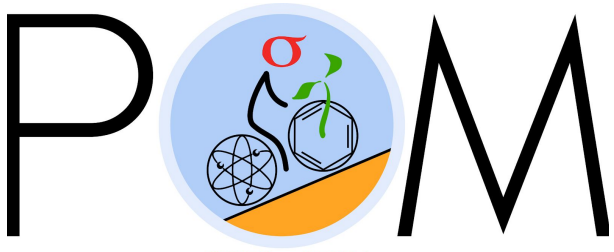
PROBLEM OF THE MONTH



October, 2020

2020 - 2021

MATHEMATICS

5 points:

Alice and Bob have independently proofread the same manuscript. Alice, the more experienced editor, has found 600 misprints while Bob managed to find only 400. When they compared their proofs they discovered that 300 of those misprints had been found by both of them. Estimate how many misprints in the manuscript are **not** found by either Bob or Alice. Please try to explain the assumptions you used in your estimate.

Hint: Assume that there are N misprints in the manuscript. Compute the probability for Alice to find a misprint and similarly for Bob. What is the probability that a given mistake is found by both of them?

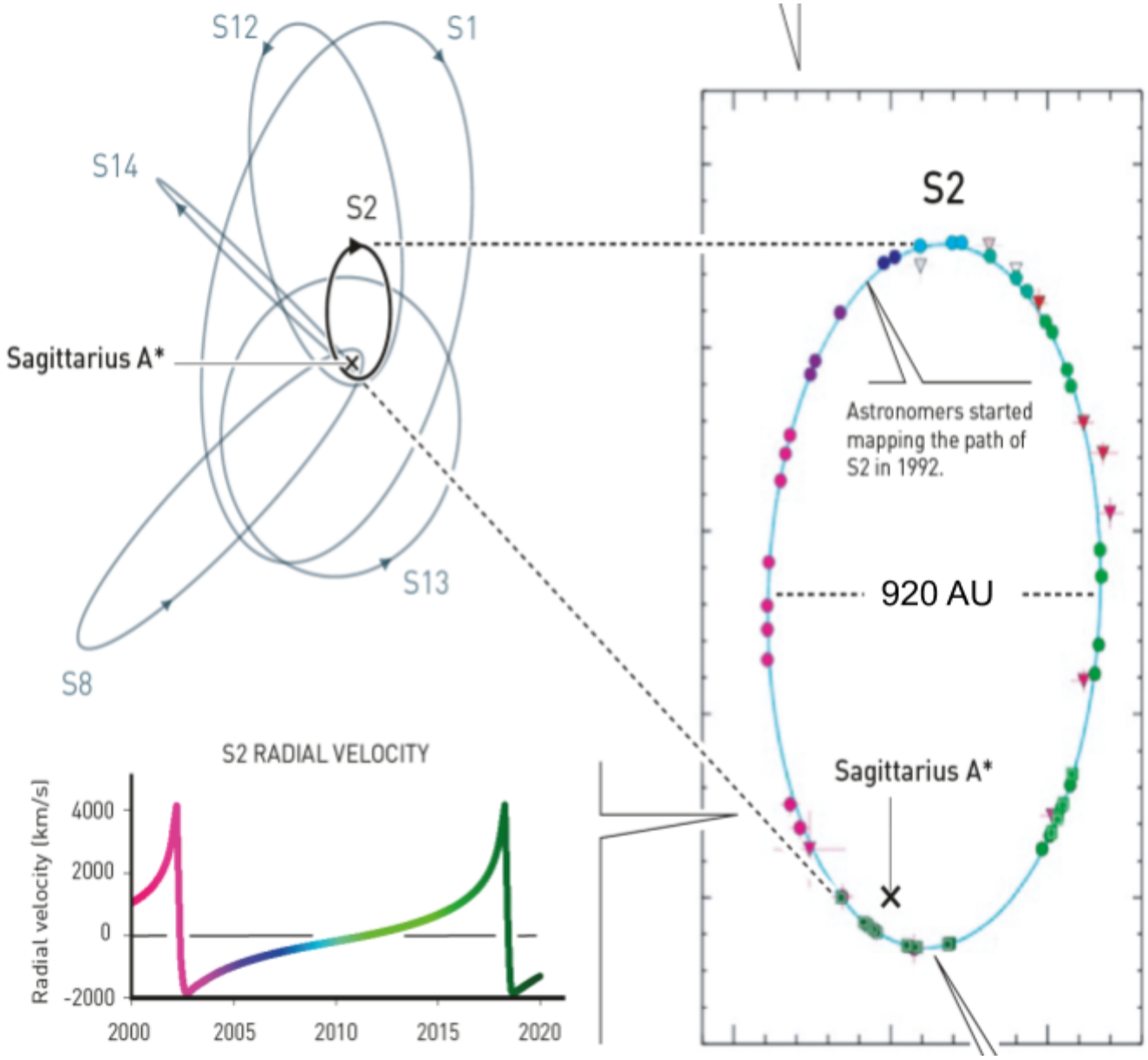
10 points: Humpty and Dumpty found a treasure chest with 140 coins. They decided to split it by playing the following game. They toss a fair coin and write down the results “Heads (H)” and “Tails (T)”. If they have three heads in a row the game is stopped and Humpty wins. If they get three tails in a row before heads, Dumpty is declared the winner. The winner of the game takes all 140 coins. Unfortunately, the game was interrupted by all the king's men when they had the following sequence of tosses: HTTHTHTT. Help Humpty and Dumpty split the coins. Dumpty claims that he had a bigger chance of winning and should take all the coins. Humpty claims that he also had a chance to win and should get his share. How many coins should be given to **Humpty** to take into account his chances to win in case the game would not be interrupted?

Hint: What would be the probability for Humpty to win the game if they continued to play? Try to relate such probabilities for sequences ending with HTT, HT, TH, THH.

PHYSICS

5 points: Sila and Numan are a pair of asteroids that orbit our Sun. In addition, they orbit each other, making one full turn around their common center of mass every 12.5 days. The distance between them is 2800 km. Find the total mass of both asteroids from these data. Gravitational Constant is G=6.67×10^{-11} N kg^{-2} m^2.

Hint: Let R be distance between the asteroids, and let their masses be m_1 and m_2. How far is each of them from their center of mass (which is the center of each ordit)?



10 points: 2020 Nobel prize in Physics was awarded (in part) for the discovery of a supermassive black hole Sagittarius A* in the center of our galaxy, Milky Way. Observations of several stars orbiting it, particularly star S2, have been used to determine the mass of this object. Use the figure above and information given on that figure to estimate that mass in terms of the mass of our Sun. Note that one astronomical unit (AU) is equal to the radius of Earth's orbit (when it is approximated by a circle). Assume that S2 orbit in the Figure is drawn to scale.

Hint: For a circular orbit, find how the period depends on the mass of the star (or a black hole) in the center, and on radius. Based on Kepler's third law, which parameter of the elliptical orbit determines the period?

CHEMISTRY

This month, the topic is: **Electrolytic dissociation**.
IMPORTANT! In this PoM season, we do an experiment: each month, an online lecture will be given. This lecture may be helpful for those who want to solve Chemistry PoMs, although it is not supposed to provide direct hints.
This month, the lecture will be on Oct 18 morning. At 10:30, a Zoom conference will start where September PoM solutions will be discussed. After that, approximately at 11:00, the lecture starts.
To join the Zoom conference, use this link:
https://us02web.zoom.us/j/4817690592?pwd=T2djSjRETEpDSHFZdWJpYlBTYzdjQT09
Meeting ID: 481 769 0592
Passcode: 879615
If you are unable to connect, email to mark.lukin@gmail.com

5 points:

This problem describes a real case that took place in one research laboratory at Stony Brook University. A researcher was preparing artificial seawater to maintain the culture of marine algae using that can be found here: http://cshprotocols.cshlp.org/content/2012/2/pdb.rec068270.full , and she found that there is no magnesium sulfate in the lab. That was a serious problem, because the seawater was needed urgently, otherwise the algal culture would die.
The researcher decided to modify the recipe and to make the same artificial water using a somewhat different set of chemicals. That worked fine, and the algal culture was saved.
What other common chemicals did the researcher use to prepare seawater of exactly the same composition? What is the amount of each component in grams per one liter of the solution?

Hint:

All components of the artificial seawater are electrolytes. That is a hint.

10 points:

Thymol blue and methyl orange are indicator dyes that change color in the presence of acids. You have three bottles with 0.1M solutions of HCl, HF, and acetic acid, and 0.01M solution of phosphoric acid. The bottles have no labels. Is it possible to figure out which is in each bottle using there two indicator solutions? Which acids are possible to discriminate, and which are not? Why?

Hint:

pH of the solution of an acid can be calculated from its dissociation constant and concentration.

BIOLOGY

5 points:

Genetically modified crops have been developed that produce a protein that makes the plant resistant to insect pests. Other genetic modification makes the crops more resistant to chemicals that kill plants (herbicides). Describe three potential biological risks of large-scale cultivation and use of such genetically modified plants. Describe three potential benefits of large-scale cultivation of such a plant (this may include potential for the biosphere, for humans, or for some specific biological species).

10 points:

In the case of COVID-19, public health experts at the CDC often make predictions about how different public policy decisions might influence outcomes over the next weeks or months. How do they do this? In epidemiology, mathematical models are often used to probe how different factors ("parameters") affect viral transmission, over an entire population, over time. The power of "models" is that, by running simulations, we can see the impact of multiple hypothetical scenarios--essentially running experiments "in silica."

Using this link, you can access a GUI and visualization of one viral model. The parameters of this model, for some particular virus, include: (1) the density of the population, (2) the population turnover (i.e., death & reproduction), (3) the degree of immunity, the degree of infectiousness (or transmissibility), and (4) the duration of infectiousness.

Assuming a closed community of 100 individuals, with a highly-infectious virus (80%), and an infectiousness duration of one year:

a) What happens to the total population if the virus is completely lethal (Virus X)?
b) What happens if the virus is only 50% lethal (Virus Y)?
c) In the 30-50% range (Virus Z), what kind of behavior do you see in terms of the viral population? Why.
d) As public health officer, with limited resources and with the sole perspective of saving the maximum number of lives, would it be a more effective one year strategy to reduce viral infectiousness by 20% (e.g., by promoting behaviors like social distancing and mask use) or by reducing viral mortality by 20% (e.g., by developing more effective medical treatments). Use the model to provide evidence for your public health policy. Would you make the same recommendation for both Viruses X and Y?

LINGUISTICS

5 points:

Consider the translations of the following phrases from an Indo-Iranian language:

1. *dusti hubi hamsoyai shumo* – “a good friend of your neighbor”
2. *hamsoyai dusti hubi shumo* – “a neighbor of your good friend”
3. *hamsoyai hubi dusti shumo* – “a good neighbor of your friend”

Determine the translations of individual words and explain your reasoning.

10 points:

Below you’re given the words in one of the ancient alphabets and their pronunciations.

𐰴𐰍𐰉	yagɨ	‘enemy’
𐰆𐰃𐰴𐰣	kɨlɨn	‘to make’
𐰃𐰇𐰴𐰞	bilge	‘wise’
𐰀𐰘	adak	‘leg’
𐰴𐰇	eki	‘two’
𐰃𐰋𐰀	kara	‘black’
𐰀𐰆𐰀𐰆	balbal	‘statue’
𐰇𐰇𐰀𐰴	ingek	‘cow’
𐰍𐰍𐰉	tag	‘mountain’
𐰉𐰚	yer	‘earth’
𐰘𐰴	ɨd	‘to send’
𐰋𐰀𐰆	bars	‘tiger’
𐰇𐰃	isig	‘labor’
𐰴𐰣𐰉𐰇	kelti	‘they came’
𐰉𐰴𐰃	esir	‘defeated’

a) Please write the following words:

elt	‘lead’
bark	‘house’
kan	‘blood’
ab	‘hunting’
altɨ	‘six’
bes	‘five’

birle ‘together’
kalɨn ‘fat’
bilig ‘title’
yablak ‘bad’

b) Figure out in how many ways the following word can be read and list them all.

COMPUTER SCIENCE

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files with specified in the problem names
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.
  No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!

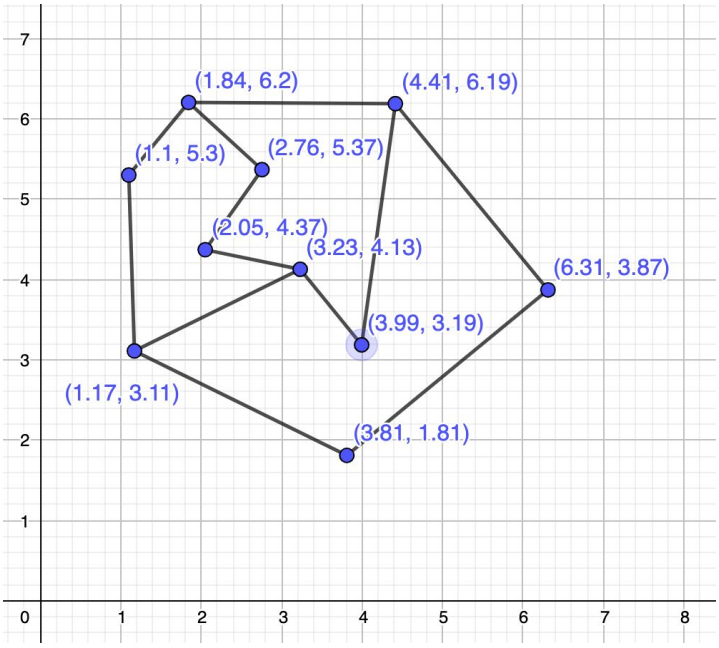
Introduction:

To maintain proper law and order in SigmaCity, Sigma police force is split into a set of precincts. Each police precinct serves an area in the shape of a polygon, which is defined as a set of (x,y) coordinates of the polygon's vertices (x and y are floating point numbers). Precinct areas do not overlap and completely cover the whole SigmaCity, which also has the shape of a polygon. Each precinct is assigned a unique number (positive integer).

5 points:

Your program should find the area of the largest police precinct in SigmaCity.

The program should read the input file **input.txt**. First line contains an integer number of police precincts in Sigma City. It is followed by that number of rows, one row per Sigma police precinct. Each row contains a series of space-separated values, starting with the precinct number and followed by pairs of (x,y) floating point coordinates of vertices defining the precinct area. Example input file:




```
3
1 1.17 3.11 1.1 5.3 1.84 6.2 2.76 5.37 2.05 4.37 3.23 4.13
2 1.84 6.2 4.41 6.19 3.99 3.19 3.23 4.13 2.05 4.37 2.76 5.37
3 3.99 3.19 4.41 6.19 6.31 3.87 3.81 1.81 1.17 3.11 3.23 4.13
```

Your program should produce the output file **output.txt**, which consists of a single floating point number representing the area of the largest precinct.

10 points:

Your program will be finding which police precinct covers a particular location in Sigma City given its coordinates.

The program should read the input file **input.txt**. First line contains an integer number of police precincts in Sigma City. It is followed by that number of rows, one row per Sigma police precinct. Each row contains a series of space-separated values, starting with the precinct number and followed by pairs of (x,y) floating point coordinates of vertices defining the precinct area. Finally, the last row contains x, y coordinates of the given location in Sigma City.
Example input file:

```
3
1 1.17 3.11 1.1 5.3 1.84 6.2 2.76 5.37 2.05 4.37 3.23 4.13
2 1.84 6.2 4.41 6.19 3.99 3.19 3.23 4.13 2.05 4.37 2.76 5.37
3 3.99 3.19 4.41 6.19 6.31 3.87 3.81 1.81 1.17 3.11 3.23 4.13
4.12 4.55
```

Your program should produce the output file **output.txt**, which consists of a single integer number representing the number of the precinct covering the given location or 0 if the given coordinates lay outside of Sigma City (i.e. outside of all its police precincts).