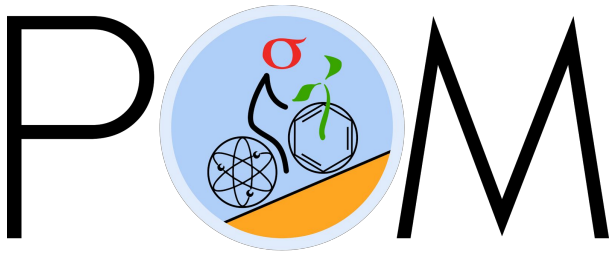
PROBLEM OF THE MONTH



November, 2015

MATHEMATICS

5 points:

Imagine that you live in a 4D world. You are asked to make a wire model of the 4D cube (called hypercube) of size 10x10x10x10 cm. How much wire is needed? Wire is only used to make one-dimensional edges of the hypercube.

Hint:

Introduce a coordinate system, and find the coordinates of all vertices of the hypercube. This will help you to count the number of edges

10 points:

Imagine that you live in D dimensions. You are asked to make a wire model of a D-dimensional cube (called hypercube) of size 10x10x...x10x10 cm. How much wire is needed? Wire is only used to make one-dimensional edges of the hypercube.

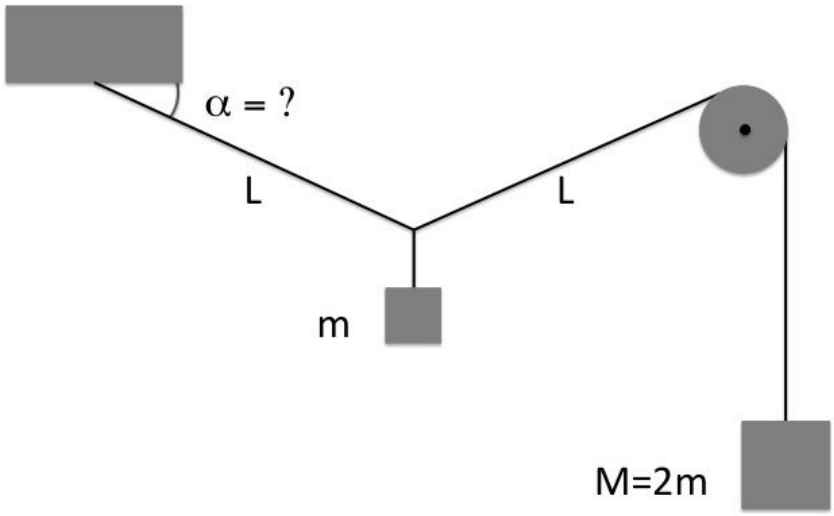
Hint:

Introduce a coordinate system, and find the coordinates of all vertices of the hypercube. This will help you to count the number of edges

PHYSICS

5 points

Two weights of masses m and M=2m are attached to an ideal weightless string. One end of the string is attached to a ceiling while the other one goes around the ideal weightless pulley. Find the angle α between the string and the horizontal line in equilibrium (see figure).



Hint: The sum of forces acting on each weight should be equal to zero in equilibrium. The tension of the rope on both sides of the pulley is the same.

10 points

A lawn sprinkler spraying water oscillates at a constant rate up-down, from 15 to 75 degrees to the horizon. The water's exit velocity from the sprinkler is 10m/s. What fraction of water will be wasted by spraying it on a wide road whose curb is at the distance 9m from the sprinkler. The road is running perpendicular to the direction of the sprinkler.

Hint: Use the range of a projectile formula to figure out which angles correspond to "wasting water".

CHEMISTRY

5 points:

Imagine you are playing the Escape the Room game. In this room, you found various things including:

- porcelain cups, glasses, and dishes;
- pair of goggles and latex gloves;
- a cotton pad;
- a horseshoe magnet;
- a sheet of dirty paper, letter size, with scrawls saying “Nothing interesting”;
- a roll of pH paper, the scale attached to the roll says it measures pH from 1.0 to 6.0;
- a book “The Bottle Imp” by Robert Louis Stevenson (it is also available online at http://gutenberg.spiegel.de/buch/the-bottle-imp-4357/1);
- a printed brochure “Making invisible ink” (also available online at http://www.rsc.org/learn-chemistry/resource/download/res00001178/cmp00002225/pdf)
- a box of matches;
- four amber glass bottles; each bottle contains some transparent odourless liquid; there are four small labels on the table near these bottles; these labels look like they were attached to these bottles, but have fallen off later; it is not possible to tell for sure which label belongs to which bottle. The labels say : “green tea + Na_2SO_4”, “NaOH”, “H_2SO_4”, “black tea + $CaCl_2$”, and “$FeCl_3$”.

You already have opened nine locks, but to exit the room you need to unlock the final lock. You need to find the code that unlocks it.
What will you do to obtain the code?

Hint: Whereas “The Bottle Imp” can hardly help you in obtaining the secret number (by writing that I do not mean it is not worth reading) the “Making invisible ink” may be really helpful. It would be also useful to remember that solutions of iron (III) salts have a yellow to brown color. If you that, and if you remember the general properties of metal salts (especially, their reactions with other salts, acids and bases) you will be able to escape the room.

10 points:

Alice and Bob, her technician, have been preparing for tomorrow's class. "Tomorrow, I am going to tell our students about the most common methods of purification of chemical compounds" - Alice said. "I plan to start with recrystallization, which is carried out in the following way: I'll take some compound, for example, some salt, which is contaminated with another chemical (for example, with some dye or with some other salt), and dissolve this mixture in a minimal amount of hot water. Then we will put it on ice for cooling. After some period, crystals of the purified compound will start to form and precipitate, whereas the admixtures remain in solution. Then you Bob will collect the crystals by filtration, and leave them to dry. " "Great", Bob said, "And which compounds should I prepare for tomorrow's demonstration?" "Maybe, you yourself have some ideas on that account?" - Alice said.

"Yes, I do", Bob replied. "Let's take sodium chloride and add a small amount of copper sulfate to it. The initial mixture will be bluish, but upon recrystallization we will obtain clear, transparent and colourless crystals of NaCl. That would be spectacular. Instead of sodium chloride we may take potassium sulfate (K_2SO_4) or cerium sulfate ($Ce_2(SO_4)_3$)."

"I am sorry, Bob, but, although you are thinking in a right direction, only one of your suggestions will work," Alice said.

Please tell if Alice was right. Which compounds proposed by Bob cannot be used to demonstrate the principle of recrystallization, and why?

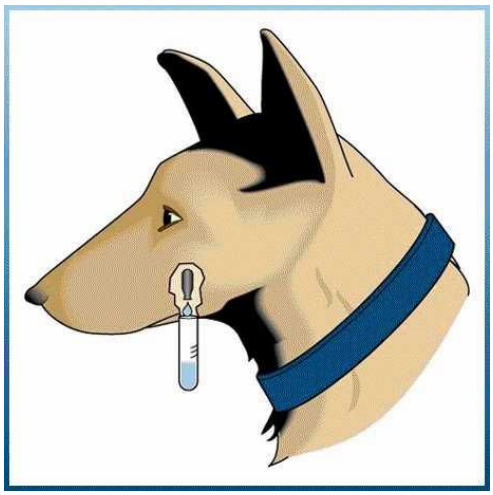
Hint: Classical recrystallization is based on the observation that solubility of most compounds increases at elevated temperature. Is that the case for the salts Bob proposed to use during the next class?

BIOLOGY

5 points:

In the second half of XIX century Russian physiologist Ivan Pavlov studied digestive system in dogs. He implanted a special tube (a fistula) into dog's cheek (see a picture) and measured the amount of saliva produced by a dog in different situations. Surprisingly, he observed dogs start to salivate not only when they are eating, but also when they see a food, or even when they hear a sound indicating the start of feeding time. Based on this and other experiments, Pavlov developed a theory of conditioned reflex, for which he was awarded the Nobel prize in 1904.

Interestingly, when scientists did similar experiments with cats, they observed cats do not start to salivate when they see food, or when they hear the signal indicating a food is coming soon. Usually, cats start to salivate only when they get a food. Can you explain why?



10 points:

In a *symbiotic* relationship, two organisms each receive benefit from living next to one another. *Endosymbiosis*, an extension of this idea, is an evolutionary theory that proposes that several of our key cell components originated as a symbiosis between separate single-celled organisms. One key example of this is mitochondria, which supply energy to our cells, and which may have originated as bacteria.

1. What are the three types of symbiosis? Give an example of each type of symbiotic relationship amongst animals, and explain why it reflects each type.

2. What evidence makes scientists think today that mitochondria may have originated as bacteria?
3. If mitochondria did originate as bacteria, would this have been a symbiotic relationship? If so, which of the three types and why?
4. How are mitochondria different from bacteria today, and what evolutionary mechanism might explain why this is so?
5. How does this relate to the recent scientific finding (July 2013) that antibiotic use damages mitochondria?

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in C, C++, Java, or Python
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Submit the problem in a plain text file, such as .txt, .dat, etc.
  No .pdf, .doc, .docx, etc!

5 points:

Your friend is a bridge engineer, tasked with designing 1000 unique bridges out of ASCII characters. The following are the building blocks:

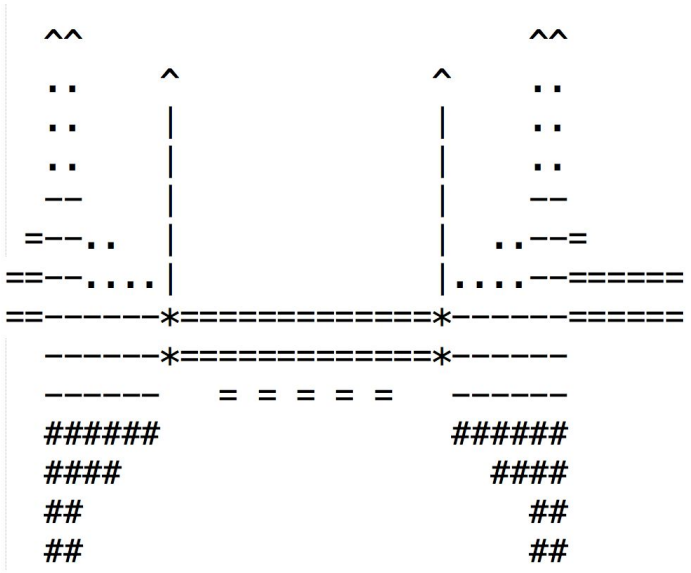
```
^
.
.
.
–
–
–       ^
– .     |       =
– .     | ^     =
– – . . | |     =
# – – * | |     X X
# – – . | | =   X *
# # – * * * = = X X
# # # * * * = = X *
a b c d e f g h i j
```

The engineer constructs the bridge from left to right by sandwiching the building blocks together, controlling how much to offset each block from the bottom. Two blocks cannot be put on top of each other.

Your friend is asking you to write him a program that would allow him to type in a string specifying how to arrange the blocks, and the program would draw the bridge. He proposes you do it as follows: Each block is denoted by its letter (a-j), followed by a number which specified the vertical offset (must be 0 or greater). Thus, the following string:

h6g6a1a0a0b2b2c3c3e5h5h5g4h5g4h5g4h5g4h5tg4h5h5e5c3c3b2b2a1a1g6h6h6h6h6h6

should generate the following output on the screen:



Please write the script to make your friend's life easier. If you feel like it, make a random bridge generator also :)

10 points:

One of important focuses of computer science in the real world is storing data efficiently. For example, here is a simple variant of a real-life problem:

You are tasked with developing a way to "save to disk" the game of checkers. The requirements are as follows:

1) The whole game be stored as a string of text. A string of text is effectively a 1D array of characters.

2) The array should contain information sufficient to re-play every step of the game.

Your solution will be scored by the efficiency of your storage, that is, how few characters you can use to encode an entire game (of course, it varies by length of the game). As a hint, think about how much "information" happens at every step of the game.

As a bonus, write a program to demonstrate how your data can be decoded to display every step of the game. (You could generate text output on screen by letting "." stand for an empty white space, "*" for empty black space, '1' for player 1’s regular pieces, '2' for player 2’s regular pieces, etc.., and draw the board that way).