y bonds is 90º.
A transforms to B in the reaction:

$$A \rightarrow B$$

Both A and B are gases at room temperature. Assuming that the energy of the X-Y bond does not change in that reaction, which process is this process more likely to be endothermic or exothermic at room temperature?
(To solve that problem, it is highly desirable to use the information that will be given in the lecture on 31st of January).

Hint:

What is the number of degrees of freedom in linear and angular molecules?

BIOLOGY

This month, the topic is: **Biology and topology**
IMPORTANT! In this PoM season, we do an experiment: each month, an online lecture will be given. This lecture may be helpful for those who want to solve Biology PoMs, although it is not supposed to provide direct hints.
This month, the lecture will be on Saturday, January 30. At 13:00, a Zoom conference where we will tell you about DNA topology and the enzymes that cause topological transformations in DNA (so called topoisomerases)..
To join the Zoom conference, use this link:
https://us02web.zoom.us/j/4817690592?pwd=T2djSjRETEpDSHFZdWJpYlBTYzdjQT09
Meeting ID: 481 769 0592
Passcode: 879615

A recording is available here: https://www.youtube.com/watch?v=BEi8NQgxhEA

5 points:

Bob was involved in a study that required preparation of a significant amount of a certain circular DNA (a.k.a. “plasmid”). This type of DNA is usually grown in bacteria, and they can be prepared in a pure form. When prepared in optimal conditions, they look like relaxed loops (as shown at the Fig. 1).

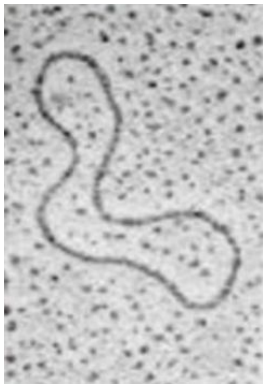


Fig. 1. An electron micrograph of circular DNA

However, when Bob checked a sample of the plasmid he prepared, the electron micrographs of his DNAs looked as shown at Fig 2 and 3: they formed knots and/or concatenated rings.

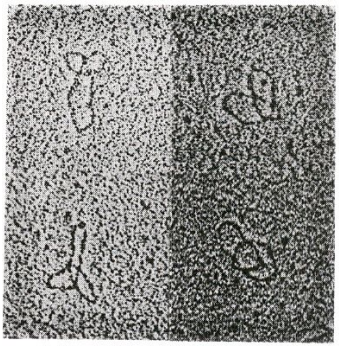


Fig 2. DNA knots

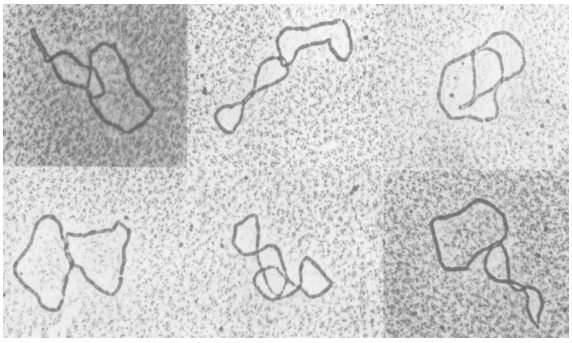


Fig. 3. Concatenated DNA rings

Bob was very disappointed, and he asked his friend Alice if she knew what was the reason.
“It looks like one important enzyme is not working properly in your bacteria. That enzyme is called “topoisomerase””, she answered.

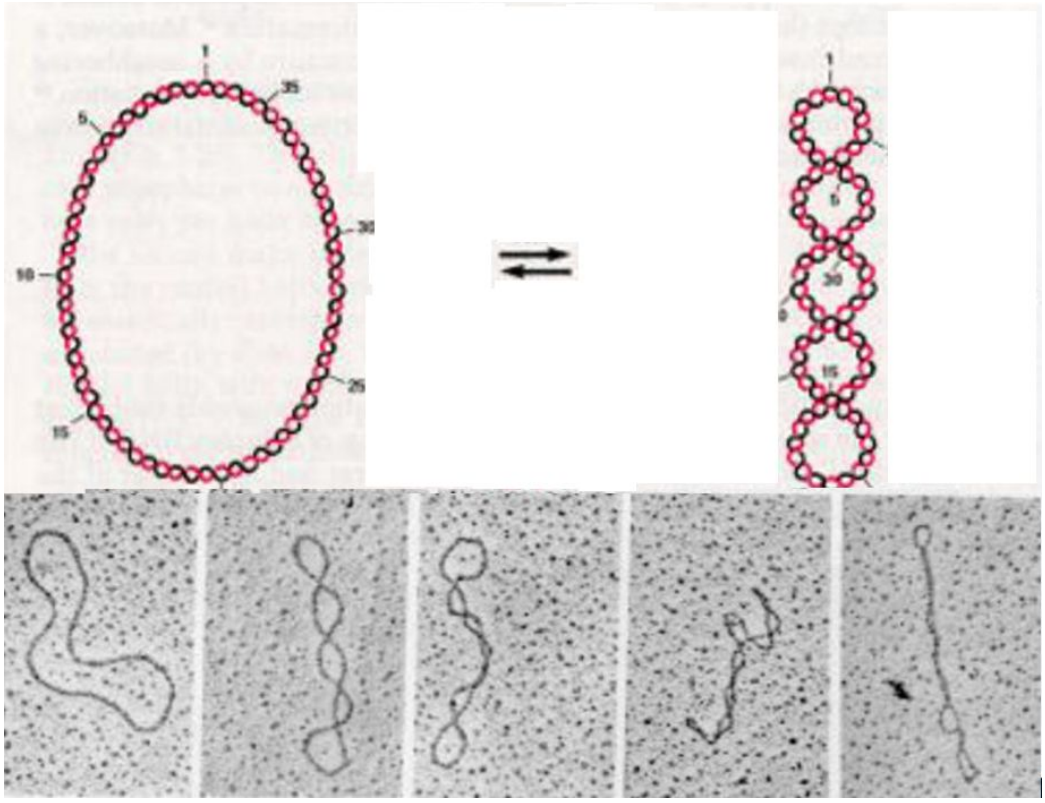
“I don’t think so,” - Bob argued. “If topoisomerase were not working in my bacteria, there would be no DNA replication at all, and I wouldn’t be able to obtain any plasmid at all.”
“Bob, you are working on a PhD in biology, and you always should keep in mind that living cells are much more complex than people usually think. In reality, there are …”
Continue Alice’s words. What did she mean?

Hint:

“… two main groups of DNA topoisomerase” (further continuation will be the answer).

10 points:

Usually, it is considered that one of the main functions of DNA topoisomerases is to untwist the DNA that becomes twisted during the DNA replication process. In other words, they convert twisted DNA as shown at the figure below (the series of electron micrographs of differently twisted DNA are shown at the bottom, and most topoisomerases from right to left):



Relaxes ↔ Moderately twisted ↔ Highly twisted

However, a group of DNA topoisomerases exist that work in an opposite direction: they conwert a relaxed (untwisted) DNA into a twisted form (the forms similar to the one shown at the right side of the picture). Is there any physiological reason for that? Explain.

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Hint:

Some important processes, such as transcription, require local melting of DNA (in that context, “melting” means local separation of DNA strands, which happens via opening base pairs without breakage of the DNA’s backbone).

LINGUISTICS

5 points:

Consider the following sentences and their translations from an unnamed language.

Karapo muba "The dog eats"
Nerip turio "The cat sleeps"
Karapo tarak hisa "The man eats a lot"
Karapo hisa babon "The old man eats"
Karapo turio tarak "The tiger eats"
Soret babon muba kiun "The puppy walks for a long time"
Karapo hams muba “The bear eats the dog”

Question 1: Translate the following sentences into this language. Make educated guesses about the words that you have not encountered in the given data.

1) The kitten eats a little
2) The wolf eats a lot
3) The tiger sleeps for a long time
4) The chief eats a lot of tiger

Question 2: Translate the following sentences into English

1) *Nerip babon hisa kiun*
2) *Soret tarak turio babon*

10 points:

Below is a set of artificial sentences:

She took the glove off from her 1.
Her 1 was covered with a hat.
Around her 1, she tied a scarf.

In this case, the number 1 is a placeholder for a word meaning “the body part on which the mentioned article of clothing is usually worn”. The words “hand”, “head”, and “neck” are assumed to be replaced by the number 1 in the sentences above, respectively. Knowing the definition of the numerical placeholder, it’s easy to determine the meaning of the sentence.

Try to come up with as many categories of replaceable words as you can in the sentences below, and substitute them with numbers. Then, **describe what type of words your placeholder numbers represent**. Keep in mind that anyone should be able to understand the sentences below if they know the meaning of the numbers you choose.

He made a horrible mess.

The cow mooed, while her calf whined and butted against me.
She eradicated the deadly disease.
His song waned.
The cat meowed.
My aunt knitted the boy a cozy sweater.
A blazing light shone.
Warm showers rained on us.
They cancelled their immediate order.
You started the great schism!
The light dissipated.
The powerful rain stopped, and the heat wave came.
She built a house.
The strong wind blew.
The horse neighed and kicked, just like its chestnut foal.
She demolished her house.
A flame ignited.
The professor yelled as he argued.
He designed an infographic.

COMPUTER SCIENCE

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files with specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.
  No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!

5 points:

Sigma Camp decided to come up with its own flag. They decided to make the flag a constellation of σ signs, as many as there are campers admitted to the camp, arranged in an aesthetically appealing rectangular formation, just like stars in the American flag. Formation could have the same number of signs in each row, just like the USA flag circa 1912, for example:

σ σ σ σ σ σ
σ σ σ σ σ σ
σ σ σ σ σ σ
σ σ σ σ σ σ

or alternating number of stars in each row: n, n-1, n, n-1.... As in the current USA flag. For example:

σ σ σ σ σ σ σ
 σ σ σ σ σ σ
σ σ σ σ σ σ σ
 σ σ σ σ σ σ
σ σ σ σ σ σ σ

To be aesthetically appealing , the aspect ratio of the star pattern (ratio of the number of columns to the number of rows) must be between 1.3 and 1.8.

Your program will receive the number of admitted campers as an input in **input.txt** file. It should write the output to output.txt file in the following format:
Each row in the flag will be represented by a number of σ signs in the corresponding row of the flag. For example, the flags above would be represented as:

6 6 6 6
and
7 6 7 6 7
correspondingly.

If the number of admitted campers cannot be represented in an aesthetically appealing flag, the program should write IMPOSSIBLE to output.txt.

10 points:

A group of Sigma counselors was working on turning a map of the Silver Lake campus into a simple jigsaw puzzle. To do that, they split the rectangular map of the campus into NxM square pieces and cut the map into those little squares. The counselors had to break their activity to attend a staff meeting. At that time some pranksters decided to meddle with jigsaw making and mixed all the squares into a pile. They did not know that counselors, being smart, anticipated such meddling and meticulously marked all the squares the following way:

- they marked the connections for each square, recording connections on top, right, bottom, left
- connections were marked as pairs of corresponding positive/negative numbers, 1/-1 to 10/-10, so that if one side of the connection would be marked *k* the opposite side of the connection would be marked *-k.* Therefore 1 would connect to -1, 2 to -2, etc.
- assignment of the connection number pairs was somewhat arbitrary, but guaranteed that each square had a unique “connection signature”
- lack of connection to another square (for edge squares) was denoted by number 0

Your program will receive its input from the file **input.txt** of the following structure:

The first row will contain 2 space-separated numbers N and M denoting the number of rows and columns the map has been split into.
This is followed by N*M rows, each containing a definition of a square in the following format: a square number, from 0 to (N*M-1), immediately followed by a colon (':') and then followed by 4 comma-separated integers denoting up, right, down, left connections.

Assume squares are rotated in the correct direction.
An example of input file is :

```
2 2
0:0,1,2,0
1:0,0,2,-1
2:-2,0,0,2
3:-2,-2,0,0
```

Your program needs to help the counselors to reassemble the map. Its output should be written to **output.txt** file, which would contain N rows of M space-separated numbers of squares in the right sequence.

For the example above, the output file would contain:

```
0 1
3 2
```