Problem
of the
Month

P�M

November, 2022

2022-2023

MATHEMATICS

5 points: In SigmaLand, there is a very large grocery store that sells apples and oranges. At the start of the day, it stocks 1729 apples and 2022 oranges on its shelves (with a limitless supply in a back room). The SigmaLand customers always come in and blindly grab exactly 2 randomly chosen fruits off of the shelves. (This does not mean they are equally likely to grab apples and oranges. They are more likely to grab the fruit type that there is more of.) The owner, Sigman, restocks the shelves after every customer. Specifically, if a customer buys two apples or two oranges, Sigman adds an apple to the shelves. If the customer buys two different fruits, Sigman adds an orange to the shelves. At the end of the day, when only one single piece of fruit is left, Sigman notes the type of fruit that remains. What is the probability that the remaining fruit is an apple?

10 points: In OmegaLand, there is a street with an address for every real number from 0 to 1. Each address contains an infinite grocery store that sells an infinite amount of apples and oranges. At every store, the portion of all fruits that are apples is precisely equal to the address of the store. For example, store 0 sells only oranges, store 1 sells only apples, and store 0.5 stocks them in equal amounts. Sigman decides to go buy some fruit. He starts by picking a store to go to uniformly at random. Once there, he chooses 2465 fruits to buy by randomly grabbing them off of the shelves, just like the SigmaLand customers. What is the probability that he buys 2022 apples (and 443 oranges)?

PHYSICS: SPECIAL RELATIVITY

Preamble

Don't be intimidated by the topic this month! Many budding physicists (rightfully so!) have a fascination and reverence for the ideas of special (and general) relativity. The unintuitive nature of the foundational concepts in this field are a gateway to some of the weirdest and most bizarre phenomena in the universe, and bizarre phenomena are precisely our raison d'être as physicists. However, in order to not get confused reasoning about physics we don't have much experience with in everyday life, we need the proper mathematical scaffolding to guide our reasoning. General relativity, unfortunately, requires the full machinery of differential geometry - something a bit difficult to pick up in a matter of weeks. Many ideas in special relativity, however, require nothing more than the language of linear algebra.

The most intense math that this month will require is matrix multiplication - this is a **very** important skill to learn across all STEM disciplines (some of you may remember being emphasized in the Quantum Computation semilab last summer), and all future months will require it in some shape or form. It's not difficult to learn - it's an algorithm that can be memorized by working through a few simple examples in an afternoon.

Special Relativity

I highly recommend watching this short introductory series on special relativity courtesy of Minutephysics, as well as this nice elucidation of matrix multiplication by 3blue1brown.

Some General Background You Don't Actually Need for the Problems:

The development of the theory of electricity and magnetism, culminating in Maxwell's equations for the dynamics of the electric and magnetic field, immediately suggested a surprising result. An oscillating electric field generates an oscillating magnetic field, and an oscillating magnetic field generates an oscillating electric field. Moreover, the speed of the resulting wave can be calculated, and it is always $c = 2.997 \times 10^8$ meters per second. Moreover, this value nearly precisely agreed with Rømer's empirical measurement of the speed of light from nearly two centuries earlier.

This marked an incredible breakthrough in our understanding of the world, and immediately offered one of those tantalizing curiosities that often lead to scientific revolutions - a paradox. Maxwell's theory suggested that light *always* moves at speed c, regardless of the observer in question. This find is fundamentally incompatible with Galilean relativity, which would suggest that an observer running at 1m/s in the same direction as the light should observe the beam traveling at $c - $ 1m/s instead. This lead to a proliferation of theories about the aether, a notion finally debunked by Michelson and Morley's experiment.

The resolution to this puzzle came in Einstein's 1905 paper - On the Electrodynamics of Moving Bodies. It suggested we must give up the fundamental assumption of how velocities add when observers move relative to each other. To understand what this means, we must introduce the notion of a *coordinate system*. We are used to the coordinate system (x, y, z) in everyday life - the three numbers x, y and z tell us where along the three coordinate axes a particular object resides. Suppose I have two objects, one sitting at (x_1, y_1, z_1) and another at (x_2, y_2, z_2). We calculate distances d between objects via the relationship

$$\Delta d^2 = \Delta x^2 + \Delta y^2 + \Delta z^2,$$

where $\Delta x = x_2 - x_1$, $\Delta x = y_2 - y_1$, and $\Delta x = z_2 - z_1$. The distance Δd between objects should be independent of what coordinate frame we choose, as this is a fake choice of description made by an observer. If my friend had chosen a different set of coordinates in which the objects were instead at positions (x'_1, y'_1, z'_1) and (x'_2, y'_2, z'_2), these two positions *must* satisfy the relationship

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 = \Delta d^2 = \Delta x^2 + \Delta y^2 + \Delta z^2.$$

In special relativity, we add in a new coordinate - time. Every vector we consider now has *four* elements, and so are referred to as four-vectors. These vectors are written as (t, x, y, z). The crucial observation is that if the speed of light is constant, in any reference frame, then for any two points along a light beam must always satisfy

$$c^2 \Delta t^2 = \Delta x^2 + \Delta y^2 + \Delta z^2.$$

The similarity of this expression to the Pythagorean distance formula suggests a new object that must be independent of coordinate description we choose - the *interval*

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2.$$

It turns out that from this simple assumption, all of special relativity follows! The rule for coordinate transformations must always preserve the interval Δs^2, including the formula for translating between the coordinate descriptions, must *always* preserve the interval.

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2 \Delta t'^2 = \Delta d^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$$

More specifically, if we define a new coordinate description (t', x', y', z') from the old one (t, x, y, z) via the relationship

$$\begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \Lambda^0_0 & \Lambda^0_1 & \Lambda^0_2 & \Lambda^0_3 \\ \Lambda^1_0 & \Lambda^1_1 & \Lambda^1_2 & \Lambda^1_3 \\ \Lambda^2_0 & \Lambda^2_1 & \Lambda^2_2 & \Lambda^2_3 \\ \Lambda^3_0 & \Lambda^3_1 & \Lambda^3_2 & \Lambda^3_3 \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}$$

This turns out to be quite constraining on the matrix elements Λ^μ_ν. In particular, our usual low-speed intuition suggest that for an observer moving with velocity v in the positive x direction, we should be able to consider the transformation

$$t' = t$$
$$x' = x - vt$$
$$y' = y$$
$$z' = z$$

However, it is easy to check that this transformation does not preserve the interval $\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$. The correct form of the transformation turns out to be

$$t' = \gamma(t - vx/c^2)$$
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

Where γ is an incredibly useful symbol defined as

$$\gamma := \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Note that γ is always strictly greater than 1, and goes to ∞ as v approaches c. Note that we encounter problems if $v \geq c$.

Some Facts that are Crucial to the Problems:

There are a few facts that immediately follow form Lorentz transformations

Length Contraction: An object that is of length L in its rest frame moving at speed v (in the same direction as its length) will have its length contracted to $L' = L/\gamma$. Objects moving at speeds near the speed of light will appear compressed in the direction of motion.

Time Dilation: Objects that move at rapid speeds slow down. If it takes me time T to solve a Rubik's cube at rest, then if I'm in motion then a stationary observer will observe me taking time $T' = \gamma T$ time. Said differently, if observer A is moving at near the speed of light with respect to observer B, A's clock will tick much slower from B's perspective.

Simultaneity: Let's apply a Lorentz transformation to two events that both occur at $t = 0$. The first event takes place at x_1, and the second at x_2. Naively, they both occur at the same time. After a Lorentz transformation, we find

$$t'_1 = \gamma(0 - \frac{vx_1}{c^2}) = -\gamma\frac{vx_1}{c^2}$$

$$t'_2 = \gamma(0 - \frac{vx_2}{c^2}) = -\gamma\frac{vx_2}{c^2}$$

We can immediately see that $t'_1 = t'_2$ *only* if $x_1 = x_2$. If two events occur at the same time in one reference frame, they might not occur at the same time in a different frame!

Addition of Velocities: while this fact can be worked out from Lorentz transformations, it is useful to know that if a train is moving at velocity v_1 in the x direction, and a toy car is moving at velocity v_2 in the x direction *in the rest frame of the train*, then the total

velocity of the toy car would not be $v' = v_1 + v_2$ as predicted by Newtonian kinematics, but instead will be

$$v' = \frac{v_1 + v_2}{1 + v_1 v_2 / c^2}$$

Slower than naively expected!

5 points:

Two ants race around a circular track in opposite directions, as pictured below. These ants are monstrous, and move at significant percentages of the speed of light. Ant 1 moves at speed v, and Ant 2 moves at speed $2v$. Each ant carries a clock which marks off the ant's age. Ant 1's age according to its clock is t_1, and Ant 2's age is t_2. By how much does the difference $t_1 - t_2$ change every time the ants see each other?
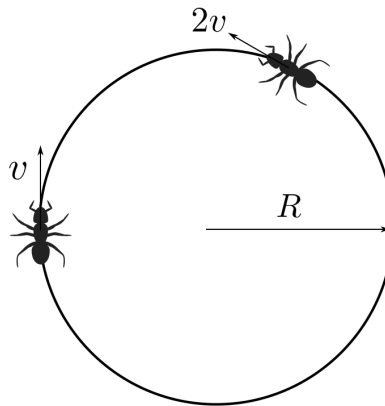
10 points:

It turns out that it's not just the space and time coordinates that transform nicely under Lorentz transformations. Energy and momentum also do! In particular, suppose an object is moving at momentum p_x in the x direction, and has total energy E. The transformation properties of these physical quantities for an observer moving at speed v will be

$$p'_x = \gamma(p_x - \frac{vE}{c^2})$$
$$E' = \gamma(E - vp_x)$$

These look quite similar to the transformation properties of x and t! This suggests that just like we arrange space and time into the four-vector (t, x, y, z) we should arrange energy and momentum into the four-vector (E, p_x, p_y, p_z). Just like the interval Δs is conserved, there will be a new invariant quantity we can build from the energy-momentum four-vector:

$$E^2 - c^2(p_x^2 + p_y^2 + p_z^2) = m_0^2 c^4$$

The quantity m_0 is the **rest mass** of the object. If the object is at rest, we have the equality

$$E^2 = m_0^2 c^4 \rightarrow E = m_0 c^2$$

Einstein's most famous equation! Just like time dilation, we can read off a useful property of special relativity that an object of rest mass m_0 moving at speed v will have a new total energy

$$E' = \gamma m_0 c^2$$

The Problem:

The Large Hadron Collider fires beams of particles at each other so that they may collide at relativistic speeds. Suppose the particles are all identical and each have rest mass m_0.

Let us restrict our attention to two particle colliding. One particle is moving with speed v in the positive x direction, and the other particle is moving with speed v in the negative x direction.

Part (a) - what is the momentum of *each* particle?

Part (b) - what is the total energy of both particles?

Part (c) - what is the total momentum and total energy of particle 2 in the rest frame of particle 1?

Challenge Problem: submit here

We didn't end up posting a challenge problem last month, so this month's problem uses both the ideas of special relativity and electromagnetism!

One of the cool features of special relativity is that it can also tell you how electric and magnetic fields transform into each other under changes of reference frame! The key step is to arrange the electric and magnetic fields into the following *matrix*, called the Electromagnetic tensor:

$$F = \begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

The correct transformation law under a Lorentz transformation is to multiply from *both sides*

$$F' = \Lambda F \Lambda$$

Part 1) Suppose in one reference frame the world is filled with a constant electric field and zero magnetic field. Find the configuration of electric and magnetic fields seen by an observer traveling in the x direction at speed v.

Part 2) Suppose there is a wire at rest, stretching along the z axis. The wire is neutral. Along this wire flows a current I. We know that the magnetic field due to the wire will be

$$\vec{B}(z) \;=\; \frac{\mu_0 I}{2\pi r}\hat{z}$$

(a) Write down the electromagnetic tensor explicitly
(b) Find a Lorentz transformation that takes you to a new reference frame with a *nonzero* electric field
(c) Electric fields can only be generated by a nonzero charge. If the wire is neutral, how is it possible that an observer moving with some velocity observes a nonzero electric field?

CHEMISTRY

5 points:

When Alice came to her lab, Bob, her technician, was looking at three small bottles with barely visible labels. In the first bottle, there was some semitransparent, white and waxy material, in the second bottle, there was a dark red sticky powder, and a small amount of black lustrous crystals were on the bottom of the third bottle.

"Hi, Alice, I am going to discard these three chemicals, because they are unlabeled, but I am not sure about a proper procedure. Can you please advise me how exactly I should dispose of them? It seems at least one of them may be poisonous."

"Yes, Bob, you are right, every chemical without a label or with an unclear label should be disposed of as chemical hazard waste where it will go through analysis. However, it seems I have an idea on what these three chemicals are, so I propose you to do some experiment. Since they have no labels, we cannot use them for anything serious, but we can use them for fun, can't we? Look." Alice filled three conical flasks with oxygen (there was an oxygen cylinder in her lab) and labeled them as #1, #2, #3. After that, she put waxy material from the first bottle into a special iron spoon, ignited it and quickly put into the conical flask #1. The waxy material started to burn, the fire was bright and intense, and thick white smoke quickly filled the flask. When the reaction has ceased, Alice poured a little bit of water into the flask and shaked it vigorously. In a few minutes the smoke disappeared, and the liquid remained clear and transparent.

"Did you see it? Now let's do the same with the other two bottles." To a big Bob's surprise, when they repeated the same operations with the materials from the second bottle (they used the flask #2 for that), they observed exactly the same phenomena: violent combustion, white smoke etc. The third experiment (with black powder from the third bottle) gave the same result.

"Great. Now, Bob, please, take a pH paper and test water in each flask. I am sure these liquids are pretty acidic"

Bob put a pH strip into each liquid, and the paper became red-orange.

"How did you know that, Alice?"- Bob asked.

"I know Chemistry", - Alice replied. After that, she took a sheet of paper, wrote something quickly and put it on the laboratory bench face down.

"Bob, please, do one thing. Take calcium hydroxide solution and barium chloride solution from the shelf, and mix a few drops of each of them with liquids from the flasks numbered 1-3. Do that in separate test tubes. After that, record your observations and compare with what I wrote."

Question: What did Bob observe when he was adding calcium hydroxide or barium chloride solution to the liquids from the flasks ##1-3, and what was written on Alice's paper?

10 points:

We all know that because of carbon's unique ability to form stable carbon-carbon bonds (which are nearly as stable as C-H, C-O, or C-N bonds), including double and triple bonds, and because carbon is tetravalent, isomerism is a very common phenomenon in organic chemistry. Thus, two different compounds with the formula C_4H_{10} can exist, butane and isobutane (a linear and a forked molecules)
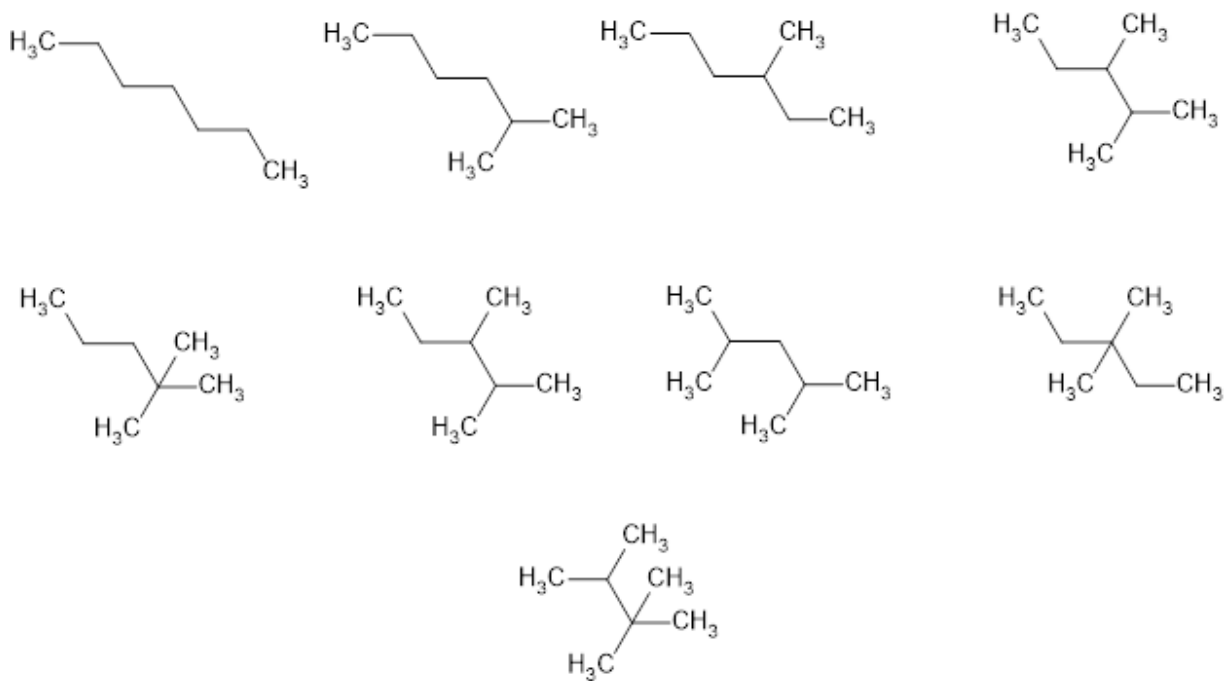
These two compounds belong to the group called "alkanes", and their general formula is $C_nH_{(2n+2)}$. It is easy to see that the number of isomers grows rapidly when n increases. Thus, it is 2 for $n=4$, but for $n=5$ it is 3:
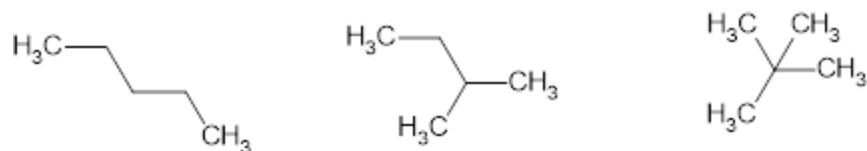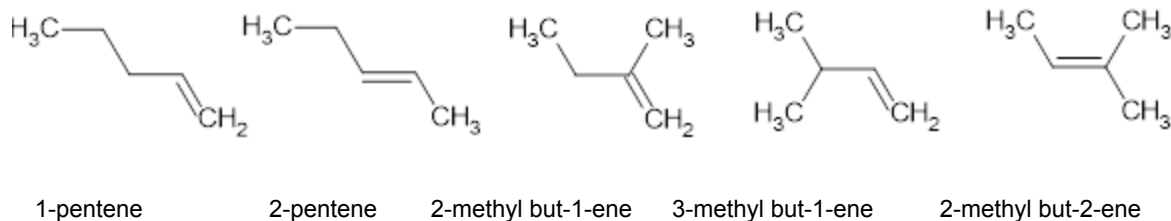
For $n=6$, it is 5:
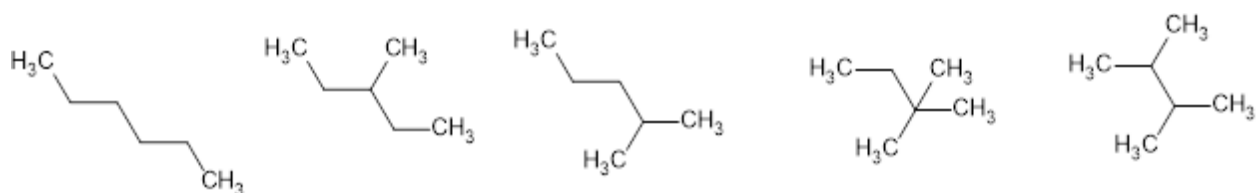
For $n=7$, it is already 9:

And decane (n=10) has 75 isomers!

Now consider another class of hydrocarbons with a general formula C_nH_{2n}. One representative is pentene (note "e" in the middle). It has 5 isomers:

1-pentene   2-pentene   2-methyl but-1-ene   3-methyl but-1-ene   2-methyl but-2-ene

But that is not the end of the story. 2-Pentene can exist in two isomeric forms (E and Z):

and these isomers have different chemical and physical properties. In addition, other hydrocarbons with the same empirical formula (C_5H_{10}) can exist, so called cycloalkanes, cyclopentane, methyl cyclobutene and two isomeric dimethyl cyclolpropanes:
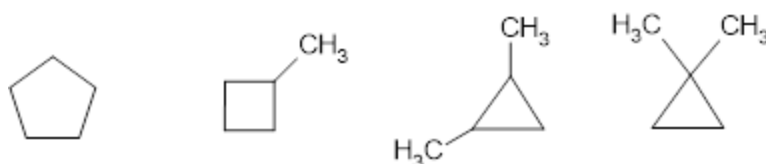
As we can see, there are *at least* 10 different hydrocarbons with a formula C_5H_{10}, whereas pentane (C_5H_{12}) has just three isomers, so removal of just two hydrogens dramatically increases diversity. It is natural to propose that if we take away two more hydrogens (i.e. we are talking about hydrocarbons with a formula $C_nH_{(2n-2)}$) diversity will increase further.

However, if we take too many hydrogens away, the number of possible isomers may decrease. Thus, only three isomers are conceivable for the hydrocarbon with a formula C_5H_4:
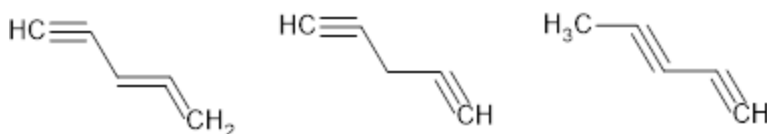
(It is necessary to keep in mind that small cycles (8<), with a triple bond cannot exist, and the cycles smaller than 7 carbons with two adjacent double bonds do not exist either, so no cyclic compounds can be proposed for C_5H_4.)

Question:

Keeping in mind that cycle formation, and branching are not the only kinds of isomerism, estimate the value of m that provides a maximal number of possible isomers in hydrocarbons with a general formula $C_nH_{(2n+2-m)}$. In your analysis, limit yourself with hydrocarbons having 6, 7, and possibly 8 carbons (n=6, 7, or 8).

BIOLOGY

5 points:
A group of researchers was monitoring the ecosystem of some small river with cold still water, which was populated predominantly by four different species of small fish. The first species (species A) was characterized by a pretty uniform phenotype, which seemed to be quite capable of withstanding potential environmental changes to a degree, and, due to their optimal size and other body parameters, this species was dominating in the ecosystem. The second species (species B) demonstrated a much higher degree of phenotypic heterogeneity, so there were too few survivors in each generation to make species B dominant in the ecosystem.
The third species (species C) was remarkable due to its plasticity over time: the researchers observed significant phenotypic changes in species C in a response to relatively minor variations of environmental conditions. The species D was remarkable because although the species was sedentary, a significant number of individuals demonstrated unusual migratory behavior.
Due to the construction of a dam, the habitat has changed drastically. As a result, one species went extinct, one species survived in the changed habitat, and other two found new habitats: one species moved to a basin of another small river, whereas another species migrated to semi-saline waters. Name these species, and explain which mechanisms lead to these changes.

10 points:
Anar was traveling for the holidays, and as a result, he caught a cold. He performed a couple of tests and figured out that the cold is of a bacterial nature. To heal himself, he took anti-bacterial antibiotics in the morning and started feeling much better by the evening of the next day. Encouraged, he came to the lab and had a long discussion with his supervisor in his office. The morning right after that, Anar felt sick again, and he immediately informed his supervisor about that.
The supervisor also got sick the next day. He took the same anti-bacterial medicine, and it was efficient … at the beginning. However, the next day, the same symptoms came back, similar to what happened with Anar.
Intrigued, Anar decided to do genomic analysis of his nasal microflora, and he found that both his samples and samples taken from his supervisor show genetic heterogeneity: a fraction of the bacterial population has genes that were sensitive to antibiotics, but some fraction showed resistivity.
Anar was puzzled: whereas bacteria are known to be capable of developing resistivity towards antibiotics, why antibiotic treatment worked well for Anar's supervisor? He asked his friend, who was a microbiologist, but it seems he was busy, so he just mumbled something like "dunno, maybe, bet-hedging?", and left.
What did Anar's friend mean?

LINGUISTICS

We all know that English is a language, and you probably know that American English is different from Australian English or British English, but did you know that American English itself contains many different dialects? Some are dialects found among cultural groups, and some are dialects found in different geographical regions. Some are even both! These dialects can differ between each other both in vocabulary and in pronunciation (or *phonology*).

The International Phonetic Alphabet (IPA) is a standardized method of representing the different speech sounds (or *phones*) of human language. You may have seen the IPA in use before, in a dictionary or at the beginning of Wikipedia pages! Here is a handy interactive guide to the IPA: https://www.internationalphoneticalphabet.org/ipa-sounds/ipa-chart-with-sounds/

5 points:

Given an audio clip of 5 different words said by the same speaker, use the IPA to transcribe the words (as you hear them - meaning the transcription should reflect the regional dialect of the speaker). Please explain your process, and cite any sources that you use. Phonetic spelling without using the IPA will receive partial credit.

The words said are: "square, porridge, mirror, mouth, goose"
The audio clip is located here:
https://drive.google.com/file/d/1bSn05eJ3MOzk-pB70lT73ZiJFn5-2rpU/view

10 points:

Given audio clips of 5 people from different U.S. regions reading from the same short story, use the IPA to transcribe the words "mirror" and "animal" from each speaker, and match the speakers to the region that they are from. There is only one speaker per region. Please explain your process, and cite any sources that you use. Phonetic spelling without using the IPA will receive partial credit.

Regions: California, Mid Atlantic, New England, New Orleans, New York City

Speaker 1: https://drive.google.com/file/d/17BJnMX4DKTLJvQbikAH29L8noDpgW1zs/view
Speaker 2: https://drive.google.com/file/d/1-HO2g4-qoSQJZiR4RSwZ3ETUm25-2Xbx/view
Speaker 3: https://drive.google.com/file/d/1gEOiiiiV1xvRe_febaazsld99DTfxR0n/view
Speaker 4: https://drive.google.com/file/d/1f-7GEkzwzX7o1oqnBrzd_4Z5DKunI5ei/view
Speaker 5: https://drive.google.com/file/d/1rb_LoW8H4ZWFq6pG31TdfxjcRKdLaCUt/view

COMPUTER SCIENCE

Thank you to everyone who attended our lecture, and we're so excited to announce our second lecture: **Robotics and Control Theory** by Anna Rosner.

What exactly is control theory? How do you go from an input to a meaningful output? Come learn about how control theory is used to make a system do what you want. No advanced math or computer science knowledge is required!

The lecture will be at **2 PM EST on Saturday, December 3rd**, and you can join here: https://sigmacamp-org.zoom.us/j/83088797287.

Recordings from the last lecture will be posted soon, and we apologize for the delay! The CS POM team has also put together a list of their favorite CS resources, from beginner to advanced, which you can find here, with some recently added resources!

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files named as specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.
  No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!

All elements in chemistry can be represented as symbols consisting of one or two characters, where the first letter is capitalized and the second letter (if it exists) is always lowercase. For instance, oxygen's chemical symbol is O, iron's is Fe, and seaborgium's is Sg. Multiple elements can be combined to make molecules, and their symbols are combined to make a chemical formula that represents the amount of each element in that molecule. For example, the formula for table salt, which contains equal amounts sodium (Na) and chlorine (Cl), is NaCl.

When an element appears more than once in a molecule, it can either be repeated or a number can be appended to the element's chemical symbol to indicate how much of this element is present. For instance, HHO is the same as H_2O. Additionally, there might be multiple of one molecule or element, in which case the coefficient is placed at the front of the chemical formula. For example, $2H_2O$ is equivalent to HHO+HHO, and 3O is equivalent to O+O+O.

Molecules and elements can be combined together in chemical reactions to result in a different set of molecules at the end of the reaction, which can be described by a chemical equation. However, the number of elements on both sides of the chemical equation representing the reaction must remain balanced. The same amount of every element must appear in the reactants (the left side of the equation) and products (the right side). To balance an equation, the coefficient at the front of a molecule or element may be changed, but the chemical formula cannot be changed. The following is NOT a balanced chemical equation:
$CH_4+Cl_2=CCl_4+HCl$

This is not balanced because the number of H and Cl atoms is not equal on both sides (left side has four H and two Cl atoms, the right side has one H and five Cl atoms). The following is a balanced equation:

CH4+4Cl2=CCl4+4HCl
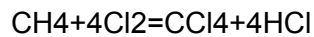
By changing the coefficients of Cl2 on the left side and the coefficient of HCl on the right side to 4, the equation is now balanced.

5 points:

Write a program that receives the reactants and products of an equation and determines whether or not the equation is balanced. Both the reactants and products will only contain elements with one letter symbols, and molecules that contain more than one instance of an element will repeat that element's symbol (e.g. HHO instead of H2O). Your program should receive the input file **input.txt**, which will contain the reactants and products separated by plus signs on two separate lines.
Example input file:

```
CCCCCCHHHHHHHHHHHHOOOOOO+6O
6CO+6HHO
```

Your program will produce the output file **output.txt**, which will contain "VALID" if the equation is balanced, and "INVALID" if the equation is unbalanced.
Example output file:

```
VALID
```

In the example above, both sides of the equation contain 6 carbon (C), 12 hydrogen (H), and 12 oxygen (O), so it is balanced.

10 points:

Write a program that receives a list of reactants and products and determines if the reactants could be combined to produce the products (i.e. can the chemical equation be balanced). Products and reactants may contain two letter elements, and if an element appears more than once in a compound, it will be followed by the number representing its amount. Your program should receive the input file **input.txt**, which will contain the reactants and products on two separate lines, with each molecule or element separated by spaces.
Example input file:

```
C6H12O6 O2
CO2 H2O
```

Your program will produce the output file **output.txt**, which will contain "VALID" if the reactants can produce the products, and "INVALID" if it is impossible.
Example output file:

```
VALID
```

In the example above, the reactants contain 6 carbon (C), 12 hydrogen (H), and 8 oxygen (O). The products contain 1 carbon (C), 2 hydrogen (H), and 3 oxygen (O). The balanced form of this equation is C6H12O6+6O2=6CO2+6H2O.