Problem of the Month

2022-2023

January, 2023

MATHEMATICS

Alice, a counselor at Sigma Camp, is handing out gloves for a chemistry semilab. She has *b* different boxes of gloves, each with gloves of a different color. Each camper (there are *c* campers) has a preference for the color and the size (small, medium, or large), but Alice is particularly tired, and randomly selects the color and size she gives to each camper (the two gloves in the pair are of the same color and size).

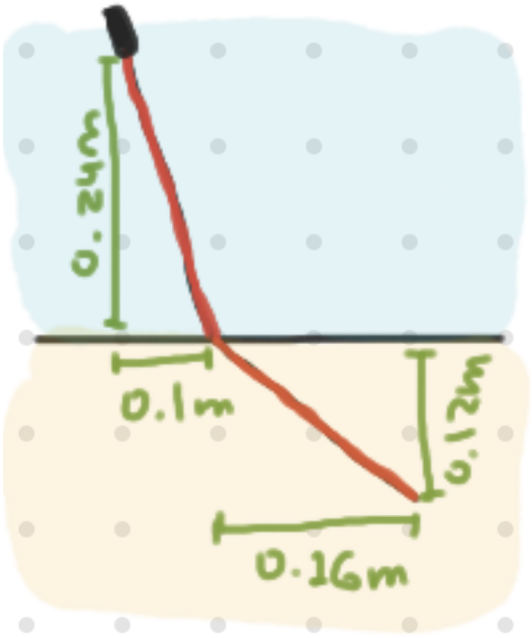
5 points: What is the probability that every camper gets either their preferred color or their preferred size (or both)? (Don't worry: Alice will never run out of gloves)

10 points: On average, how many colors does she end up handing out?

Challenge problem (no points given - do not submit your solution): Alice makes all the campers write their names on their lab coats (no two campers have the same name), but the next day, all the campers randomly select a lab coat. Alice makes each camper stand next to the person who took their lab coat, forming several circles of people. What is the expected number of circles formed? For instance, if there are 4 campers (Alex, Bob, Charlie, and Dave), and Alex took Charlie's coat, Bob took Alex's, Charlie took Bob's, and Dave took his own, then the two circles that would be formed would be Alex-Bob-Charlie and Dave.

PHYSICS

5 points (Snell's Law): Alice shines a laser through two substances as shown below. What is the ratio in the indices of refraction?



10 points (Imaginary Images): A **convex** lens can produce what is known as a 'real' or an 'imaginary' image. An image is called 'real' because it corresponds to the focusing of rays at a physical point on the lens's opposite side with respect to the source (for a mirror, the reflected rays can also form a 'real' image if they coalesce, but it would be on the object's side of the mirror and not the opposite side). For example, if you use a magnifying glass to focus rays from the sun to burn a piece of paper, you are using a real image to do so.

(a) Use ray tracing diagrams to describe the difference between real and imaginary images. Clearly explain the phenomenon of an imaginary image. What determines whether a produced image is imaginary or real?

(b) Describe the process by which, as you change a parameter (for example, the distance from the object to the lens), an image goes from real to imaginary. Can you describe it as a continuous process, relating it to the geometric concept of the point at infinity?

CHEMISTRY

When Alice came to the lab, she noticed Bob, her lab assistant, who was reading an old book.
“Where did you find this old folio, Bob?” Alice asked.
“In our library. That is a captivating reading! Look, they describe a recipe of Philosopher's stone. Read this:”

The text was as follows:

Oh, thou, my faithful servant! Heed well the wisdom I impart unto thee, as well as my warnings, for the path thou art treading upon can lead to great accomplishments, but this road is fraught with secrets and dangers, especially if thou turnest thy thoughts away from the Lord. I shall reveal unto thee my greatest secret, and my greatest achievement, the one that lifted me to the heights and brought me low. And may this tale serve as a lesson to thee. I shall tell thee of the preparation of the Lesser Elixir, that is, the Minor Philosopher's Stone, which doth not grant eternal life, doth not cure diseases, and doth not transform mean metals into gold, but only doth turn Venus into the Moon.

This is how it is done: on the night when retrograde Mercury changeth its course and hasteneth after the Moon, immediately after the last rooster crows, take a large retort, and an alembic, and flasks, and small vessels, and ascend to the open rooftop, and, having gazed upon the heavens, offer a prayer to the Blessed Virgin. It is of utmost importance to behold the heavens during thy Anabasis, for I have discovered that demons love closed chambers, and they may reward thee with a headache, fever, and cloud thy mind if thou secludest thyself from the World of God in thy cellar. But if thou dost thy experiments under the shadow of the Heavens of God, then the Blessed Virgin shall protect thee.

Now, take two pounds of common saltpeter, grind it carefully in a mortar, and place it in a large flask. Then take two pints of vitriol oil, which thou hast previously prepared by distilling blue vitriol, and add it carefully to the saltpeter. Shake the flask, attach the alembic to it, and put in on the oven, and kindle a small fire in the oven. Heat everything carefully until noon, and if the Savior and the Blessed Virgin are with thee, thou shalt collect about a pint of Agua Fortis. Next, put three ounces of finely ground cinnabar into a small retort and add a little Aqua Fortis to it, so that the latter completely wets the cinnabar. Place the retort on a small fire, and heat it until the first Red Dragon is expelled from the vessel. If thy efforts are unsuccessful, offer up a prayer to the Blessed Virgin, and add one spool of salt to the retort, and the oil of vitriol, quantum satis. After the first Dragon devours his tail, remove the retort from the fire and add more Aqua Fortis to it. Heat the retort again. Repeat these actions for several days and nights tirelessly, until all the dragons devour their tail and leave the vessel. Then carefully heat the mixture to let most Aqua Fortis evaporate and leave the retort. But do not heat too much, lest thou obtain the Caput Mortum, which is by no means thy goal. Remove the retort from the heat and let it cool. Place the resulting residue in a regular pot and let the moisture soak into

the clay. In the morning, take what is left, and it shall be the Lesser Elixir. Pulverize it, and to test its power, take half an ounce of the Elixir, mix it with water, and rub it on common coppers. Before thou hast time to recite the Lord's Prayer, thy copper coins shall turn into pure silver, and they shall shine brighter than the full Moon.

This discovery is the main achievement of my life, my main happiness, and my disaster, my incredible luck, and my terrible punishment. For I realized that my labors had turned mine heart away from God, from the Most Holy Trinity and the Virgin Mary, and that the sight of that silver had drained mine soul. And for this sin, our Lord punished me with a terrible punishment: mine hands were covered with scabs, and they no longer obey me, so I have to dictate this manuscript to the scribe. Mine vision is clouded, and mine mind is leaving me. Mine chest is tight, I am breathing with difficulty, and I am feeling that I will soon stand before the gates of St. Peter, and I will hear the inquiring voice of our Heavenly Father. And in mine last hour, I conjure: O Thou, mine unknown follower, the one who reads these lines! May thy thoughts be pure on thy path, and may our Lord and all the Holy Saints be with thee. Trust in them, and do not be overwhelmed by passions and pride. And may thine be more successful than mine. Amen."

"Interesting", - Alice noted after having read the text. "It seems old Alchemists already knew about basic safety rules. But I think you are not going to reproduce this experiment in our lab, aren't you, Bob?"

5 points:

What did Alice mean when she mentioned "safety rules", and why didn't she want Bob to reproduce this experiment in her lab? Is there any rational explanation for God's punishment inflicted on the author of the recipe, and if yes, explain what caused his suffering?

10 points:

Explain all chemical transformations described in this document. What did he mean under "Red Dragon"? Why was salt needed? Which operations and precautions were necessary, and which were not? What does "but only doth turn Venus into the Moon" mean?

BIOLOGY

Beogulf the Bald, king of Orimentalia, was tormented by doubts. He had to choose a wife for his beloved son Tirstan. Marriage to Marilda, daughter of Botton XIV, the powerful king of Laurasia, promised to be the basis for a powerful alliance against the evil Lamblia (an aggressive and annoying kingdom that dreams of tearing away and annexing the southern provinces of Orimentalia). But instead of Marilda, Thirstan could have married Aminalia, princess of Lamblia, a wealthy kingdom that controlled important trade routes to the southern lands rich in species, gold and ivory.
But more importantly, Beogulf was worried about the future of his dynasty. The current Orimental dynasty was almost interrupted because Beogulf's grandfather, Geowulf III the Thin-Legged, suffered from a disease causing his blood to not clot; so even a minor cut could cause severe bleeding, which was very difficult to stop. He was living surrounded by the best doctors, but, despite all conceivable precautions, he died at the age of 19, after he accidentally pierced his finger with an ordinary fork. Fortunately, Geowulf's only son Tuvalett I (Beogulf's father) turned out to be quite healthy. Nevertheless, doctors advised Beogulf to pay special attention to the health of Tirstan's future sons, and to this end, pay special attention to the health of his future wife.
Yesterday, Beogulf the Bald received two messages from his spies in Lamblia and Laurasia, and they were not encouraging. Spies reported that the Count of Carran-d'Aché, the younger brother of Botton XIV (and Marylda's uncle), had two sons, and one of them had recently died of the same illness that killed Geowulf. Another spy reported that Aminalia's cousine (a daughter of a sister of Aminalia's mother) has three sons, and one of them appears to be suffering from the same condition.

5 points:, If you were Beogulf's medical adviser, considering the nature of the disease and the family history, whom (Marilda or Aminalia) would you recommend? Explain your answer.

10 points:
Beogulf the Bald was greatly disappointed by this news and sent several envoys to find a healthier bride for his beloved son. Very strict instructions were given to every ambassador to carefully ascertain the medical pedigree of each candidate and to pay special attention to the disease that had killed Beogulf's grandfather. The reports of the ambassadors revealed a sad picture: this disease appeared quite often in the royal dynasties of nearby kingdoms. However, several candidates looked very promising. Thus Princess Eutectica, the only daughter of Prince d'Ammoniac, had a healthy pedigree, although old archival records showed that the brother of Guinerva (she was

the Countess of Gurgon and the mother of Eutectica's mother) suffered from the same strange disease. Another candidate was Princess Allosterica, the third daughter of Graf Ottoman von Schwarzwaldburgerkingkrighsmarinenhof. The House of Schwarzwald-burgerkingkriegsmarinenhof had a fairly healthy family history, with the only exception being Ottoman's cousin, whose third son is currently battling this strange disease.

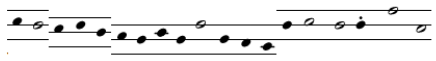
Calculate the probability of Tirstan's children (boys and girls) getting this disease depending on if he marries Marilda, Aminalia, Eutectica or Allosterica.

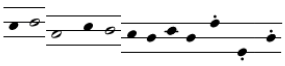
LINGUISTICS/APPLIED SCIENCE

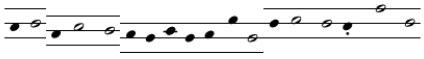
5 points:

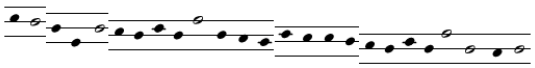
Bob and Alice are friends and like to pass notes to each other during class. They got tired of other students (and sometimes the teacher) intercepting and reading their messages so they invented their own language to keep everything super secret. Match the following English sentences pertaining to Bob and Alice's juice box negotiations to the sentences in their secret language.

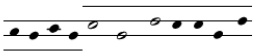
1. Put the blue box there.
2. Put the yellow box inside the house.
3. The box is outside the house.
4. Open the blue box and the red box.
5. Close the red box.
6. I want the bottom box here.
7. Do you want the red box?
8. I have no boxes.
9. You have two boxes.
10. She has the most boxes here.

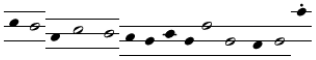
A 

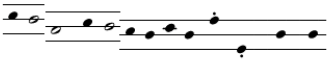
F 

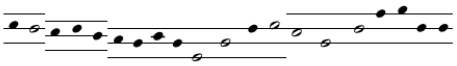
B 

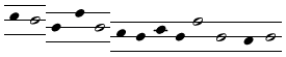
G 

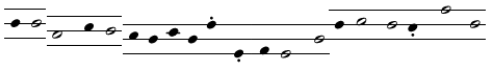
C 

H 

D 

I 

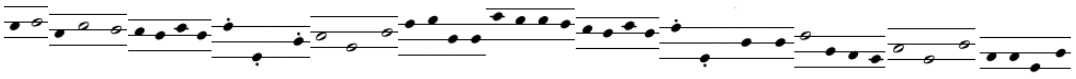
E 

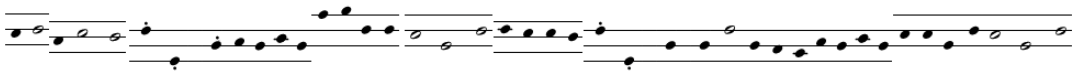
J 

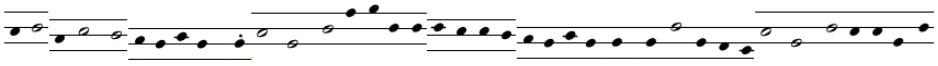
10 points:

Bob wishes to write a message at home in the secret language they made up. However, he left the rulebook at school and can't remember all of the grammar that went along with his vocabulary list. He's drafted three potential sentences to send to Alice, but

unfortunately none of them look right. Explain to Bob what's wrong with each sentence, and write Bob's message in the secret language correctly.

(1) 

(2) 

(3) 

COMPUTER SCIENCE

This month, we're excited to offer another lecture: **Defensive Programming** by Yuri Salkinder. When it comes to unexpected scenarios, our code doesn't always behave the way we thought it would! Come learn about how to protect your code against tricky test cases and minimize points of failure. No advanced computer science knowledge required!
This lecture will be offered at **3 PM EST on Sunday, January 22nd**, and you can join here: https://sigmacamp-org.zoom.us/j/7678155593.

Additionally, you can find some of our favorite resources here if you're looking for great places to get some new knowledge!

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files named as specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.
  No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!

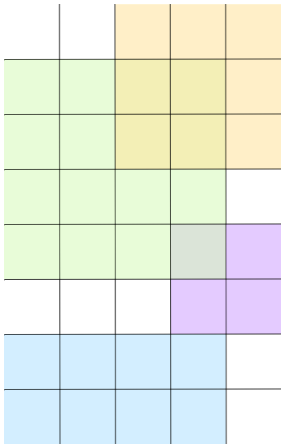
Alice has a table with width w and length l. ALice has arranged t tablecloths on the table, where each tablecloth starts at position (x_i, y_i) and ends at position (i_i, j_i). For every tablecloth, $x_i < i_i$ and $y_i < j_i$ and x_i, i_i, y_i, j_i are all integers. All tablecloths are rectangles.

This month, the 5 and 10 point problems will use the same input format. Your program should receive the input file **input.txt**, which will consist of t + 2 lines. The first line will contain the width and length of the table separated by a space, and the second will contain the number of tablecloths. The subsequent lines will contain x_i, y_i, i_i, j_i for each tablecloth, all separated by spaces, and not necessarily sorted in any way.

Example input file:

```
5 8
4
0 1 4 5
0 6 4 8
2 0 5 3
3 4 5 6
```

This input file would look like the following table setup:



5 points: Write a program that receives the input as described above and determines whether or not the entire table is covered. Your program should produce the output file **output.txt**, containing 1 line with “YES” if the table is covered, and “NO” if it is not.
Example output file:

```
NO
```

In the example input, the table is not completely covered.

10 points: Write a program that receives the input as described above and calculates the maximum area of the table that can be covered if Alice removes one tablecloth. All tablecloths are removable. Your program should produce the output file **output.txt**, containing 1 line with the maximum number of squares of the table that can be covered.
Example output file:

```
29
```

In the example input, a maximum of 29 squares can remain covered with the removal of one tablecloth.