## MATHEMATICS

5 points:

Seven friends are deciding who is going to run to the store to get bagels. They have a traditional die with six faces numbered 1 to 6. How could they choose one of them fairly using that die?
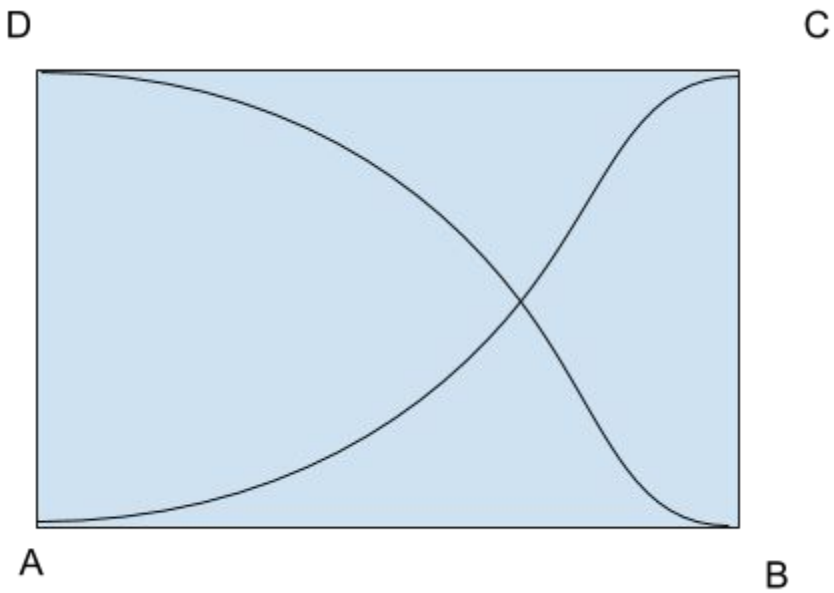
10 points:

A cake that has the shape of a rectangle ABCD is cut along a curved line that goes from point A to point C, as shown in the figure, always going up. You are allowed to cut the cake along any vertical line and take the two pieces that contain points B and D, respectively. Assuming that you are not on a diet and want to get as much of a cake as possible, describe how should you cut it to obtain the maximum combined size of the two pieces, S1+S2, and substantiate your answer.
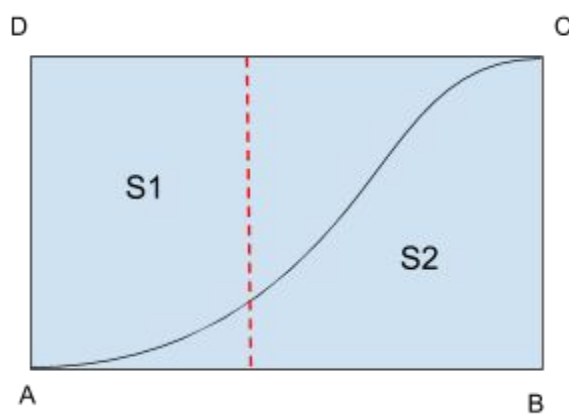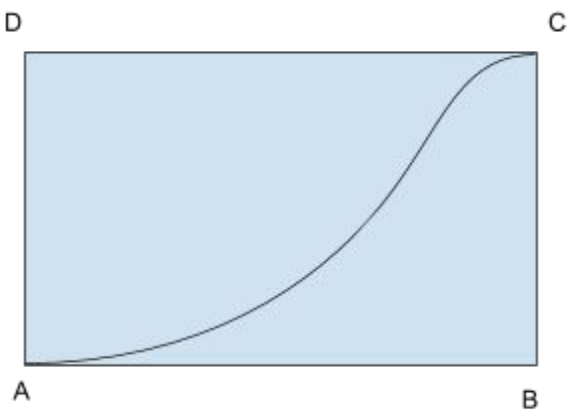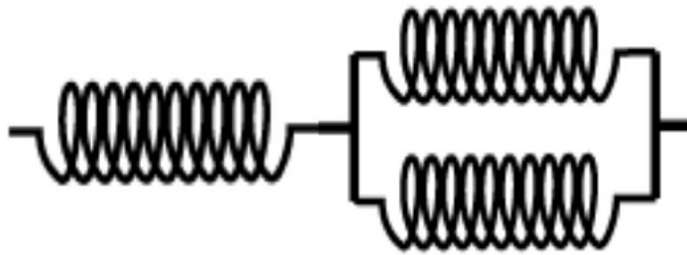
Hint:

Draw a line from D to B which is symmetric to the one connecting A and C, as shown in the Figure.   Now, using this Figure, try to reformulate the problem so that your pieces of cake contain points D and C.

PHYSICS

5 points:

Three identical springs having spring constant k are connected as shown in the figure. Find the effective spring constant of the system.

Hint: Fix one end of the system and apply the force F to the other end. Find the tension of every spring. What is the distance X by which the system is stretched?

10 points:

The vessel has a form of a solid of revolution (this means that any horizontal cross-section of the vessel is a circle of some radius, r; this is e. g. the case for a typical glass, or a bowl). A small hole is punched at the very bottom of the vessel. It is known that the level of water in the vessel is decreasing at constant rate due to the water leak from the hole. Find the shape of the vessel.

Hint:

1. See solutions to September Physics POM problems
2. Knowing the exit velocity of the fluid from the hole of a given size try to calculate the rate at which the water level is decreasing.

CHEMISTRY

5 points:

Alice and Bob, her technician, have been preparing for tomorrow class devoted to the study of the properties of bases and acids. Alice asked Bob to prepare 1% solution of sodium hydroxide. Upon having examining lab's shelves, Bob found the bottle with solid sodium hydroxide was empty. "Alice, we have a problem", he said. "We have no sodium hydroxide, no potassium hydroxide, no hydroxides of other alkaline metals. What will we do? Should we cancel tomorrow's class?"

"Yes, Bob, that is a problem. An interesting problem," - Alice replied. "That is a good opportunity for us to check if we are good problem-solvers. Let's see what we have on our shelves. Sodium chloride, barium nitrate, calcium iodide, manganese acetate, barium hydroxide, sodium phosphate, sodium carbonate - Bob, we have enough chemicals to prepare sodium hydroxide solution!" She took a sheet of paper and wrote one formula on it. "You take this, dissolve in water…" "Stop, Alice, please!" - Bob exclaimed. "Don't explain it! Now I myself realize which chemicals I should take. You may go home, I'll prepare sodium hydroxide solution."

Which formula did Alice write, and how will Bob prepare sodium hydroxide?

Hint:

A reaction between a soluble salt of a metal X and a soluble hydroxide of a metal Y yields a hydroxyde of the metal X if the second product of this reaction is ….
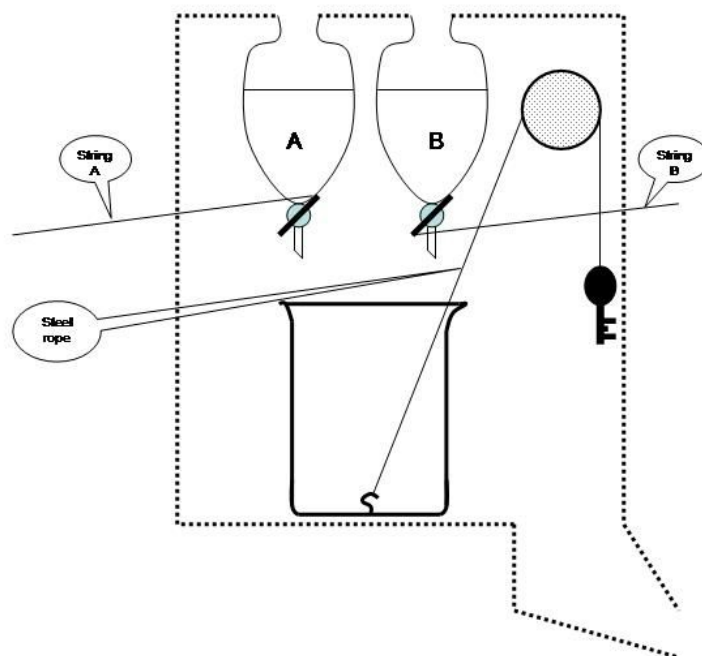
10 points:

Imagine you are playing the Escape the Room game. The key that (probably) opens the door is in the transparent plastic box shown on the figure below. The key is attached to a steel rope that hangs down a freely rotating wheel, and its opposite end is connected to the hook at the bottom of a glass beaker. Upon examining the box you realize the only way to get a key is to break the rope to let the key to fall down through the window at the bottom of the box. There are two dropping funnels (i.e. glass vessels with stopcocks at the bottom) filled with some unknown liquids. By pulling the string A you open a stopcock in the funnel A. By pulling the string B you open a stopcock in the funnel B. Once a stopcock is open, you cannot close it, so all the liquid from the funnel will be poured to the beaker. The necks of both funnels are open, as shown on the picture, so you can drop some small objects into the funnels (or take something from them).

The plastic box shown on the picture is attached to the table, you cannot move it.

In addition, in the room you found various things including:
- porcelain cups, dishes, and teapot;
- a pair of goggles and latex gloves;
- a silver spoon and German silver forceps;
- a bar magnet;
- two pipettes; they are long enough to withdraw a liquid from funnels A or B;
- a plastic model of the molecule of sulfuric acid; one oxygen atom was labeled with an asterisk;
- a book "Black holes and time warp" by Kip Thorne, there was a bookmark between pages 110 and 111, and the figure 2.5 on the page 110 was underlined with a yellow marker;
- a lighter;
- a candle;
- a bottle of vinegar;
- a flyer "SigmaCamp 2013"
- 10 pennies, two Canadian dollars, one 10 Russian roubles coin, one 500 Costa Rican colones coin;
- a globe where Australia, France and China capitals have been marked with a red marker;
- two small jars with metal shavings; both metals look light, silverish, and non-magnetic; your attempt to ignite the shavings from one jar was successful: the shavings burned producing bright white solid; when you withdrew (using a pipette) some liquid from each of two funnels and dropped it onto the shavings from the second jar, both liquids reacted with shavings to produce bubbles of some gas; when you withdrew some amount of the liquid from the funnel A, poured it to a cup, and added some amount of a liquid from the funnel B, the cup became warm.
- an odd box with 32 switches, not connected to other devices;
- a bottle of Poland spring sparkling water;
- and many other potentially useful things.

Propose the best strategy to get the key. 3 extra points may be given to those who solves the problem after obtaining a hint (8 pts maximum).

Hint:

The fact that the two liquids when mixed together produce heat means they are two different substances. Since both liquids react with metal shavings, these two liquids most likely are the solutions of some acid and some base, accordingly (two different acids very rarely react with each other; and one category of metals exists that reacts both with acids and bases). Unfortunately for us, iron reacts only with acids. Therefore, to dissolve the steel rope and to release the key, you need to pour an acid to the beaker. If you pull a wrong string, a base solution will be poured into the beaker, and it will be senseless to pull the second string after that, because the base (which is already in the beaker) will neutralise the acid, and the rope will not dissolve. Therefore, you have to decide which solution is an acid. To do that, try to guess what the shavings in the jars are. Which metals react both with acids and bases, and which metals react only with acids? Which metals can burn? May be, if you identify these metals you will know what to do.
PS. It is highly likely that the shavings in the jars are not exotic or precious metals, but something very common and inexpensive....

BIOLOGY

5 points:

When you send your DNA to the sequencing service, like 23&me, they will often tell you that your ancestors from your mother's side have a particular heritage, and your ancestors from your father's side comes from a different heritage. How can they tell if genetic material came from your mother or father? Can this be determined for any genes in your genome?

10 points:

Under conditions of acute fear, the body prepares for fight or flight by activating the hypothalamic-pituitary-adrenal (HPA) axis, releasing a cascade of stress hormones such as adrenaline and cortisol. If chronically elevated through long-term emotional stress or sleep deprivation, stress (via cortisol) will actually cause people to increase their storage of abdominal fat. Likewise, people with a larger body-fat percentage generally produce more cortisol. What is the mechanism by which cortisol and fat-production are related? And how does this mechanism aid the fight or flight response?

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in C, C++, Java, or Python
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Submit the problem in a plain text file, such as .txt, .dat, etc. **No .pdf, .doc, .docx, etc!**

5 points:

You are given a string of letters (ex: "asdkdjdhddjskqjjqoiu"). The string is entered interactively from the terminal. The task is to output the longest palindrome (a string that reads the same right to left and left to right) that you can form by rearranging (and/or removing some of) the letters in the string.

For example, if the input is: *aabbccddefg*

A valid output is: *abcdedcba*

Another valid output is: *bcdagadcb*

10 points:

N pirates found a treasure chest. There are M individual pieces inside (gold coins, silver medallions, pearls, diamonds, etc), and the pieces do not necessarily have the same value (for example, a pearl costs 1000 times less than a diamond). The pirates need to distribute the treasures as fairly as possible. Write a program to help them.

On the input, you receive the number of pirates, number of pieces of treasure, and the value of each piece:

N = 5, M = 1000, values = [10, 1, 1004, 12, 7, 34, 234 ...]

Feel free to type all values of treasure pieces directly into your program instead of reading them from user input or a file.

[Hint: begin by deciding for yourself: How do you define "fair"?]