Problem of the Month

POM

2022-2023

January, 2023

MATHEMATICS

Alice, a counselor at Sigma Camp, is handing out gloves for a chemistry semilab. She has *b* different boxes of gloves, each of a different color. Each camper (there are *c* campers) has a preference for the color and the size (small, medium, or large), but Alice is particularly tired, and randomly selects the color and size she gives to each camper (the two gloves in the pair are of the same color and size).

5 points: What is the probability that every camper gets either their preferred color or their preferred size (or both)? (Don't worry: Alice will never run out of gloves)

Hint: No Hint

Answer: $(⅓)^c+(1/b)^c-(1/(3b))^c$

Solution: The probability of an individual camper getting their preferred color is 1/b, and the probability of an individual camper getting their preferred size is ⅓. If there are c campers, the probability of every camper getting their preferred color is $(1/b)^c$ and the probability of every camper getting their preferred size is $(⅓)^c$. Size and color are independent, so we can add these probabilities - but make sure to subtract out the probability of the campers all getting exactly what they want as this is double-counted. So we get the answer of $(⅓)^c+(1/b)^c-(1/(3b))^c$.

10 points: On average, how many colors does she end up handing out?

Hint: No hint.

Answer: $b – b(1 – 1/b)^c$

Solution: We can think of the number of colors Alice ends up handing out as the sum of n random variables, where each one represents if Alice handed out a particular color (set to 1 if the color is selected and 0 if it is not). The probability that Alice does not select a particular color is $(1 – 1/b)^c$, so the expected value of each random variable is $1 – (1 – 1/b)^c$. The total expected value is therefore $b – b(1 – 1/b)^c$ using linearity of expectation.

Challenge problem (no points given - do not submit your solution): Alice makes all the campers write their names on their lab coats (no two campers have the same name), but the next day, all the campers randomly select a lab coat. Alice makes each camper stand next to the person who took their lab coat, forming several circles of people. What is the expected number of circles formed? For instance, if there are 4 campers (Alex, Bob, Charlie, and Dave), and Alex took Charlie's coat, Bob took Alex's, Charlie took Bob's, and Dave took his own, then the two circles that would be formed would be Alex-Bob-Charlie and Dave.

Challenge solution: This problem is equivalent to the problem of finding the average/expected number of cycles in a permutation of {1, …, n}. Write the number of cycles N as sum_{i=1}^n 1/X_i, where X_i is the length of the cycle that contains i.
Then use linearity of expectation:

E[N] = E[\sum_{i=1}^n 1/X_i]

= sum_{i=1}^n E[1/X_i].

We then calculate:

E[1/X_i] = sum_{k=1}^n 1/k P(X_i = k)
= sum_{k=1}^n 1/k * (n-1)*...*(n-k+1) *(n-k)! / n! (the (n-1)*...*(n-k+1) picks the k-1 other elements of the cycle that i is in)
= sum_{k=1}^n 1/(nk)

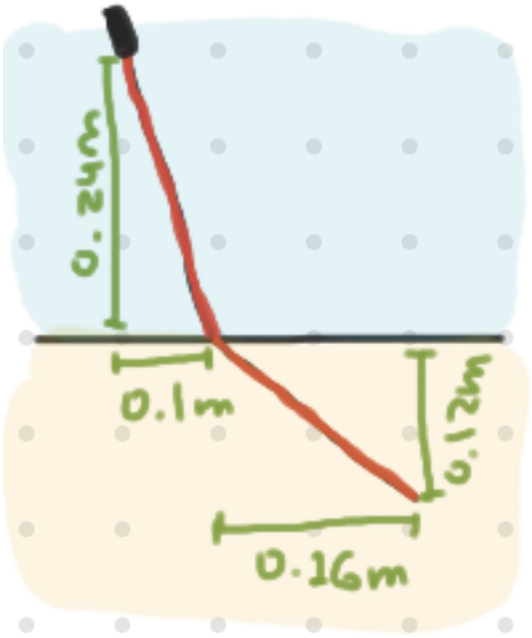
So:

E[N] = sum_{k=1}^n n/(nk)
= sum_{k=1}^n 1/k,

which is the n’th harmonic number.

PHYSICS

5 points (Snell's Law): Alice shines a laser through two substances as shown below. What is the ratio in the indices of refraction?



Solution: We remember Snell's law: when light crosses between media of optical densities n_1 and n_2, its path will satisfy

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Sine is opposite over hypotenuse. Remembering the Pythagorian triples (5,12,13) and (3,4,5) we can further deduce

$$\sin\theta_1 = \frac{0.1}{\sqrt{0.1^2 + 0.24^2}} = \frac{5}{13}, \quad \sin\theta_2 = \frac{0.16}{\sqrt{0.16^2 + 0.12^2}} = \frac{4}{5}$$

Allowing us to write

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{52}{25}$$

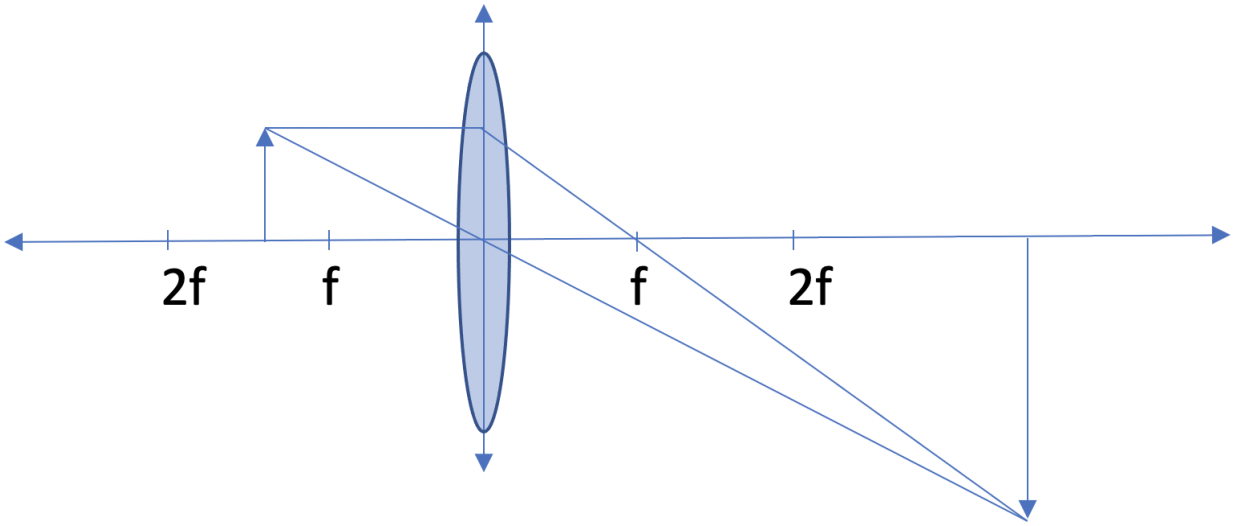
10 points (Imaginary Images): A **convex** lens can produce what is known as a 'real' or an 'imaginary' image. An image is called 'real' because it corresponds to the focusing of rays at a physical point on the lens's opposite side with respect to the source (for a mirror, the reflected rays can also form a 'real' image if they coalesce, but it would be on the object's side of the mirror and not the opposite side). For example, if you use a magnifying glass to focus rays from the sun to burn a piece of paper, you are using a real image to do so.

(a) Use ray tracing diagrams to describe the difference between real and imaginary images. Clearly explain the phenomenon of an imaginary image. What determines whether a produced image is imaginary or real?

(b) Describe the process by which, as you change a parameter (for example, the distance from the object to the lens), an image goes from real to imaginary. Can you describe it as a continuous process, relating it to the geometric concept of the point at infinity?

Solution:

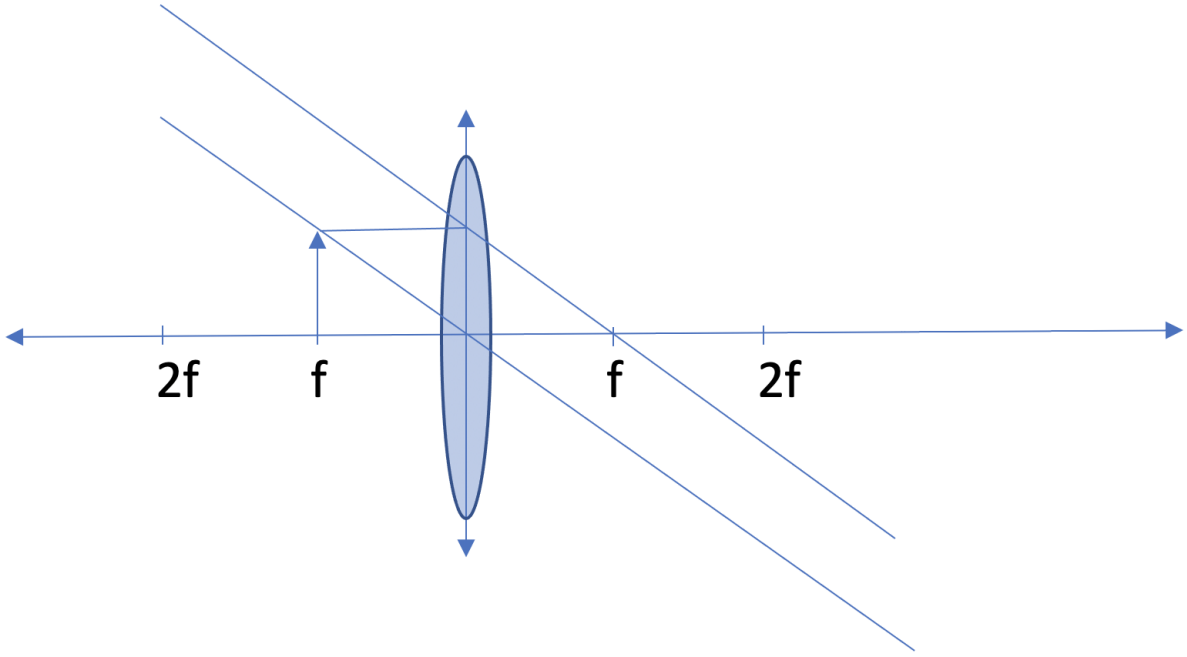
A ray tracing diagram for a convex lens with a real image:



In a real image, the rays from one point will come together at a point (intersect). We are assuming a lens is an ideal lens. This means that to find where all rays will cross, we just need to check two cases to find a crossing point, and the other rays will also converge to that point by the nature of the correctly-shaped lens. There are two natural rays to chose to find where all rays will cross. First ray is the one that is parallel to the axis of the lens. We know that parallel rays focus at the focal plane (distance f away from the lens). By

symmetry, we then know the ray parallel to the lens axis will go through the point distance f away on the lens axis. The other “easy” choice for the second ray is through the center of the lens: Assuming the lens is thin and we can ignore the displacement due to refraction in the center of the lens, the ray travels straight through the lens. You can see above the image forms on the other side of the lens. It is worth noting there is also a special point at 2f, where the image formed will be real and exactly the same size as the object. Repeating the same ray tracing diagram for an image closer than f to the lens, we see the rays do not intersect at all, but, from the observer on the other side of the lens, appear to cross on the same side of the image. Since the rays do not actually cross, the image is called imaginary, since no physical concentration of light at a point actually takes place.

As we sweep the object through the focal distance (from d > f), we start with a real image, and the image size is growing and getting further and further away as d approaches f. When d equals f, the real image is infinitely large and far away, and at the same time one can say the imaginary image forms, infinitely large and far away, but flipped and on the other side.



The point at infinity is the mathematical concept of the infinitely-far-away point where parallel lines cross. Actually, there are two points, on either side at infinity. One corresponds to the real image, one to the imaginary.

If you are interested in the point at infinity, may find the following blog post on the real projective plane interesting:

https://pointatinfinityblog.wordpress.com/2016/04/11/points-at-infinity-i-projective-geometry/

CHEMISTRY

When Alice came to the lab, she noticed Bob, her lab assistant, who was reading an old book.
“Where did you find this old folio, Bob?” Alice asked.
“In our library. That is a captivating reading! Look, they describe a recipe of Philosopher's stone. Read this:”

The text was as follows:

Oh, thou, my faithful servant! Heed well the wisdom I impart unto thee, as well as my warnings, for the path thou art treading upon can lead to great accomplishments, but this road is fraught with secrets and dangers, especially if thou turnest thy thoughts away from the Lord. I shall reveal unto thee my greatest secret, and my greatest achievement, the one that lifted me to the heights and brought me low. And may this tale serve as a lesson to thee. I shall tell thee of the preparation of the Lesser Elixir, that is, the Minor Philosopher's Stone, which doth not grant eternal life, doth not cure diseases, and doth not transform mean metals into gold, but only doth turn Venus into the Moon.

This is how it is done: on the night when retrograde Mercury changeth its course and hasteneth after the Moon, immediately after the last rooster crows, take a large retort, and an alembic, and flasks, and small vessels, and ascend to the open rooftop, and, having gazed upon the heavens, offer a prayer to the Blessed Virgin. It is of utmost importance to behold the heavens during thy Anabasis, for I have discovered that demons love closed chambers, and they may reward thee with a headache, fever, and cloud thy mind if thou secludest thyself from the World of God in thy cellar. But if thou dost thy experiments under the shadow of the Heavens of God, then the Blessed Virgin shall protect thee.

Now, take two pounds of common saltpeter, grind it carefully in a mortar, and place it in a large flask. Then take two pints of vitriol oil, which thou hast previously prepared by distilling blue vitriol, and add it carefully to the saltpeter. Shake the flask, attach the alembic to it, and put in on the oven, and kindle a small fire in the oven. Heat everything carefully until noon, and if the Savior and the Blessed Virgin are with thee, thou shalt collect about a pint of Agua Fortis. Next, put three ounces of finely ground cinnabar into a small retort and add a little Aqua Fortis to it, so that the latter completely wets the cinnabar. Place the retort on a small fire, and heat it until the first Red Dragon is expelled from the vessel. If thy efforts are unsuccessful, offer up a prayer to the Blessed Virgin, and add one spool of salt to the retort, and the oil of vitriol, quantum satis. After

the first Dragon devours his tail, remove the retort from the fire and add more Aqua Fortis to it. Heat the retort again. Repeat these actions for several days and nights tirelessly, until all the dragons devour their tail and leave the vessel. Then carefully heat the mixture to let most Aqua Fortis evaporate and leave the retort. But do not heat too much, lest thou obtain the Caput Mortum, which is by no means thy goal. Remove the retort from the heat and let it cool. Place the resulting residue in a regular pot and let the moisture soak into the clay. In the morning, take what is left, and it shall be the Lesser Elixir. Pulverize it, and to test its power, take half an ounce of the Elixir, mix it with water, and rub it on common coppers. Before thou hast time to recite the Lord's Prayer, thy copper coins shall turn into pure silver, and they shall shine brighter than the full Moon.

This discovery is the main achievement of my life, my main happiness, and my disaster, my incredible luck, and my terrible punishment. For I realized that my labors had turned mine heart away from God, from the Most Holy Trinity and the Virgin Mary, and that the sight of that silver had drained mine soul. And for this sin, our Lord punished me with a terrible punishment: mine hands were covered with scabs, and they no longer obey me, so I have to dictate this manuscript to the scribe. Mine vision is clouded, and mine mind is leaving me. Mine chest is tight, I am breathing with difficulty, and I am feeling that I will soon stand before the gates of St. Peter, and I will hear the inquiring voice of our Heavenly Father. And in mine last hour, I conjure: O Thou, mine unknown follower, the one who is reading these lines! May thy thoughts be pure on thy path, and may our Lord and all the Holy Saints be with thee. Trust in them, and do not be overwhelmed by passions and pride. And may thine be more successful than mine. Amen."

“Interesting”, - Alice noted after having read the text. “It seems old Alchemists already knew about basic safety rules. But I think you are not going to reproduce this experiment in our lab, aren’t you, Bob?”

5 points:

What did Alice mean when she mentioned “safety rules”, and why didn't she want Bob to reproduce this experiment in her lab? Is there any rational explanation for God's punishment inflicted on the author of the recipe, and if yes, explain what caused his suffering?

Solution:

To answer this question, we need to keep in mind that cinnabar is a mineral composed mostly of mercury sulfide. Mercury is extremely toxic, and its toxicity is due to its capability of making a tight chemical bond with sulfur and similar residues in the proteins our body is composed of.

In cinnabar, mercury atoms are already bound to sulfur, so the mercury's affinity to sulfur has already been saturated. That makes cinnabar absolutely non-toxic.
However, Alice correctly noted that the experiments described in that old manuscript yield some soluble mercury compound, and this compound may contaminate the lab space for a very long time, because traces of mercury are very hard to get rid of. That is why she prohibited Bob doing that experiment.
The symptoms of "God's punishment" described in the manuscript are very similar to the symptoms of mercury poisoning, which was quite expectable: the unknown alchemist took no precautions during preparation of his *lesser elixir*.

10 points:
Explain all chemical transformations described in this document. What did he mean under "Red Dragon"? Why was salt needed? Which operations and precautions were necessary, and which were not? What does "but only doth turn Venus into the Moon" mean? What was the true reason for the health problems described in this manuscript?

Solution:

Conversion of a copper coin to a "silver" coin is a well known experiment, which was very popular in the 20th century. Now people have become much more cautious, and most experiments that involve mercury are prohibited.

In this experiment, mercury is displaced from mercury chloride by copper, so the copper goes to the solution, and mercury metal precipitates on the coin surface.

$Cu + HgCl_2 \rightarrow CuCl_2 + Hg$

Copper and mercury form an amalgam (a solution of some metal in mercury), which may be a solid, which covers the coin surface with a thin silverish film. It looks like a fresh silver surface. Alchemists believed that only seven metals existed (gold, silver, mercury, copper, iron, tin, and lead, and they knew about seven celestial bodies (Sun, Moon, Mercury, Venus, Mars, Jupiter, and Saturn). They believed each metal is associated with its planet, and Venus was linked with copper, whereas the Moon was linked with silver. That is why conversion of copper to silver was described as "transmutation of Venus to the Moon".

To make mercury chloride (or mercury nitrate, or some other salt), it is not sufficient to treat cinnabar with some strong acid. This reaction:

$HgS + 2HCl \rightarrow HgCl_2 + H_2S$

does not go: sulfur binds to mercury too tightly, so the equilibrium is shifted to the left side of the equation. To transform cinnabar into a soluble mercury salt, we need to use an acid AND some oxidizer that will oxidize H_2S, thereby removing it from the reaction mixture. Theoretically, nitric acid (HNO_3) can meet these conditions, for it is both an acid and an oxidizer.

However, even nitric acid is insufficiently active to efficiently oxidize cinnabar. Even if mixed with sulfuric acid, this reaction is very slow. Some stronger oxidizer is needed. In the 16th century, a Swiss-born alchemist Paracelsus discovered that cinnabar actively dissolves in *aqua regia,* a mixture of nitric and hydrochloric acids. *Aqua regia* is a very corrosive liquid that reacts even with gold. That is because a nitrogen oxychloride (NOCl) forms when HCl and HNO_3 react:

$2HCl + 2HNO_3 \rightarrow 2NOCl + O_2 + 2H_2O$

Nitrogen oxychloride reacts with cinnabar much more actively than HNO_3.

$HgS + 2NOCl \rightarrow HgCl_2 + H_2SO_4 + NO$

NO forms as a byproduct, but this oxide is not stable, so it immediately reacts with an atmospheric oxygen:

$2 NO + O_2 \rightarrow 2 NO_2$

Nitrogen dioxide (NO_2) is a gas with intense red color, and its stream looks like a dragon. That is what the alchemist meant under "the dragon".

Our anonymous alchemist didn't add HCl to the nitric acid, which he prepared from saltpeter (i.e. potassium nitrate) and vitriol oil (i.e. sulfuric acid). Instead, he added table salt instead. NaCl reacts with HNO_3, so some HCl formed directly in the reaction vessel:

$NaCl + HNO3 \rightarrow HCl + NaNO_3$

This HCl reacted with excess HNO3, as described above. Both acids are volatile, so they evaporate after prolonged heating. The remaining solid was a mixture of $Hg(NO_3)_2$, $HgCl_2$, and $NaNO_3$. This solid is what the unknown alchemist called *lesser elixir*.

Of course, this *lesser elixir*, as well as “silver” coins were highly toxic, so the “God’s punishment” is mercury poisoning.

BIOLOGY

Beogulf the Bald, king of Orimentalia, was tormented by doubts. He had to choose a wife for his beloved son Tirstan. Marriage to Marilda, daughter of Botton XIV, the powerful king of Laurasia, promised to be the basis for a powerful alliance against the evil Lamblia (an aggressive and annoying kingdom that dreams of tearing away and annexing the southern provinces of Orimentalia). But instead of Marilda, Thirstan could have married Aminalia, princess of Lamblia, a wealthy kingdom that controlled important trade routes to the southern lands rich in species, gold and ivory.
But more importantly, Beogulf was worried about the future of his dynasty. The current Orimental dynasty was almost interrupted because Beogulf's grandfather, Geowulf III the Thin-Legged, suffered from a disease causing his blood to not clot; so even a minor cut could cause severe bleeding, which was very difficult to stop. He was living surrounded by the best doctors, but, despite all conceivable precautions, he died at the age of 19, after he accidentally pierced his finger with an ordinary fork. Fortunately, Geowulf's only son Tuvalett I (Beogulf's father) turned out to be quite healthy. Nevertheless, doctors advised Beogulf to pay special attention to the health of Tirstan's future sons, and to this end, pay special attention to the health of his future wife.
Yesterday, Beogulf the Bald received two messages from his spies in Lamblia and Laurasia, and they were not encouraging. Spies reported that the Count of Carran-d'Aché, the younger brother of Botton XIV (and Marylda’s uncle), had two sons, and one of them had recently died of the same illness that killed Geowulf. Another spy reported that Aminalia's cousine (a daughter of a sister of Aminalia’s mother) has three sons, and one of them appears to be suffering from the same condition.

5 points:, If you were Beogulf's medical adviser, considering the nature of the disease and the family history, whom (Marylda or Aminalia) would you recommend? Explain your answer.

Answer:
The symptoms of the disease suggest that it is hemophilia. It is a X-linked recessive inherited disorder. It happens when the person has a dysfunctional gene that encodes the protein responsible for blood clotting. Normally, each gene is present in two copies, because we have a double set of chromosomes. And if one copy of the blood clotting factor has a defect, another copy is still functional, so the person stays healthy. That is why this disorder is considered “recessive”. The word “X-linked” means that the gene that causes hemophilia is located in the X-chromosome.

A girl can have hemophilia only if her mother is a carrier (has at least one X-chromosome with a dysfunctional clotting factor) and her father is sick (has hemophilia). This situation is extremely rare.

To summarize: *only males suffer from hemophilia, and only a sick man can be a carrier of this disease. A man transfers hemophilia only to daughters, and the daughter becomes a carrier with a 100% probability. Females do not suffer from hemophilia unless their mother was a carrier and father was sick. If a healthy woman is a carrier, she transfers the disease to her daughter with 50% probability, and her son may be sick with 50% probability.*

A situation with males is different. Males have only one X-chromosome, which is inherited from a mother. If a mother has one X-chromosome with a dysfunctional blood clotting factor, her son may be sick with a probability of 50%. And, obviously, since the father does not pass his X-chromosome to his son, a man cannot transfer this disease to his male offspring.

In this problem, the disease Geowulf III was suffering from is irrelevant: males do not transfer hemophilia to their male offspring. Indeed, Tuvalett I was healthy, and he was not a carrier of the mutant gene. The news about Carran-d'Aché's son is irrelevant too: hemophilia is maternally transmitted, which means Carran-d'Aché's son got hemophilia from his mother, who is not Marylda's relative. That means it is quite safe to marry Marylda.

Aminalia's case is more worrying: since a daughter of her mother's sister has a son with hemophilia, it is likely Aminalia may be a carrier.

Therefore, the answer seems obvious: Tristan should marry Marylda.

10 points:

Beogulf the Bald was greatly disappointed by this news and sent several envoys to find a healthier bride for his beloved son. Very strict instructions were given to every ambassador to carefully ascertain the medical pedigree of each candidate and to pay special attention to the disease that had killed Beogulf's grandfather. The reports of the ambassadors revealed a sad picture: this disease appeared quite often in the royal dynasties of nearby kingdoms. However, several candidates looked very promising. Thus Princess Eutectica, the only daughter of Prince d'Ammoniac, had a healthy pedigree, although old archival records showed that the brother of Guinerva (she was the Countess of Gurgogne and the mother of Eutectica's mother) suffered from the same strange disease. Another candidate was Princess Allosterica, the third daughter of Graf Ottoman von Schwarzwaldburgerkingkrighsmarinenhof. The House of Schwarzwald- burgerkingkriegsmarinenhof had a fairly healthy family history, with the only exception being Ottoman's cousin, whose third son is currently battling this strange disease.

Calculate the probability of Tirstan's children (boys and girls) getting this disease depending on if he marries Marilda, Aminalia, Eutectica or Allosterica.

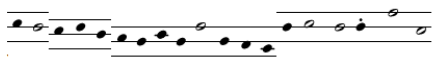
Answer:
As we already know (see the problem #1), the probability that Marylda and Tristan may have a son with hemophilia is 0 (assuming that his other female ancestors were healthy).
To calculate a probability for Aminalia, we must keep in mind that if a mother is a carrier, she has one healthy X-chromosome and one mutant X-chromosome. The probability to pass a mutant X-chromosome to her daughter is 50%. Accordingly, the probability of passing this mutation from a grandmother to a granddaughter is 25%, and so on.
We know that a sister of Aminalia's mother has a son who is sick. That means Aminalia's cousine is a carrier. She got this mutation from her mother, and her mother got it from her mother (who is Aminalia's mother too). That means there is a 25% probability that Aminalia is a carrier. The probability that a mother who is a carrier of the hemophilia gene can have a sick son is 50% (because the probability to inherit each of two X-chromosomes is equal). If a mother is a carrier with a probability of 25%, her son may be sick with a probability of 12.5%. Therefore, for Aminalia, the answer is 12.5%.
The probability that Eutectica may have a sick son can be calculated keeping in mind that Guinerva (Eutectica's grandmother) has a sick brother. That means Eutectica grandgrandmother (Guinerva's mother) was a carrier. From that, it is easy to calculate that Eutectica is a carrier with a probability of 12.5%, and her son may be sick with a probability of 6.25%.
With regard to Princess Allosterica, she, like Marylda, has no maternal ancestors with hemophilia, so the probability to have a sick son is zero.

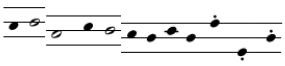
LINGUISTICS/APPLIED SCIENCE

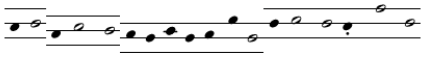
5 points:

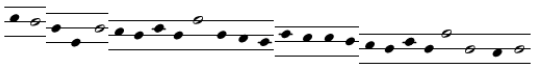
Bob and Alice are friends and like to pass notes to each other during class. They got tired of other students (and sometimes the teacher) intercepting and reading their messages so they invented their own language to keep everything super secret. Match the following English sentences pertaining to Bob and Alice's juice box negotiations to the sentences in their secret language.

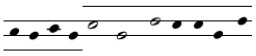
1. Put the blue box there.
2. Put the yellow box inside the house.
3. The box is outside the house.
4. Open the blue box and the red box.
5. Close the red box.
6. I want the bottom box here.
7. Do you want the red box?
8. I have no boxes.
9. You have two boxes.
10. She has the most boxes here.

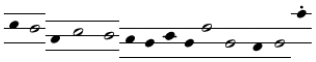
A 

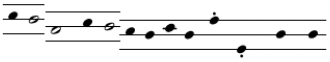
F 

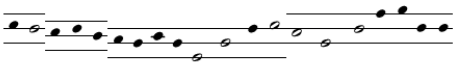
B 

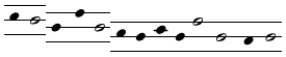
G 

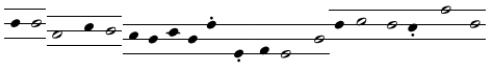
C 

H 

D 

I 

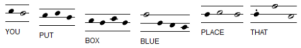
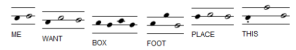
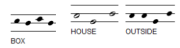
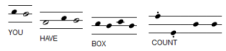
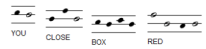
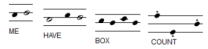
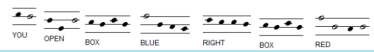
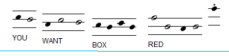
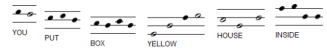
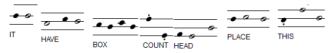
E 

J 

Solution:

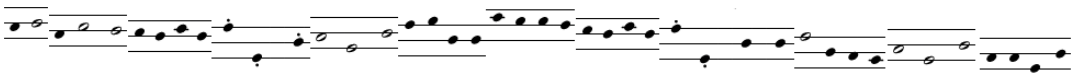
This problem uses a constructed language called Moss, which uses combinations of two to four music notes as words. Moss also has a limited vocabulary. Here are the

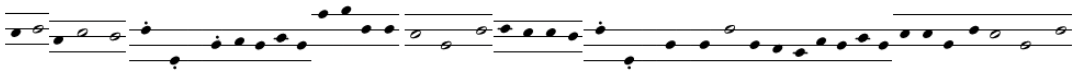
sentences in Moss, their literal translations, and the matching proper English translations

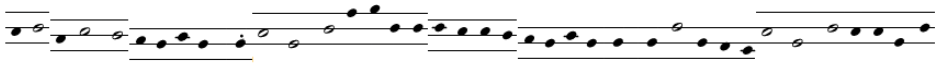
A		1. Put the blue box there.
B		6. I want the bottom box here.
C		3. The box is outside the house.
D		9. You have two boxes.
E		5. Close the red box.
F		8. I have no boxes.
G		4. Open the blue box and the red box.
H		7. Do you want the red box?
I		2. Put the yellow box inside the house.
J		10. She has the most boxes here.

10 points:

Bob wishes to write a message at home in the secret language they made up. However, he left the rulebook at school and can't remember all of the grammar that went along with his vocabulary list. He's drafted three potential sentences to send to Alice, but unfortunately none of them look right. Explain to Bob what's wrong with each sentence, and write Bob's message in the secret language correctly.

(1) 

(2) 

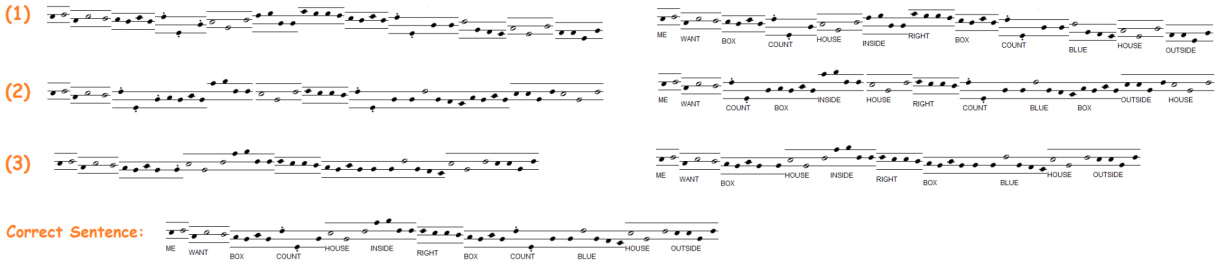
(3) 

Solution:

Sentence 1 misplaces the heights of each word - words in a sentence in Moss go down by half steps, except for adjectives and prepositions which are level with the nouns they are modifying, and conjunctions like ‘and’ (literally translated as ‘right’) which go up by a whole step.

Sentence 2 swaps the nouns and adjectives/prepositions - in Moss, those words go after the nouns they’re modifying

Sentence 3 does not properly use the numbers system - in Moss, numbers are denoted by the word ‘count’ and then a repeated middle note that repeats as many times as the number is (1 to 9)



COMPUTER SCIENCE

This month, we're excited to offer another lecture: **Defensive Programming** by Yuri Salkinder. When it comes to unexpected scenarios, our code doesn't always behave the way we thought it would! Come learn about how to protect your code against tricky test cases and minimize points of failure. No advanced computer science knowledge required!
This lecture will be offered at **3 PM EST on Sunday, January 22nd**, and you can join here: https://sigmacamp-org.zoom.us/j/7678155593.

Additionally, you can find some of our favorite resources here if you're looking for great places to get some new knowledge!

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files named as specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.
  No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!

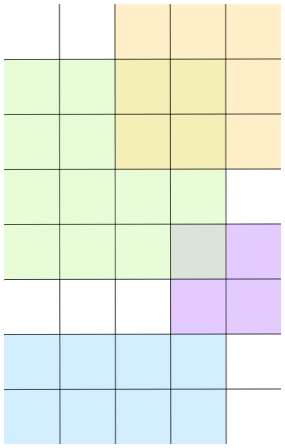
Alice has a table with width w and length l. Alice has arranged t tablecloths on the table, where each tablecloth starts at position (x_i, y_i) and ends at position (i_i, j_i). For every tablecloth, $x_i < i_i$ and $y_i < j_i$ and x_i, i_i, y_i, j_i are all integers. All tablecloths are rectangles.

This month, the 5 and 10 point problems will use the same input format. Your program should receive the input file **input.txt**, which will consist of t + 2 lines. The first line will contain the width and length of the table separated by a space, and the second will contain the number of tablecloths. The subsequent lines will contain x_i, y_i, i_i, j_i for each tablecloth, all separated by spaces, and not necessarily sorted in any way.

Example input file:

```
5 8
4
0 1 4 5
0 6 4 8
2 0 5 3
3 4 5 6
```

This input file would look like the following table setup:



5 points: Write a program that receives the input as described above and determines whether or not the entire table is covered. Your program should produce the output file **output.txt**, containing 1 line with “YES” if the table is covered, and “NO” if it is not.
Example output file:

```
NO
```

In the example input, the table is not completely covered.

Solution:

```
with open("input.txt") as input_file:
    width, length = [int(n) for n in input_file.readline().split()]
    num_tableclothes = int(input_file.readline())

    dimensions = []
    for _ in range(num_tableclothes):
        dimensions.append([int(n) for n in input_file.readline().split()])

covered = [0 for _ in range(width * length)]
for x0, y0, x1, y1 in dimensions:
    for x in range(x0, x1):
        for y in range(y0, y1):
            covered[x + y * width] = 1

answer = "NO" if 0 in covered else "YES"
```

```
with open("output.txt", "w") as output_file:
   print(answer, file=output_file)
```

10 points: Write a program that receives the input as described above and calculates the maximum area of the table that can be covered if Alice removes one tablecloth. All tablecloths are removable. Your program should produce the output file **output.txt**, containing 1 line with the maximum number of squares of the table that can be covered.
Example output file:

```
29
```

In the example input, a maximum of 29 squares can remain covered with the removal of one tablecloth.

Solution:

```
with open("input.txt", 'r') as file:

   # width, length, and number of tablecloths, to be set later.
   w, l, t = 0, 0, 0

   # x_i, y_i, i_i, j_i for each tablecloth, to be set later.
   left, right, top, bottom = [], [], [], []

   # Count up the total area
   area_so_far = 0

   # a 2D array where covers[y][x] is the number of tablecloths covering
   # the square (x, y)
   covers = []

   for current_line, line in enumerate(file):

       # read in w, l
       if current_line == 0:
           w, l = (int(x) for x in line.split())
```

```
        # fill covers with 0s
        for i in range(l):
            covers.append([0 for _ in range(w)])

    # read in t
    elif current_line == 1:
        t = int(line)

    # read in x_i, y_i, i_i, j_i
    else:
        dimensions = line.split()

        left.append(int(dimensions[0]))
        top.append(int(dimensions[1]))
        right.append(int(dimensions[2]))
        bottom.append(int(dimensions[3]))

        # update area_so_far and covers
        for x in range(left[-1], right[-1]):
            for y in range(top[-1], bottom[-1]):

                # if (x, y) was previously not covered, then increment area
                area_so_far += (covers[y][x] == 0)

                covers[y][x] += 1

# To determine the maximum area after removing one tablecloth, we will
# iterate through each tablecloth and count the number of squares that
# are only covered by this tablecloth (when it is removed, that square will
# be uncovered). This is equivalent to counting the number of 1s in the
# bounds of the squares that the tablecloth covers. We find the minimum of
# the number of 1s, then subtract it from our total area we found earlier.
min_num_ones = 0


for i in range(t):
    num_ones = 0

    # add up the number of 1s
```

```
    for x in range(left[i], right[i]):
        for y in range(top[i], bottom[i]):

            # If covers[y][x] == 1, increment num_ones.
            num_ones += (covers[y][x] == 1)

    if i == 0 or num_ones < min_num_ones:
        min_num_ones = num_ones

# write to file
f = open("output.txt", "w")
f.write(str(area_so_far - min_num_ones))
f.close()
```