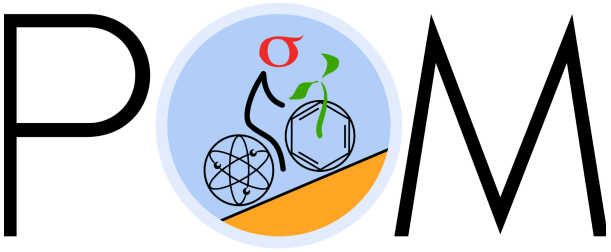
PROBLEM OF THE MONTH



November, 2021

2021 - 2022

MATHEMATICS

10 points:

Find the number of positive integers n larger than 2021 that divide $1^n + 2^n + \ \ldots \ + (n-1)^n$. Show/prove why.

PHYSICS

The Lorentz force describes how electric and magnetic fields act on charged particles: the Lorentz force comprises two components, F_E and F_B. The force from the electric field with magnitude E on a particle with charge q is:

$$F_E = qE,$$

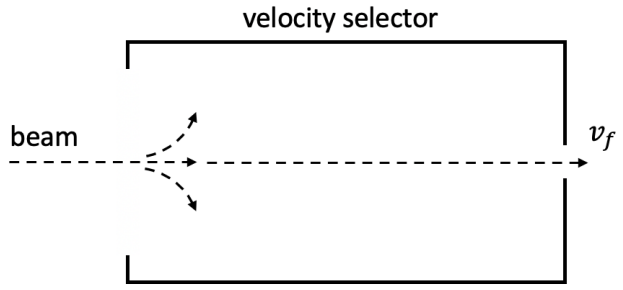
acting parallel to the direction of E. The force on the particle moving with velocity v perpendicular to a magnetic field with magnitude B is:

$$F_B = qvB,$$

acting perpendicular to both v and B (see right-hand rule).

5 points:

Consider a particle accelerator which generates a beam of atoms of different species and a range of velocities. Assume all atoms are singly-ionized, i.e. have lost one electron. A *velocity selector* apparatus is shown in the diagram below. The device consists of a tube with a slit at the end, and uniform magnetic and electric fields are created in the tube. For an appropriately chosen field combination, only one velocity of particles (for any mass) will go through the slit. What must the magnetic and electric fields be (their magnitude and direction) to obtain a specific speed, v_f?



10 points:

Once you have filtered the beam so that all particles are moving with the same speed, how can you use a combination of magnetic and electric fields to sort particles based on their mass? Specifically, your apparatus should translate mass m into a position coordinate x (one can then scan an electric charge meter across a range of positions x and from the electric current infer the mass composition of the beam of particles).

Suppose now that the beam of particles contains a small fraction of doubly-ionized atoms in addition to the majority singly-ionized atoms. If you are trying to detect singly-ionized Carbon-14 atoms, which doubly-ionized atom would give a false-positive on your detector?

CHEMISTRY

5 points:

Diamond and graphite are two allotropic forms of carbon, which can be converted into each other under some specific conditions. Would it be possible to develop a hypothetical enzyme or chemical catalyst for conversion of graphite to diamond at room temperature and normal pressure? Explain your answer.

10 points:

An article in SciNews (http://www.sci-news.com/othersciences/chemistry/liquid-gallium-carbon-dioxide-conversion-10164.html) describes the results of recent studies published in a reputable journal “Advanced materials” (Tang et al. Liquid-Metal-Enabled Mechanical-Energy-Induced CO_2 Conversion. Advanced materials 2021, 2105798). According to those studies, a catalytic transformation of CO_2 back to gaseous oxygen and carbon is possible, and the whole process requires as little as 230 kWh of energy for conversion of one ton of CO_2. Assuming that the heat of combustion of carbon is 32.8 MJ/kg, and the efficiency of fossil fuel power plants is ~ 40%, estimate if that newly discovered process can be implemented for generation of electricity with a zero carbon footprint. Verify your conclusion in terms of general conservation laws.

BIOLOGY

5 points:

The genomic DNA fragment shown below is known to encode a C-terminal fragment of some protein sequence (In other words, peptide synthesis ends somewhere in that fragment). The starting nucleotide is not known. Read the amino acid sequence and present it in the form of three and single letter codes. The sequence shown below is a sense strand, a.k.a. a coding strand.

- .....tttccatcggtatggcttctgccatgccctaattcttgatattcttggggtgg …. -

10 points:

Below, the segments of aligned genomes of several closely related species are shown. These segments encode for the same protein, which is pretty conserved among those species. It is very sensitive to mutations, so replacement of even one amino acid usually leads to a significant loss of the protein's functionality, which has a detrimental effect on viability.

Read the sequence encoded by that DNA segment. Similar to the previous problem, it is a coding strand, and a reading frame is unknown.

ttgagccaatcggggagaatgagaccatctgctcctt

agagcctatcggggagaatgagacgatctgctccaa

aagaacccattggtgaaaacgaaacaatttgttctt

ggacccaataggtgaaaacgagacgatttgttcgcc

gagccaatcggagagaatgagactatctgctcgct

ggagccaatcggcgagaacgagactatatgctcgcc

cgagccaataggtgagaacgagacgatttgttcgg

gggacccaataggtgaaaacgagacgatttgttcga

gaacctattggcgaaaacgaaactatttgttcttaa

LINGUISTICS

5 points:

The following sentences have been translated from English to a certain constructed language from the early 20th century. The translations are given without punctuation or capitalization and in a random order:

was the scientist in the house?
i am not in the house.
is the baby with you?
the scientist will not have been with the baby.
i will have been with the baby and the scientist.
i will be in the town with the baby.

ab nela in at dubal
ab eta ir at ragab ud at rilica
ab ema in at ducaf ir at ragab
at rilica neta ir at ragab
wela ragab ir ac
weka at rilica in at dubal

Match the sentences to their English translations, then translate the following sentences:

will the scientist have been with you?
i was in a town with the scientist.
you are not in the town.

10 points:

This language was built around the concept that words that are closely related in meaning should also be closely related in spelling. Given some sentences in this language and their English translation, fill in the blanks in the following column of words and their translations.

Sing a song of six-pence, a pocket-ful of rye.
Four and twenty black-birds baked in a pie.
When the pie was opened the birds began to sing.
Wasn't that a dainty dish to set before the king?
The king was in his counting house, counting out his money.
The queen was in the parlour, eating bread and honey.
The maid was in the garden, hanging out the clothes.

When down came a blackbird and pecked off her nose!

Miqadec ap misoda iv zal-gerdac, ap pegap-gisob iv luraf.
Zaf ud zacax bodom-mula pokicef in ap polar.
Avit at polar jeboded at mulaz jekibef migadeb.
Weka ni am ap lagom dogab dabiseb ik at rajad?
At rajad eka in ashe zojer-dubal, zojer ashe gerac.
At rajoda eka in at dugaf, ligiber polab ud pojaq.
At ramibaf eka in at dalida, paniker at dibacz.
Avit oj kepelef ap bodom-mula ud ligibef of ashe meraq!

Fill in the vocabulary table below (each ? represents one letter, each _ represents a full word)
song - _
house - _
bird - _
kingdom - ???oda
prince - ??kmad
baked - _
to bake - _
number - ??
four - _
five - ???

COMPUTER SCIENCE

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easygui in Python
- All the input and output should be via files named as specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py
- **No .pdf, .doc, .docx, etc! Programs submitted in incorrect format will not receive any points!**

Six Degrees of Separation

The famous adage states that all people on Earth are at most 6 social connections away from each other, which means that a chain of "friend of a friend" statements can be made to connect any two people in a maximum of six steps.

This month's problems will have a common input format. Each line in the input file **input.txt** will contain a representation of “A is a friend of B,” with A and B separated by space. For example:

```
Alice Bob
Alice Ben
Alice Carry
Bob Ben
Ben George
George Alice
Alice Laura
Carry Laura
```

Note that this set of connections can be depicted as a non-directed graph, where people are nodes (called *vertices*), and their friendships are links between the corresponding nodes (called *edges*).

5 points:

Given an input as described above, your program should find the person with the most friends of her friends. The name of the person should be written to the **output.txt** file.

10 points:

Given an input as described above, your program should find the pair of people who have the largest **number of paths with 6 edges** connecting them. Your algorithm should be able to handle graphs with thousands of nodes and edges in a reasonable amount of time (say at most a minute). The names in the pair should be written to the **output.txt** file in sorted order in a space-separated format.

Note: for simplicity, it is allowed for a path to pass through the same node more than once.