PROBLEM OF THE MONTH

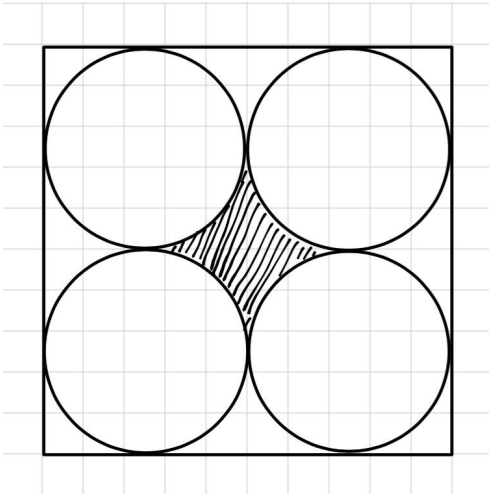
POM

2016-2017

October, 2016

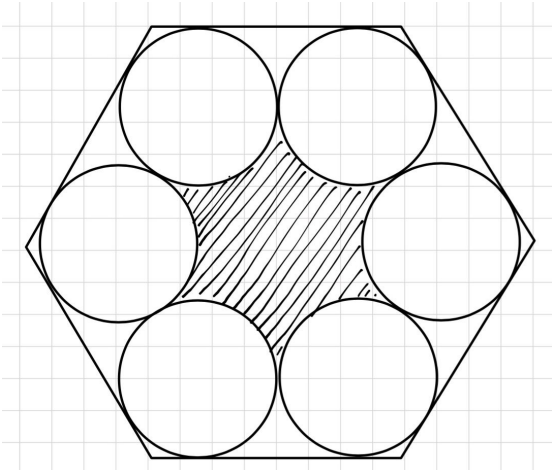
MATHEMATICS

5 points: Find the area of the star-shaped region constructed from the square with side a as shown in the figure.



Hint: Connect the centers of circles and consider the obtained square.

10 points: Find the area of the star-shaped region constructed from the regular hexagon with side a as shown in the figure.

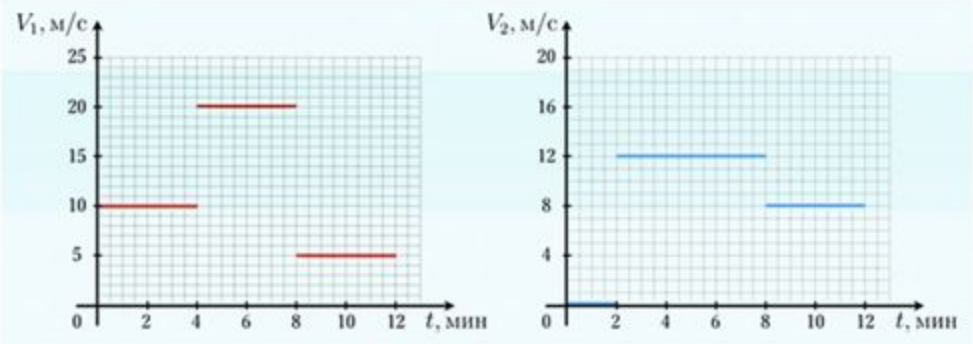


Hint: Connect the centers of circles and consider the obtained hexagon.

PHYSICS

5 points:

The two graphs below represent speeds of two friends biking in opposite directions. Biker #2 started moving two min later and biked 7 minutes until he met his partner. Find the initial distance between the bikers at t=0.



Hint: Find how much each biker travelled over each time interval when the speed was constant, up to the moment *t=9min*.

10 points:

42 freight carriages of the same mass initially stand still on rails spaced by equal gaps of 10 cm. At certain moment, the first carriage is pushed and starts moving towards the next one with initial speed $v = 36\ km/h$. When a moving carriage hits the next one, a coupling mechanism is engaged, and they get rigidly connected. This repeats for all the subsequent collisions between the carriages. Find the time between the first and the last such collisions.

Hint: Note that at each collision momentum is conserved, but energy is not. This means that when *n* carriages are moving together, they have the same momentum as the first carriage in the very beginning. Can you find their speed? The time it will take for them to collide with the next carriage?

CHEMISTRY

5 points:

(Halloween problem)

We all know that will-o'-the-wisp, or *ignis fatuus,* a ghost light travellers see at night over swamps, marshes and bogs is a standard element of fairy tales and scary stories. It is generally believed *ignis fatuus* is the result of combustion of the mixture of methane with small traces of phosphine that form during anaerobic decomposition of organic materials. However, this theory seems to contradict with what we know about combustion of methane. Indeed, methane-air mixtures that contain more than 17% of methane are too methane rich to ignite: there is not enough oxygen to support combustion, so there simply will be no burn. The mixtures containing less than 4.4% of methane also cannot burn, because there is not enough methane there. However, when the methane concentration is in between 4.4% and 17%, combustion does not start spontaneously; at least a small spark is needed for that. Once such combustion has started, it continues violently, in a form of a violent explosion, and it doesn't look like *ignis fatuus.*
Please, explain (i) what initiates the combustion of the swamp gas (remember, methane-air mixture cannot ignite spontaneously), and (ii) why fire *ignis fatuus* look like faint slow light, not violent explosions.

Hint:

Try to read more about properties of phosphine, and about the phenomenon of "cold fire"

10 points:

In "The Terminator" film, Kyle buys and brings to the hotel the moth balls, corn syrup, and ammonia. Using this set of chemicals, is it possible to make a weapon that will destroy the Terminator? If yes, explain how. If not, explain what else should Kyle buy in the store. Limit yourself with the materials available for purchase in common stores.
Of course, we don't need to know the exact recipe, we discuss it from a purely theoretical point of view.

Hint:

To kill the Terminator, Kyle had to make some explosive (he mentioned nitroglycerol, but it is more likely he prepared something more simple and safe). The explosion (if it is not

a nuclear explosion) is a liberation of energy during some chemical reaction. The most violent reactions are the reactions between a strong oxidizer and some reducing agent. A lot of materials available in stores can serve as reducing agents, for example, sugar or charcoal. However, Kyle also needs an oxidizer. What could it be?

BIOLOGY

5 points:

(Halloween problem) In the early 20th century, scientists attempted to provide a viable explanation/reasoning for various manifestations of vampirism. For example, actual reason behind the legend about vampire's intolerance of garlic smell was the allergic reaction of people believed to be vampires to garlic. This reaction could be severe and frequently lead to asphyxiation. Please try to give a medical/scientific explanation for other tendencies associated with vampires, such as:

- fear of daylight
- avoidance of contact with silver
- repulsion of rooster call
- tendency to sleep in burial vaults and tombs
- desire to consume fresh blood
- very pale skin
- avoidance of contact with aspen wood

10 points:

In 2011, health officials noticed that children in Germany, Austria, and Switzerland who grow up drinking raw milk don't get asthma and other allergies (e.g., hay fever). If you wanted to figure out why, how would you design the best experiment that you can think of in order to test what mechanism is responsible? Be careful that your design provides a way to rule out other factors that have nothing to do with the milk. Link your experiment to specific hypotheses.

Hint: Since the commercial milk is assayed and pronounced "clean" in order to be sold, it must be that it's not so much that the commercial milk has something bad, but rather that the raw milk has something good/protective that is being killed off by pasteurization. Control conditions need to eliminate other factors that have nothing to do with the milk; for example, we would need to control for children who grow up on farms versus other (less healthy) environments.

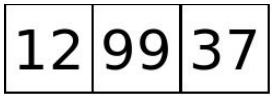
COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc.
  No .pdf, .doc, .docx, etc!

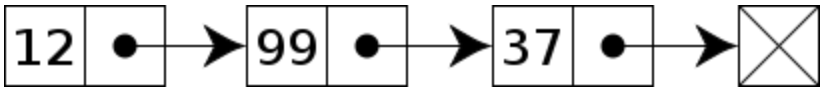
Introduction: Lists

(If you already know what single-dimensional and multidimensional arrays are, you are welcome to skip straight to the problems below)

You are already familiar with a concept of an array - a data structure used by computer languages to store a sequence of values. **List**, or more formally, **linked list**, is another data structure used to store a sequence of values. If in an array all the values are stored in array elements located next to each other in *contiguous* area of memory, like this:



lists consist of a collection of nodes, with each node containing a single value plus a *link* to the next node in sequence:



A linked list whose nodes contain two fields: an integer value and a link to the next node. The last node is linked to a terminator used to signify the end of the list.

Nodes of a linked list do not need to be in contiguous area of memory. The size of the arrays is fixed, so we must know the upper limit on the number of elements in advance. Also, generally, the allocated memory is equal to the upper limit irrespective of the

usage. On opposite, the size of the lists is not fixed - lists can grow or shrink dynamically as the new nodes are deleted or removed.

Inserting a new element in an array of elements is expensive, because room has to be created for the new elements, and to create room existing elements have to shifted. For example, suppose we maintain a sorted list of IDs in an array ids[].

ids[] = [1000, 1010, 1050, 2000, 2040, …..]

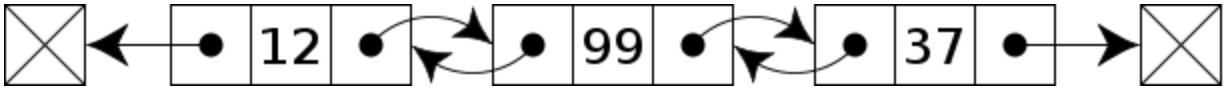
And if we want to insert a new ID 1005, then to maintain the sorted order, we have to move all the elements after 1000 (excluding 1000). Deletion is also expensive with arrays. For example, to delete 1010 in ids[], everything after 1010 has to be moved.

With lists, and insertion (or deletion) is simple: a space for a new node needs to be allocated, and then to insert the new node in the list all what needs to be done is to adjust the links around the area of insertion.

However, compared with arrays, lists have the following drawbacks:

- We can't simply access N-th node of a list as we access N-th element of an array. We have to access elements sequentially starting from the first node.
- Extra memory space is required in each node of the list to store a link.

By the way, list nodes may contain not one, but two links: one to the previous node of the list and one to the next node. Such a list is called doubly linked list:



A doubly linked list whose nodes contain three fields: an integer value, the link forward to the next node, and the link backward to the previous node

Different languages have different syntax for lists. Look up the list documentation for the language of your choice to learn how to implement them in your code.

5 points:

Card players traditionally "cut" the deck before each game: after the cards are shuffled one of the players is offered to take an arbitrary number of cards from the top of the deck and place them at the bottom. We will emulate this process in our assignment. For simplicity our cards will contain integer numbers. Your program should enter N cards

constituting a deck. Then it should ask the user how many cards (M) he or she would like to cut. In response, first M cards should be moved to the end.
For example, if original deck was [1,2,3,4,5,6,7,8] after cutting 3 cards resulting deck is [4,5,6,7,8,1,2,3]. Your program should print the resulting deck. Use a list to implement the procedure.
Additionally, please calculate how many how many swap or move operations would be required if you would have implemented your program using an array.

10 points:

For this month's assignment you will need to write a program to transform one string into another string, which consists of the same characters as the original string, but in a different order. You need to figure out what is the minimal number of operations necessary, if the **only** operation permitted is to take one of the characters in the string and move it to the front. For example, to transform ANT into TAN only one move in necessary: T to the front. To transform ABCD into ACBD two moves are needed: ABCD -> CABD -> ACBD. Your program should request an original and a target strings and then should print all the necessary transitions to transform the former into the latter. If transformation is not possible, your program should report so.