Problem of the Month

October 2022

2022-2023

MATHEMATICS

5 points:

Alice and Bob have been given a riddle by a conniving evil wizard. The wizard places a hat on each of their heads and writes a natural number on each hat. Alice and Bob can see each other's numbers but not their own. The wizard tells them that, without communicating, they must both write down a *finite* list of guesses for what they think the natural number on their own hat is. How can Alice and Bob guarantee that at least one of them will guess the number on their own hat among their list of guesses?

10 points:

Now the wizard invites Alice and Bob's friends to rise to his challenge. He explains the rules of the challenge, and gives the n players some time to discuss a strategy. He erases Alice and Bob's numbers, places hats on the friends' heads, and writes natural numbers on all n of the hats. The players are allowed to look at the other hats but not discuss what they see. At the wizard's signal, they must all simultaneously guess the *number of players that have the same number as them on their hat (including themselves)*. How can the friends come up with a strategy that will guarantee that at least one person will guess correctly? (As an example, if the numbers on the hats are 17, 100, 17, 17, 100, and 51, then the six players want to guess the values 3, 2, 3, 3, 2, and 1, respectively. Note that only one of them has to get it right to solve the wizard's riddle.)

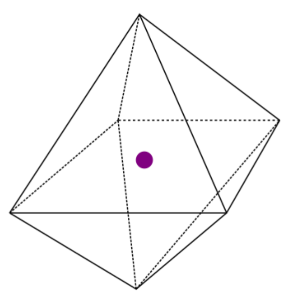
PHYSICS

October's topic is **Electromagnetism!**

Electromagnetic phenomena have been known to humanity for at least three thousand years, but the first (recorded) correct mathematical model didn't start taking shape until the 18th century. In 1785, Coulomb first published the inverse-square law of the electric force. Nearly a century later, the study of classical electromagnetism reached a crescendo with the crystallization of Maxwell's Equations - a set of twelve coupled linear partial differential equations (or 15 nonlinear ones if you want to include matter). Luckily, we will only need a small subset of them in their simplest forms. These equations tell you how changing electric fields create magnetic fields, changing magnetic fields create electric fields, and how charges and currents can generate electromagnetic fields.

5 points:

A point charge of charge q sits in the center of an octahedron of side length a. The octahedron has $total$ charge Q spread evenly around its surface.



(a) What is the total force due to the charge on $each$ face of the octahedron? You may find Gauss's law very useful.

(b) Suppose we allow the octahedron to deform, but we don't allow the material making it up to expand (i.e. to change in surface area). When the system achieves equilibrium, what is the total force with which the charge is pushing the sphere away from it?

10 points:

There's a neat trick we can employ sometimes when dealing with conductors. Conductors are defined as regions of space where the electric potential is required to be constant. For example, consider a point charge sitting next to an infinite conducting plane:

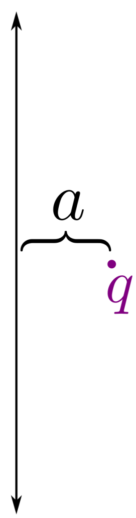


Figure Ph.2

We would like to calculate the electric field due to the charge everywhere to the right of the plane, meaning we would like to find a valid field configuration where everywhere along the plane we have $V(\vec{r}) = constant$. A cheeky hack to accomplishing this is to put a second charge of equal magnitude and opposite sign in the corresponding position on the other side of the plane:

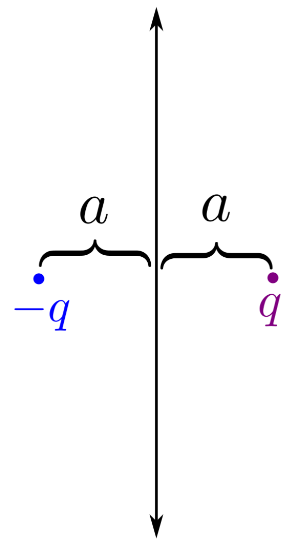


Figure Ph.3

With this charge in place, the condition that the electric potential is constant along the plane is automatically satisfied, and we can entirely forget about it! The correct answer for the electric field configuration to the right of the plane is the sum of the electric fields due to these two charges.

Part a) Prove that the configuration in Figure Ph.2 satisfies the correct boundary condition along the plane.

Part b) We instead place a charge halfway between *two* infinite conducting planes:

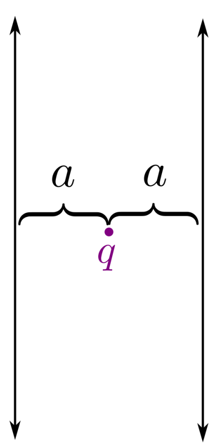


Figure Ph.5

Determine the electric potential everywhere between the two planes.

CHEMISTRY

5 points:

When Alice entered her lab, Bob was sitting before the laboratory bench and staring at three bottles. .

"Alice, I need help. I am studying the behaviour of laboratory mice, and I needed to make some incentive for them when a mouse behaves in a certain way. I used sweet water, which I prepared from glucose. I took this bottle (the bottle labelled “D-glucose”), and I prepared a 5% solution. My experiment went well, but, according to the plan of my study, the same series of experiments must be repeated three times, and they must be separated by a two week break. When I started the second series of experiments, I had to prepare a fresh solution of glucose. However, I found that the bottle of D-glucose was empty, so I took another one. The label on the new bottle was partially destroyed, but the letters "-lucose" were seen pretty clearly. To my big surprise, my mice refused to take this new solution, although I prepared the solution of exactly the same concentration. It looked like they didn't like it at all. I checked the melting point of this compounds, and it was exactly 150°. I asked Cecille to analyse it using her mass-spectrometer, and she told me that the molecular mass of this compound was 180 Da, so it would be natural to conclude it is glucose. Why don't my mice like it?”

“It seems I understand the reason, Bob”, - Alice replied.

“Wait, that is not the end of the story. Upon having realized that there is something wrong with that bottle, I took a fresh one. Unfortunately, the label on that bottle was partially destroyed too, but the letters “-ose” were clearly visible. Before making my experiment, I gave a sample to Cecile, and she told me that mass-spectrum showed a mass of 180 Da, so I decided to use this bottle. However, my mice refused to drink this solution either. To understand what is wrong with this solution, I tasted it, and it appeared to be not sweet, but slightly bitter. Can you please explain to me what is wrong with these two bottles?”

“Sure, I can do that, Bob”, - Alice replied. “But you must realize that you made TWO big mistakes, and you must promise me to *never ever* repeat them.”

What mistakes did Bob do, and what compounds were in the bottles #2 and #3?

10 points:

Next day, when Alice came to the lab's office, she found Bob reading a thick foliant.
"I decided to educate myself, and I started from the beginning. Look, *Britannica* says":

The division of a sample of a substance into progressively smaller parts produces no change in either its composition or its chemical properties until parts consisting of single molecules are reached. Further subdivision of the substance leads to still smaller parts that usually differ from the original substance in composition and always differ from it in chemical properties. In this latter stage of fragmentation the chemical bonds that hold the atoms together in the molecule are broken.

"From that, I conclude that a molecule of some substance is a smallest piece where all atoms are held together by a network of covalent bonds. Am I right?"

"No exactly, Bob. What about DNA?"

"Oh, you are right, Alice. I forgot that DNA is a *double helix*, but the two strands are held together only by hydrogen bonds. Ok, I amend my statement a little bit: *a smallest piece where all atoms are held together by a network of covalent bonds, as well as hydrogen bonds or other weak interactions.* Is it correct now?"

"You are almost right, Bob. However, are you sure that "*a network of bonds*" is an absolutely necessary condition? Is it possible that some smallest piece of matter cannot be split apart without breakage of a chemical bond, but, nevertheless not all parts of that piece are connected to each other by some type of chemical bond? In other words, is it possible that some pieces of a molecule are not connected to each other by any chemical bond or a weak interaction, but the molecule doesn't fall apart? Try to think about that. If you give me an example of such a compound category, I'll give you a can of ginger ale (one can for each type of compound)".

What is a minimal number of cans Bob can earn? Explain your answer.

BIOLOGY

Research aimed at uncovering relationships between the brain and behavior depends on an anatomical understanding of brain regions, neural pathways, and other tissues/organ systems. Neuroanatomy and methods used to study the structure of the nervous system have also informed the development of many targeted therapies to treat neurological conditions. Read through the paper here: https://www.cambridge.org/core/services/aop-cambridge-core/content/view/BC705F9216427F98212016A5B224EE7C/S2056469400003855a.pdf/div-class-title-psychiatric-neurosurgery-in-the-21st-century-overview-and-the-growth-of-deep-brain-stimulation-div.pdf on deep brain stimulation, and answer the following questions. The first question is worth 5 points, and the second is worth 10 points.

5 points:

Imagine you are a psychiatrist. Your long-time patient diagnosed with a chronic psychiatric disorder has not responded to pharmacological treatment, and the symptoms are worsening. Given your knowledge about DBS, its advantages and disadvantages, and the state of its current research in the field of psychiatry, what would be a scientific and ethical threshold for implementation of DBS as a potential treatment for your patient? Discuss your reasoning as compared to other available therapies and any ethical considerations.

10 points:

You now have two more patients who are not responding to their respective pharmacological treatments and are curious about DBS. One patient is suffering from major depressive disorder, and the other one is suffering from Tourette's syndrome. Which patient would you expect to respond better to DBS and why? Also specify which areas of the brain would be most beneficial to target for each disorder and justify your answer. Taking into account that a brain operates as a circuit, what possible risks to target its specific components can be anticipated in each of those cases?

APPLIED SCIENCES

Surprise! This POM season, Linguistics will be sharing space with a new subject - **Applied Sciences!** Applied Sciences is a very large umbrella that covers all types of engineering (electrical, mechanical, civil) and more.

In honor of the SCREEM (Sigma Camp REcreational Engineering Marathon) event, this month's Applied Science POM is an engineering challenge. A Rube Goldberg is defined by Merriam-Webster dictionary as “accomplishing by complex means what seemingly could be done simply.” For another definition, examples, and inspiration, check out rubegoldberg.org (they also hold their own Rube Goldberg contest!).

Your challenge this month is to build a Rube Goldberg that will result in creating a mark with a pencil on a piece of blank paper.
Up to 10 points points will be awarded for elements/technical parts, and up to 5 points will be awarded for creativity, for **a maximum of 15 points possible.**

For reference of how partial credit will be awarded:
5 technical points

- At least 4 distinct elements (in addition to the pencil element), including two different simple machines
- An element is defined as a process that transfers energy from a group of similar objects or a single object to another object
- You may not use a living being as an element

10 technical points

- At least 8 distinct elements
- Total of four simple machines

To submit your solution, record a video (no more than 5 minutes) of your Rube Goldberg running. Please take the video in one continuous shot, and say your name at the beginning of the video.
Submit a word document or pdf to the 10 point problem on the POM website: include a **link to your video** (upload it to youtube/google drive/your choice of filesharing service), **any explanation you would like to include** (no points will be deducted for not including it, but it may help you in earning points), and whether or not you would allow us to use your video in a compilation video (will not affect your score).

COMPUTER SCIENCE

Thank you to everyone who attended our lecture introducing basic computer science concepts! We hope you enjoyed it, and recordings will be posted soon if you weren't able to make it or you'd like to revisit a session. We plan to have another lecture next month. The CS POM team has also put together a list of their favorite CS resources, from beginner to advanced, which you can find here. As a fun note, you might've noticed that Alice and Bob pop up in a lot of POM problems, and their origin comes from a 1978 cryptography paper! If you're interested in learning more about them you can do so here.

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files named as specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.
  No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!

5 points:

After relentless begging and countless favors, Alice finally got access to her older brother's Minecraft account, and has begun making her own circuits with Redstone Dust, a conductive powder that can be placed on the ground and can transport an electric signal. Generating electricity in Minecraft is awfully easy; it can be accomplished with the single push of a button. Alice's objective is to light a lamp, which she places on another point in the grid. For a lamp to be lit, it must either be adjacent to a button, or adjacent to Redstone Dust that carries a signal. For Redstone Dust to carry a signal, it must be placed adjacent to a button, or to another Redstone Dust that carries a signal. Redstone Dust **cannot** transmit any signal diagonally. Each Redstone Dust can have **only two neighbors** in a valid circuit, counting the button, the lamp and adjacent Redstone Dusts (only horizontal and vertical adjacency counts; diagonal does not).

Alice has asked her older brother to analyze one of her circuits and determine whether the circuit will light when the button is pressed. If the button lights, the circuit is valid. On a grid with dimensions a x b, the button will be represented as "B", the lamp as "L", Redstone Dust as "#", and empty space as ".". **It is guaranteed that there will be exactly one button in the top left and one lamp in the bottom right on each grid.**

The following is a valid circuit example:

```
B...
##..
.#..
.#..
.##L
```

Note that a button can directly power a lamp if it directly beside it, therefore the following is also valid:

```
BL
```

Write a program that receives a, b, a circuit grid, and determines whether or not the circuit is valid. Your program should receive the input file **input.txt**, which will have a and b separated by a space on the first line, and the circuit grid on the following lines.
Example input file:

```
5 6
B....
###..
..#..
.....
..##.
...#L
```

Your program will produce the output file **output.txt**, which will contain “VALID” if the circuit is valid, and “INVALID” if the circuit is invalid.
Example output file:

```
INVALID
```

In the example above, there is no way for the signal to travel from the third line to the fifth, so the circuit is invalid.

10 points:

Bob is sitting in front of a red, a blue, a yellow, and a green buttons, and a screen displaying an integer. When a button is pressed, the integer on the screen is updated following these rules:

- When Bob presses the red button, the integer is subtracted by 1.
- When Bob presses the blue button, the integer is multiplied by 2.
- If Bob presses the yellow button and the integer is odd, nothing happens. If Bob presses the yellow button and the integer is even, the integer is divided by 2.
- When Bob presses the green button, the integer is incremented by the number of prior button presses. For instance, if Bob presses the green button first, nothing will happen. If Bob presses it after the 10 button presses, the integer will be incremented by 10.

Write a program to help Bob calculate the least number of button presses possible to go from N to M, where N is the initial integer on the screen and M is the target integer. Your program should receive the input file **input.txt**, which will consist of N and M separated by a space.
Example input file:

3 7

Your program should produce the output file **output.txt**, which will contain a single integer with the minimal amount of button presses.
Example output file:

2

Bob can press the blue button followed by the green button to go from 3 to 7.