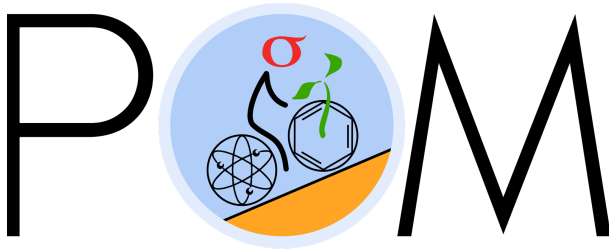
PROBLEM OF THE MONTH



October, 2017

2017-2018

MATHEMATICS

5 points: A car drove from point A to point B with average fuel efficiency of 32 miles per gallon. During the last ¼ of the trip, its fuel efficiency was 36 miles per gallon. What was the fuel efficiency during the first ¾ of its trip?

Hint: Assume that the total distance travelled is d. How much fuel is used during the whole trip? How much on each of the segments?

10 points: A car drove from a town in New York State to a town in New Jersey with an average fuel efficiency of 36 miles per gallon (mpg). While in NY, its fuel efficiency was 30 mpg, and its average speed was 50 miles per hour (mph). In NJ, its fuel efficiency was 40 mpg and average speed was 60 mph. What was the average speed of the whole trip, if the car did not visit any other state?

Hint: Let *d* be the total distance travelled, out of which *xd* NY, and *(1-x)d* in NJ. How much fuel was used on each segment? You can find x, and then express the total travel time in terms of d.

PHYSICS

This month Physics problems are on the free fall and projectile motion. You might find the following links useful.
https://en.wikipedia.org/wiki/Free_fall
https://en.wikipedia.org/wiki/Projectile_motion
http://hyperphysics.phy-astr.gsu.edu/hbase/traj.html#ffall

5 points:

A ball is thrown upwards with initial velocity $12\ m/s$. One second later another ball is thrown upwards. What should be an initial velocity of the second ball so that both balls land at the same moment? Neglect air friction. The free fall acceleration is $g \approx 10\ m/s^2$.

Hint:

What time does it take for the ball thrown upwards to land?

10 points:

A ball is thrown from the horizontal surface at an angle α to the horizon. What should be the initial velocity V of the ball so that the ball would get into a hole which is at the distance L from the point of the throw? The ball can bounce from the horizontal surface. Assume that all such bounces are absolutely elastic and neglect air friction. The free fall acceleration is g.

Hint:

What is the time between bounces of the ball?

CHEMISTRY

5 points:

Four beakers are placed on each scale of the balance. The beakers contain:
on the left scale
- water,
- sodium carbonate (powder,)
- sodium bisulfate (powder)
- potassium acetate (powder)

on the right scale
- water
- phosphoric acid (80%)
- magnesium acetate (powder)
- iron pellets

(each beaker has a volume of 250 mL and contains 50 grams of one of the above listed material)

The scales are at equilibrium.

You have to do some chemical reaction(s) with these chemicals that will shift the equilibrium less than in 1 hr after the process started (the sensitivity of the balance is sufficient to detect the mass change as low as 1 gram). The following rules must be observed:
1. No transfer of matter is allowed between the two scales during your manipulations with the beakers (for example, you can mix the content from the beaker 1 and beaker 2 from the left scale, and put both beakers back; however, you cannot mix the content of the beaker 1 from the left scale and the beaker 2 from the right scale).
2. You can take the beakers from each scale temporarily (to mix their content), but you must put them back after mixing is finished. No spillage of any material is allowed.

In your answer, predict at which direction the equilibrium shifts.

Hint: By mixing a content of some beakers you may start a reaction where gas evolves, so the total mass of the chemicals on one scale decreases. Please, keep in mind that virtually no reaction occurs between phosphoric acid and iron: phosphoric acid is not a strong acid, and iron phosphate that forms initially is insoluble, it precipitates on iron's surface thereby protecting the metal from further reaction.

10 points:

Five beakers are placed on each scale of the balance. The beakers contain:

<u>on the left scale</u>

- water,
- sodium hydrophosphate (powder)
- aluminium pellets
- potassium sulfate (powder)
- copper sulfate

<u>on the right scale</u>

- water
- silicic acid (meta, powder)
- iron turnings
- aluminium pellets
- mercuric chloride (powder)

(each beaker has a volume of 250 mL and contains 50 grams of one of the above listed material)

The scales are at equilibrium.

You have to do some chemical reaction(s) with these chemicals that will shift the equilibrium less than in 1 hr after the process started (the sensitivity of the balance is sufficient to detect the mass change as low as 1 gram). The following rules must be observed:

1. No transfer of matter is allowed between the two scales during your manipulations with the beakers (for example, you can mix the content from the beaker 1 and beaker 2 from the left scale, and put both beakers back; however, you cannot mix the content of the beaker 1 from the left scale and the beaker 2 from the right scale).
2. You can take the beakers from each scale temporarily (to mix their content), but you must put them back after mixing is finished. No spillage of any material is allowed.

In your answer, predict at which direction the equilibrium shifts.

Hint: By adding a content of some beaker(s) to one of the metals you may activate it, so it will start to absorb atmospheric oxygen. As a result, the total mass of the chemicals on one scale increases.

BIOLOGY

5 points:

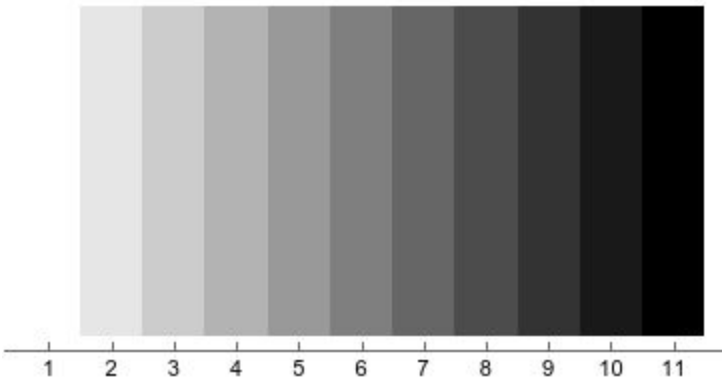
A Museum of Natural History was undergoing an extensive renovation during the summer. Upon completion of all the hard work, the construction crew's last task was to put up labels on the wall for exhibits that the newly reorganized 2nd and 3rd floors will house. They had to finish putting the labels up in the exact locations that exhibits will be located at and do so before the precision handling crew brought the exhibits out of storage rooms. On the last day of their work, during their lunch break, two construction workers discussed what they have been up to in the morning:

worker 1: I have been putting up labels in what looks like latin all morning on the 2nd floor. Now I don't know a word of latin myself, but I can say that my labels had two words in them, and some of the exhibits that would stand next to each other had identical second words in their names.
worker 2: Very strange! On 3rd floor, I had an opposite situation: sometimes labels next to each other only had their first words identical, but not the second. And sometimes, there would be names with 3 words. In that case, both first and second words would be the same for some neighbors, and the third word would be different.
worker 1: Let us come back and see what the curators have in their mind once the whole thing re-opens?
worker 2: Good idea!

Based on the naming of the exhibits and their grouping, what do you think the museum might be housing on the 2nd and 3rd floors in its newly renovated arrangement?

10 points:

Our visual system is known to play tricks on us quite frequently: we constantly encounter with or without our knowledge multitude of optical illusions that make us "see things" not exactly the way they actually are. One such example can be seen in the figure below. In a visual scene where multiple bands of brightness are packed closely together, our visual machinery tricks us into seeing each individual band of brightness (exactly 11 bands shown: from white to black) as a gradually changing pattern in itself: each band between 2nd and 10th appear to us as darker



on the left and brighter on the right. But we know that each band is in fact a single uniform level of brightness, hence exactly the same color. It is no doubt you have noticed this before.

A) Describe how the human visual system accomplishes this deception and what purpose this mechanism serves, commenting on whether seeing a gradually varying brightness where there really is a single level of brightness is advantageous to our survival or not. Make sure you define all terms used.

B) Imagine an obscure and rare primate species whose eyes and brain are anatomically similar to humans', but it isn't yet clear to the scientific community whether they also possess a visual system capable of the same trickery. Describe an experiment that would help resolve this ambiguity and establish whether their vision is as tricky as ours.

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc. **No .pdf, .doc, .docx, etc!**

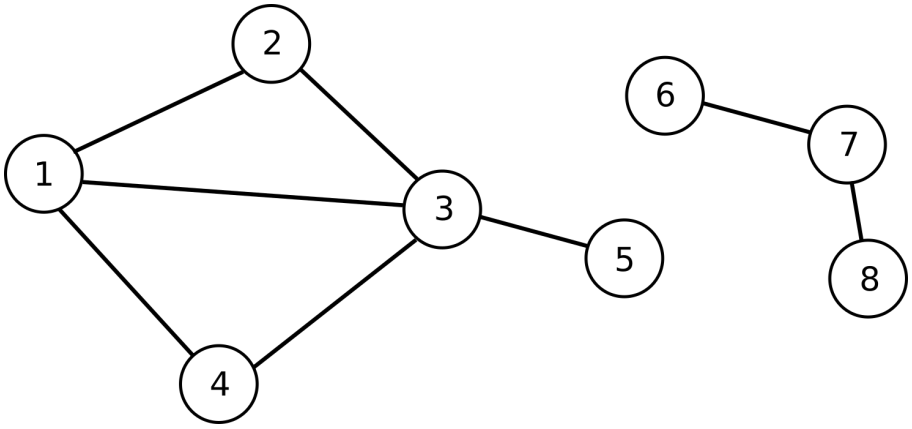
5 points:

Given a graph (https://en.wikipedia.org/wiki/Graph_(discrete_mathematics)*), your program must determine which node has the greatest number of neighbors.* The input is a 2xN array that lists each pair of connected nodes. The nodes are labeled by numbers:

For example, given the input

```
[[1,    2],
[2,    3],
[3,    4],
[4,    1],
[1,    3],
[3,    5],
[6,    7],
[7,    8]]
```

The output should be "3". The graph that corresponds to this example is:



10 points:

Given a graph (same input format as in the 5pt problem), and two nodes, determine if there is a path between the two nodes.

For example, given as input the same graph as above and nodes 1 and 5, your program should return “true”. Given input 5 and 6, your program should return “false”.