PROBLEM OF THE MONTH

POM

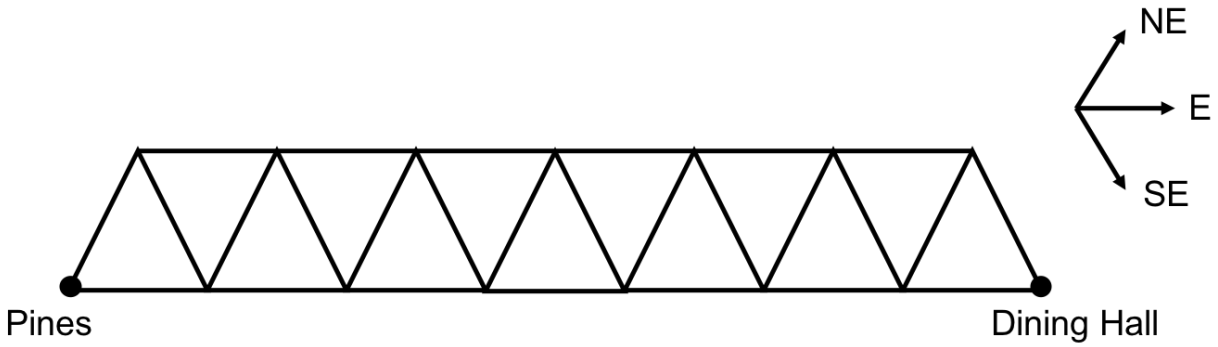
2018 - 2019

October, 2018

MATHEMATICS

5 points:

A Sigma-camper leaves Pines and walks to the dining hall along the paths shown in the image. In how many ways can he reach the dining hall if he is only allowed to walk east, northeast, and southeast?



10 points:

The Sigma robotics team built a stepping robot, positioned it at the brink of an abyss at the bank of the Silver Lake and sent it to reach a destination by following a narrow trail, where the robot can only move in one direction, either making a step forward, or a step backward. However, the robot somewhat lost its sense of direction. Instead of moving directly away from the abyss, it makes steps randomly, but a step away from the abyss is two times as likely as a step towards it.

a) What is the probability that the robot falls into the abyss before making six steps?
b) What is the probability that the robot eventually falls into the abyss?

PHYSICS

5 points:

By measuring the work that a horse can do within a certain time, James Watt defined the horsepower as 33,000 ft·lbf/min, which in modern units this is approximately 745 Watt. What is the maximum velocity that a horse of a single horsepower can pull the carriage weighting 745 kg up the road of a constant slope, which rises by 10 m per every 100 m travelled? Neglect the losses due to friction and other dissipations in the carriage.

10 points:

In times before the round wheel was invented, carriages in Sigmaland were equipped with square wheels. How many horses of 1 horsepower (745 Watt) are needed to pull such a carriage weighting 745 kg and having square wheels of 1m side with the velocity 5 km/hour on an even, horizontal road? Assume that collision of a square wheel with the road is fully inelastic.

CHEMISTRY

5 points:

Alice, a college faculty, came to her lab and found that 8 bottles of chemicals on the shelf had unreadable labels. She told Bob, her technician, that, according to safety rules, these chemicals should be discarded. According to their inventory, the solid chemicals in these bottles were: sodium chloride, potassium fluoride (dihydrate), glucose, anhydrous potassium carbonate, sodium thiosulfate, sucrose, sodium iodide, and salicylic acid.

Alice decided not to drop all chemicals into one waste container, and, to facilitate their utilization, she asked Bob to put them into two separate waste containers: one for inorganic and one for organic waste. “How can I decide which chemical should go into which container?” Bob asked. “Their labels are unreadable.”

“Bob, try to do some simple test,” Alice replied. “It is not too difficult to distinguish inorganic compounds from organic ones, especially when they are solids.”

The next day, Bob told Alice that he had separated the inorganic and organic compounds, and, according to his test, 3 bottles appeared to be inorganic, and 5 bottles were organic. “Hmm,” Alice said, “Something is wrong. According to the list, 5 compounds were supposed to be inorganic ones, and only 3 were organic. Which test did you use, Bob?”

“Alice,” Bob replied, “From my high school chemistry course, I know that ionic inorganic compounds are solids with a high melting temperature, whereas all organic solids have a low melting point. I took small samples from each bottle and started to heat them gradually. The samples from five bottles melt at temperatures below 250-300 degrees, and some of them even melted below 100 degrees. They are definitely organic compounds, I am pretty confident there is no mistake here.”

“My dear Bob,” Alice replied, “Your rationale looks correct, but you missed one point. Some of those compounds were ….”

Please, continue Alice’s statement and explain what was wrong with Bob’s method. How could his method be modified to make it work?

10 points:

"*We had two bags of ferric chloride, seventy-five ounces of sodium hydroxide pellets, five kilograms of high purity citric acid, a saltshaker half-full of aspirin, and a whole galaxy of multi-colored pH papers, rubber balloons, strings etc... Also, a quart of isopropanol, a quart of acetone, a case of Poland Spring water, a pint of raw ether, and two dozen grams of isoamyl alcohol. Not that we needed all that for our graphomaniac exercises, but once you get locked*

into a serious chemicals collection, *the tendency is to push it as far as you can. The only thing that really worried me was the ether. There is nothing in the world more helpless and irresponsible and depraved than a man in the depths of an ether binge, and I knew we'd get into that rotten stuff pretty soon.*"

Using the stuff described in this quote, can you prepare a dark ink? Which items listed here are needed for that, and how will you do it?

BIOLOGY

If you look through the history of important discoveries in biology and medicine, you may notice a very common pattern: a natural or an experimentally-induced pathology often yields insight into normal biological function. As one former Cold Spring Harbor professor has described it, half-joking, if a biologist were to study a radio, they would shoot it with a shotgun until the radio stopped working, and then identify the broken parts as obviously being critical to the radio's function. Funny as that sounds, careful analysis of anomaly and disease can turn into a very useful strategy when trying to understand normal physiology. Moreover, this approach holds true at many levels of experimental biology, from the biochemistry of single cells to the behavior of complex primates. The questions you are about to work on are all examples of just this principle.

First we will consider an example that takes place on a physiological macro-scale, focusing on the evolution of human brain anatomy.

5 points:

A patient who was involved in an accident suffered extensive trauma to the back of his head. Having imaged his brain, the doctors determined that his occipital lobe was extensively damaged, and that the patient would be blind. The hospital where the patient received treatment happened to be a research hospital, so as part of the follow up, the patient was signed up to be part of a study of vision disorders conducted on the premises. When given a normal vision test, the patient reported to be totally blind – he could not identify any shapes, colors, or movement.
During the second part of the study, images of real life objects were presented to the patient while the researchers measured the patient's skin conductance. This parameter reflects the readiness of the skin to sweat in response to arousal, part of the fight-or-flight system, which is the organism's instinctive reaction to danger. To their big surprise, the researchers found that any time the patient was shown an image containing a snake-like object, their device detected a spike in skin conductance, meaning that the patient was having an emotional response to these images. Other images produced no such response. Same experiments with healthy individuals who participated in this study as a control group demonstrated the same spikes in skin conductance when snake-like objects *as well as other generally threatening images* were presented to them, and no change in conductance was observed when neutral images were presented.

Please provide an explanation of the results observed in this study. What does this study tell us about the evolutionary history of the primate brain?

10 points:

We can learn much about a particular gene's function by studying what happens when we remove that gene from the genome. It is thanks to this approach that we have been able to identify essential and non-essential genes in simple organisms like yeast and fruit flies, and then find their parallels in our human genome.

As far as essential genes go, their loss through somatic mutations results in the death of the mutant cell, or even of the entire organism, if the loss occurs at early embryonic stages. However, the genomes of most eukaryotes contain a high number of non-essential genes, any one of which may be lost without producing catastrophic effects on the cells carrying the mutation. Below, we provide a simplified description of an experiment that has been done with yeast.

During a study of yeast, two genes (let us denote them yA and yB) were found to be non-essential, because deletion of either one of them produced viable mutant yeast. However, deletion of both yA and yB was lethal to the yeast.

Question 1: Provide a possible explanation for this observation.

Genes yA and yB were found to have analogs in the human genome (let us call them hA and hB). A study of highly-proliferating tumors in a certain population of cancer patients showed that their tumors have a mutation in the hA gene that renders hA inactive.

Question 2. Can you think of how this information, together with the previous observations from experiments with yA and yB, may be used to design cancer therapy? Can you think of a reason why this approach may not work in humans? What experiments can you think of in order to check that concern?

COMPUTER SCIENCE

- **Your program should be written in Java or Python**
- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
  Please note that *codingground* site modified its structure and now all the input for the program run is entered on a separate tab. This is convenient as the same input can be used across multiple runs without re-entry
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc.
  No .pdf, .doc, .docx, etc!

Introduction:

Sigma1D is a one-dimensional universe. Everything in that universe has natural length measured in sigmeters. Unizon is a trading giant that sells all sorts of goods in Sigma1D online. For efficiency Unizon standardized on 10 sigmeter long boxes. Unizon optimizes packing of sold items in boxes in order to ship the least number of boxes and save on shipping costs. In the problems below items will be represented by their length in sigmeters.

5 points:

Write a program that takes an arbitrarily long list of items (i.e. their length in sigmeters). Your program should figure out how many boxes could be completely filled, if you can put no more than 2 items in one box.

For example, given items 10, 9, 8, 7, 5, 2, 2, 1 three boxes could be completely filled: 10; 9+1; 8+2. Remaining items 7, 5 and 2 sigmeters long cannot fill any boxes completely.

Your program should print the number of full boxes and the items that go into each of them.

10 points:

Write a program that takes an arbitrarily long list of of items (i.e. their length in sigmeters). Your program should propose efficient packing of these items into boxes. It should output a number of rows where each row represents one box and contains a list of items.

If any of the items the program receives is too large to fit into a box, then the program should list all of them and terminate.

The program should pack the items into fewer boxes than there are items, and if such a packing is impossible, then it should indicate so by printing "Only trivial packing is possible".

For example, given items 2, 4, 7, 4, 3, 5, 2, 3, 2, 1, the following can be a solution:
2, 4, 4
7, 3
5, 2, 3
2, 1

Try to be as efficient in packing as possible. Describe the intuitive reasoning of your algorithm in the program comments.