PROBLEM OF THE MONTH

November, 2018

2018 - 2019

MATHEMATICS

5 points:

A Sigma team consisting of 10 campers forms a team to participate in the Sigma Tournament. Campers have to select 6 people out of their number to solve timed problems at the table during the tournament, so that other 4 campers are sitting on a couch solving written problems. One of the campers sitting at the table should be selected as team captain. In how many different ways one can divide 10 campers into 1 captain + 5 campers at the table + 4 on the couch?

Hint: In how many ways can you select the captain from the team of 10? In how many ways can you select 5 campers out of the remaining 9 campers?

10 points:

There are three campers: one from team alpha, one from team beta and one from team gamma. Every year at Sigma, two of the three campers are randomly chosen and they switch teams.
a) What is the chance that after 2017 SigmaCamps, they will all be on their original teams?
b) After 2018 camps?

Hint: Let us denote the original placement of students as (1, 2, 3), meaning that Student 1 is in team alpha, Student 2 is in team beta, and Student 3 is in gamma. After the first swap you can end up in (1, 3, 2), (2, 1, 3), and (3, 2, 1) with equal probability. Think about what happens with these probabilities during the second step, and so on.

PHYSICS

5 points:

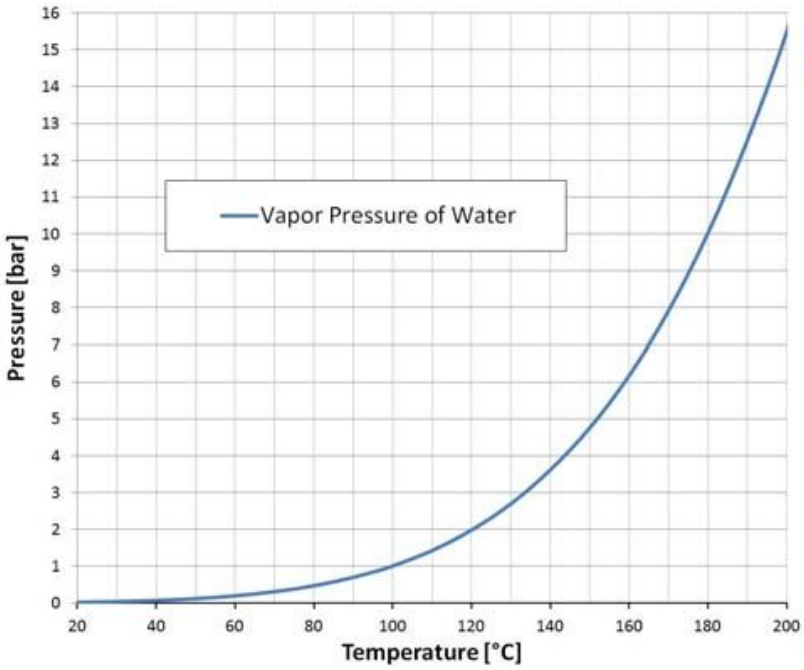
An airgun bullet is 4.4 mm in diameter and weights 0.5 g. It is propelled by a compressed air. Find the speed of the bullet as it exits the gun, if the length of the barrel is 0.5 m, and excess air pressure behind the bullet (with respect to the atmospheric pressure outside) stays nearly constant, around 300 kPa.

Hint: Use the fact that work done by the compressed air all goes to changing the kinetic energy of the bullet.

10 points:

A geyser consists of an underground cavity in which water is heated geothermally, and a channel that connects the cavity to the surface. An eruption occurs when the water in the cavity starts boiling. It stops when the temperature of the water drops below the boiling temperature. Note that the temperatures at which the boiling starts and stops are different because the channel is originally fully filled with water, which leads and additional hydrostatic pressure in the cavity. The channel is completely vertical and has a length of 90m.

Estimate the fraction of the total mass of water in the cavity which is vaporized during a single eruption. Specific heat of water is ***C=4.2kJ/kg°C,*** the latent heat of its vaporization is ***L=2230 kJ/kg***.



The figure on the right shows Pressure vs. water boiling temperature. Unit of pressure in the plot is 1 bar $=10^5$ Pa , which is close to 1 atmosphere (that's why T=100°C corresponds to P=1 bar).

Hint: 1) find the pressure in the cavity in before the eruption, and use the plot to determine the boiling temperature. 2) As the water gets converted from liquid to steam, it consumes heat and cools remaining water in the cavity.

CHEMISTRY

5 points:

"*We had two bags of Arabica coffee, seventy-five ounces of sodium hydroxide pellets, five kilograms of high purity acetic acid, a saltshaker half-full of aspirin, and a whole galaxy of multi-colored pH papers, rubber balloons, strings etc... Also, a quart of concentrated sulfuric acid, a quart of acetone, a case of Poland Spring water, a pint of raw ether, and two dozen grams of isoamyl alcohol. Not that we needed all that for our graphomaniac exercises, but once you get locked into a serious chemicals collection, the tendency is to push it as far as you can. The only thing that really worried me was the ether. There is nothing in the world more helpless and irresponsible and depraved than a man in the depths of an ether binge, and I knew we'd get into that rotten stuff pretty soon.*"

Using the stuff described in this quote, can you prepare a banana smell? Which items listed here are needed for that, and how will you do it?

Hint:

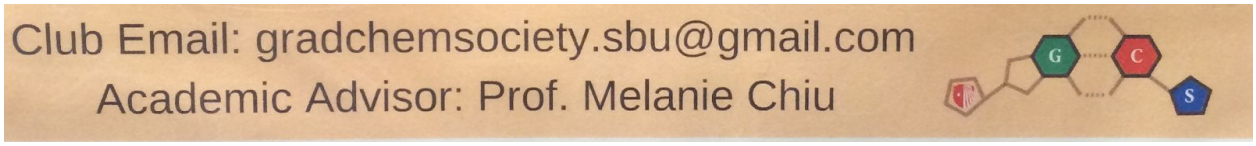
Banana smell is a compound that is an ester of acetic acid. What ester it is, and how can it be prepared? Explain.

10 points:

Alice, a college faculty, came to her lab and found Bob, her technician, reading a Molecular Biology textbook. “What are you doing, Bob”, she asked. “Hi, Alice, I’ve just spoke with one my friend, he told me a lot about recent discoveries in molecular biology. It is so interesting! Ribozymes, the RNA world hypothesis, genome editing using a CRISPR-Cas9 system - all of that sounds like a science fiction. Tomorrow, I meet my friend again, and I have a lot of questions to ask.”

“Who is your friend, Bob?”, Alice asked.

“Oh, he is a really smart guy!”, - Bob exclaimed. “He is a graduate student at Stony Brook university, and he is a member of the Graduate Chemical Society. By the way, he showed me their logo, I like it very much. It looks like some formula, it is so beautiful! Look, Alice”. And Bob showed this photo to Alice:



“Hmm,”- Alice replied, - “Those guys may be good chemists, but I am pretty sure there are no experts in nucleic acid chemistry or biology in this society. I doubt your friend can explain you anything about ribozymes or CRISPR”.

Please, tell why Alice made this conclusion?

Hint:

It seems that the logo is supposed to be an imitation of the Watson-Crick base pair (more concretely, a G-C pair, because it is a **G**rad **C**hemistry society). However, the “Watson-Crick” pair on the logo is shown incorrectly: there are *two* errors there. What these two errors are?

BIOLOGY

5 points:

A genome of some organism contains the fragment with the following sequence:
5' TTATCCATGTGGCATTAGATGTAAGG 3'
3' AATAGGTACACCGTAATCTACATTCC 5'
How many different proteins can a ribosome synthesise from the DNA that contains this segment? In your answer. assume that the start position for the synthesis of each peptide is situated in this segment. Write all possible amino acid sequences of these peptides.

HINT

Note that DNA is double-stranded, and ORF can be present in both directions.

10 points.

One of the color pattern in rats is called "hooded" - animals have dark fur on heads and top of the back, but a belly and legs are white. It has been determined that rats with hooded color pattern have an 1 kb insert in the intron of a *Kit* gene.

An intron is a DNA segment inside some gene. Introns encode no protein sequence; after RNA is synthesized from the DNA, a special cellular machinery cuts the RNA at the beginning and the end of the intron segment, removes it, and re-connects the ends of the cut RNA back; only after that procedure (it is called “splicing”) the RNA is ready for translation.
The *Kit* gene encodes some receptor for a protein which regulates cell division and migration, including migration of melanocyte precursors. We know that *Kit* is important for cell migration during early development, and the introduction of the 1 kb insert into its intron has a direct relation to the hooded color pattern in rats.

Please suggest explanations of how inserting inactive sequence in an intron of *Kit* can cause such dramatic change in phenotype.

HINT Although intron is not coding for amino acids, it can affect what happens with DNA or during gene transcription. Consider how intron can change chromatin structure or which important sequences it might introduce between coding exons.

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc. **No .pdf, .doc, .docx, etc!**

This month we are visiting Sigma2D, two-dimensional Sigma Universe. In Sigma2D, in one of the remote areas, there is a square minefield. The minefield has the shape of a grid with a side of 10, and therefore it has 100 cells in total. Each of the 100 cells can contain one mine or be empty. The minefield has markings on it: there is a number on each cell indicating how many mines there are in the 8 cells (or less, if the cell is at the edge of the minefield) immediately next to it.

5 points:

Write a program that takes on input a 10-by-10 array of 1's and 0's (1 stands for a mine, and 0 for an empty space) and returns an array of the same size, but containing numbers for how many mines there are at each cell's eight (or less) immediate neighbors. The program should terminate with a complaint if the input contains anything other than mines, separators and empty spaces (1s and 0s).

For example, for a mine array of:
1, 1, 1, 1, 1, 1, 1, 0, 1, 0
1, 0, 0, 0, 1, 1, 0, 0, 0, 1
1, 0, 1, 1, 0, 1, 0, 0, 0, 0
1, 1, 0, 1, 1, 0, 0, 0, 1, 1
1, 0, 1, 0, 1, 1, 1, 1, 0, 1
1, 1, 1, 0, 1, 1, 1, 1, 0, 1
0, 1, 0, 1, 1, 1, 1, 0, 1, 1

0, 0, 0, 0, 1, 0, 1, 0, 0, 0
1, 1, 1, 1, 0, 1, 1, 0, 0, 1
1, 0, 0, 1, 0, 0, 1, 1, 0, 0

the program should return the following numbers representing the number of mines around each cell:
2, 3, 2, 3, 4, 4, 2, 2, 1, 2
3, 6, 5, 6, 6, 5, 4, 2, 2, 1
3, 5, 3, 4, 6, 3, 2, 1, 3, 3
3, 5, 5, 5, 5, 5, 4, 3, 3, 2
4, 7, 4, 6, 5, 6, 5, 4, 6, 3
3, 5, 4, 6, 6, 8, 7, 5, 6, 3
3, 3, 4, 4, 5, 7, 5, 5, 3, 2
3, 4, 5, 5, 5, 7, 4, 4, 3, 3
2, 3, 3, 3, 4, 4, 4, 4, 2, 0
2, 4, 4, 2, 3, 3, 3, 2, 2, 1

Hint:

It is convenient for each cell to iterate through all cells with coordinates ranging from the current coordinate-1 to the current coordinate+1. In addition, a helper function for determining whether a particular pair of integers is a legitimate coordinate on the mine field might prove useful.

10 points:

For this problem, your program should perform the "inverse" of what the previous problem's solution did. The input to your program should be a 10-by-10 array containing number of mines in each cell's immediate neighborhood of cells. The program should calculate and return the corresponding array of 1’s and 0’s, depending on whether there is a mine in the current location or not. This array, when given as an input to the 5 point problem above, would have produced the input array of this problem. As an example, for an input of the array of integers shown in the 5 point problem as a sample output, your program should return the array presented in the 5 point problem as a sample input.

Hint:

When there are "uncertain" cells around the current cell, look for the situations when the number of definitively placed mines around the current cell already equal to this cell's value (and therefore all "uncertain" cells definitely do not have mines), or, on opposite, the number of definitively placed mines plus the number of "uncertain" cells equal to the cell's value (and then all "uncertain" cells definitely have mines). Keep doing it iteratively. When you can't make any more progress, start "experimenting" with placing mines in the next "uncertain"cell.