SigmaCamp's *Problem of the Month* Contest

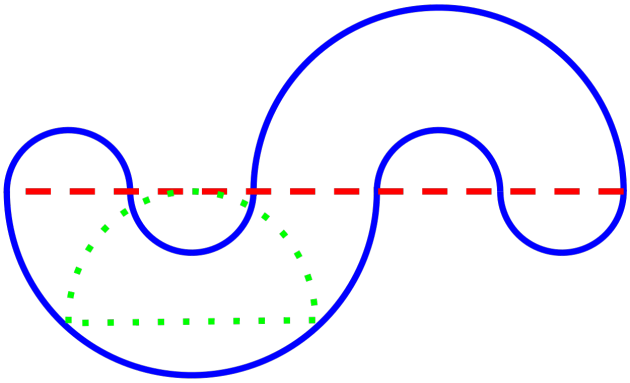
DECEMBER 2023

Mathematics

5 points:

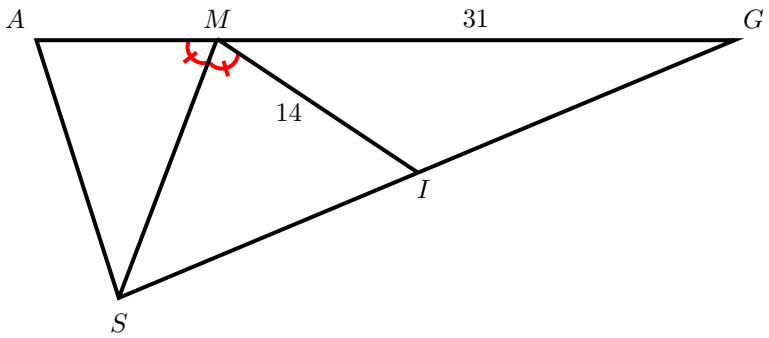
The figure below is comprised of a blue shape consisting of six blue semi-circles, along with a dotted green semi-circle. The small blue semi-circles all have the same diameter, equal to the thickness of the blue shape. The dotted red line passes through the centers of all the blue semi-circles. The dotted green semi-circle has a radius of 10, and is tangent to the dotted red line.

What is the area of the entire blue shape?

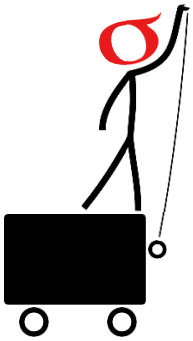


10 points:

In the following triangle, $IM = 14$ and $GM = 31$. If MS bisects $\angle IMA$, what is the ratio $SI : GI$?



Physics



5 points:

Recently, a star system was discovered where all six planets have perfectly synchronized orbits. The observed orbits go as follows:

- Planet 2 orbits the star twice for every three times planet 1 orbits.
- Planet 3 orbits twice for every three times planet 2 orbits
- Planet 4 orbits twice for every three times planet 3 orbits
- Planet 5 orbits three times for every four times planet 4 orbits.
- Planet 6 orbits three times for every four times planet 5 orbits.

Suppose planet 1 orbits the star at radius R. What are the orbital radii of all the other planets?

10 points:

Consider the same planetary system as the 5 point problem. Suppose an advanced civilization lives on planet 6, and they wish to send a probe to explore planet 1. This civilization has a very peculiar mode of launching interplanetary probes - they use a giant slingshot stationed on the planet's surface. Once a probe is launched, it has no way of correcting its orbit under its own power.

What is the minimum velocity with which the slingshot needs to fire the probe from the surface of planet 6 in order for the probe's orbit around the star to intersect with the orbit of planet 1? In which direction should the probe be launched? Neglect the escape velocity of planet 6.

In addition to the data given in the 5 point problem, take the mass of the star to be M and the orbital period of planet 1 to be T.

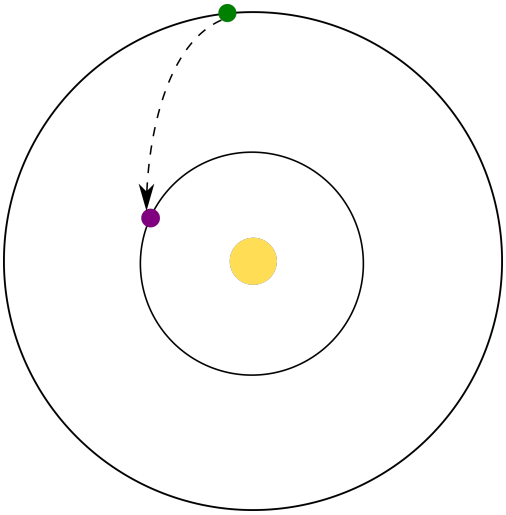
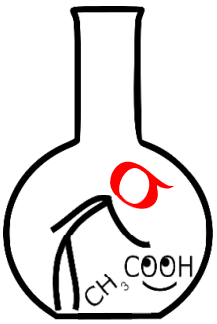


Figure 1: A depiction of a probe (the dashed trajectory) being sent from planet 6 to planet 1. The solid lines depict the circular orbits followed by the two planets around the central star. The other four planets between 6 and 1 are not pictured. While in flight, the probe is in a free orbit around the star and never accelerating under its own power.

Chemistry



5 points:

Scheme 1 (Fig. 2) shows the addition of hydrogen halides to alkenes. Although unsymmetrical alkenes can potentially yield two possible products, as shown in the scheme 1, usually only one product forms, and its structure obeys what is known as Markovnikov's rule. This rule is historically the first example of an empirical rule that predicts the outcome of an organic chemical reaction in which more than one isomer could potentially be formed.

Read about Markovnikov's rule by yourself and draw the structures of the products of the reaction between each of the five hydrocarbons shown below and HBr.

10 points:

Neutral species containing one carbon with one unpaired electron (a.k.a. free radicals) are very unstable and short living, and they are prone to recombination: two free radicals form a covalent bond by sharing their unpaired electrons. Some examples of recombination reaction are shown on Fig. 3. In addition, a cyclopropylmethyl radical **(1)** is especially unstable, and it very quickly rearrange to buten-4-yl radical **(6)**. This isomerization reaction is shown on Fig. 3.

One method to generate two different radicals simultaneously is shown at Scheme 2 (Fig. 3). This reaction is initiated by irradiating a diazo compound with UV light, as shown on Fig. 2. In this reaction, one very stable molecule of nitrogen forms, and two very active species, benzyl and cyclopropylmethyl radicals. It would be natural to expect that these two reactive species undergo recombination. However, whereas five recombination reactions are conceivable in this case (Fig. 3), in reality, when a dilute solution of the diazo compound in an inert solvent is irradiated with UV light, only four recombination reactions shown on Fig. 3 occur, whereas the fifth reaction does not.

Why?

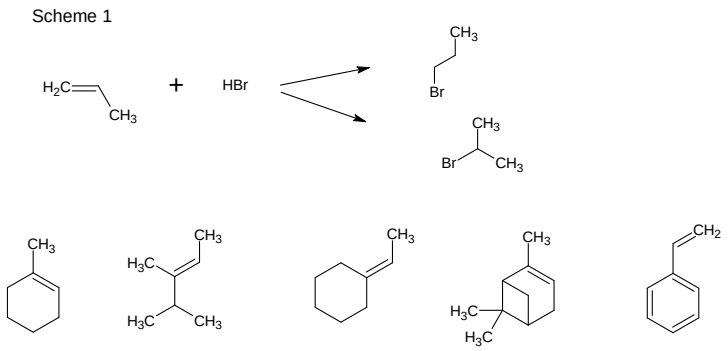
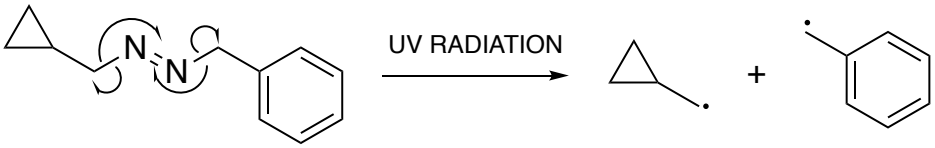
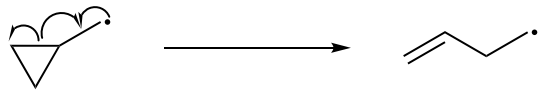


Figure 2: Addition of hydrogen bromide to alkenes.

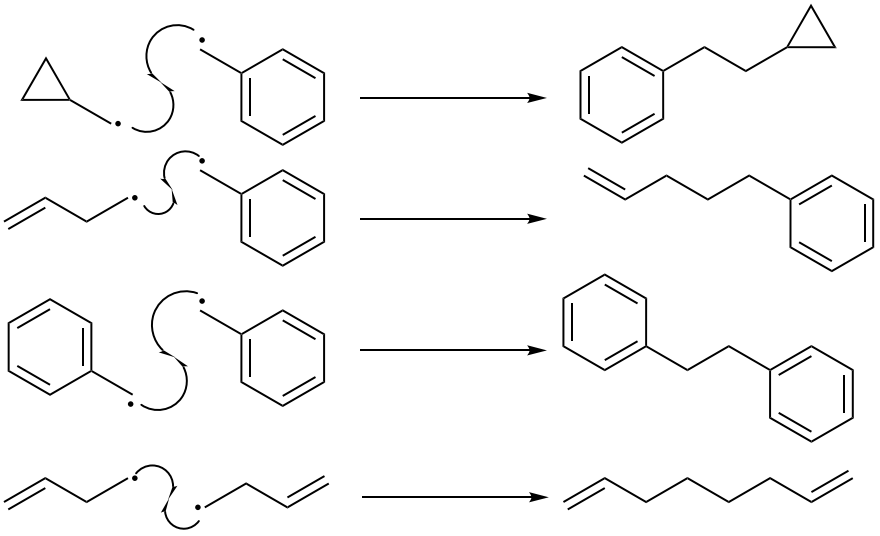
INITIATION



ISOMERIZATION



RECOMBINATION



WHY IS THIS REACTION NOT OBSERVED?

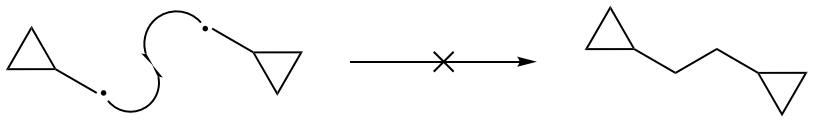
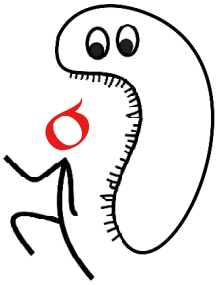


Figure 3

Biology

5 points:



All species in the Pandora ecosystem depicted "Avatar" are connected by means of Tsaheylu. According to the author, this deep connection among all living things creates a complex network of symbiotic relationships between all species. However, the existence of some categories of animals seems to be incompatible with the claim that all Pandoran species are involved in symbiotic relationships with each other. Existence of which species is incompatible with Tsaheylu?

10 points:

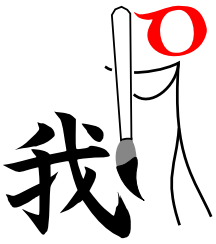
In May of 2236, Harvard University's starship Michel Foucault took off from the Logan cosmodrome in Boston. Its destination was the sixth planet in the planetary system HR16995 of the Orion nebula where the university had recently built a new high resolution neutrino observatory. During this trip, Michel Foucault made a stop at the third planet orbiting the same star to unload one passenger Sarah Tamagotchi. Sarah was a graduate student finishing their terraforming experiment. Their PhD thesis was aimed at creating a de novo ecosystem that would be devoid of any kind of violence, competition and oppression. In accordance with the concepts of tolerance, mutualism, and inclusiveness, any form of violence or oppression was strongly condemned by a society. However, since elimination of predators from the Earth's biosphere appeared to be impossible without direct or indirect violence, the Earth personkind had to tolerate predators as a necessary evil. Being an outstanding thinker, Sarah came up with a brilliant idea: if we cannot eliminate violence from our own biosphere, we can create a new one from scratch. Their highly innovative project was immediately approved, and Harvard University rented the unnamed planet in the HR16995 star system for 999 million years specifically for this study.

During their last visit, Sarah set up all the necessary parameters on the planet - which at that moment was entirely lifeless - that would be favorable for development of a new biosphere. According to their computational model, run on the Harvard's quantum computer cluster (2024 cores, 2^{2024} qubits each), an ecosystem based on the principles of symbiosis was expected to quickly give rise to multicellular life and transform the whole planet into a green paradise where all species would flourish peacefully and cooperatively. As their PhD study could not last longer than 4 years, Sarah decided to use a portable bio-accelerator (a special device that accelerates metabolic and evolutionary processes 1,000,000 times). Sarah turned it on before departure and expected that during their absence the evolution would progress enough to yield visible results.

When Sarah arrived on the planet for a second time, it was covered by various organisms that were crawling, jumping, or simply sitting on rocks/the sea floor. Some of them developed photosynthesis and became similar to Earth plants, while others were pretty mobile and looked like Earth animals. However, when Sarah started to study the metabolism of those creatures and the material transfer pathways between species, the results were far from expected.

What did Sarah observe and why?

Linguistics & Applied Sciences



5 points:

A creole language is a language that forms naturally from a combination and simplification of parts of multiple languages. For example, many creoles based on European languages developed during the period of colonization. The following creole language has a base in English, as you can probably tell. The same passage is given in this creole language and in English.

Identify three ways the grammar or syntax (not vocabulary) of the creole language differs from English, and give the creole words for the following English words: **To eat, Really, To think, Restaurant, Nine**

Passage:

O gutpela! Bipo, mi no save yusim chop stik. Sapos mi save go long ples ol i save yusim chop stik, bai mi save askim tasol long spun na fok o naip bilong kaikai wantaim. Tasol long yusim chop stik, nougat tru! Oke las taim tasol mi bin go long Singapo mi go skul na mi stap nainpela mun long Singapo. Orait long hap ya, em i hat long askim tumas fok na spun long wanem sampela taim haus kaikai i no gat fok na spun. Orait mi bin ting ating em i mo gut sapos mi lainim pasin bilong yusim chop stik. Orait nau long Singapo yet mi save nau long yusim chop stik. Chop stik i gutpela samtin tru long yusim.

Oh good! Before, I did not know how to use chopsticks. If I went to a place where they use chopsticks I just asked for a spoon and a fork or knife to eat with. But as for using chopsticks, no way! Okay the last time I went to Singapore I went to study and I was in Singapore for nine months. There it is difficult to ask all the time for forks and spoons because sometimes the restaurant doesn't have a fork and spoon. Consequently I thought, "I think it would be better if I learnt how to use chopsticks." So there in Singapore then I learnt to use chopsticks. Chopsticks are really good things to use.

10 points:

The following creole language has a base in a Romance language. Match the following sentences in the unknown creole language to their English translations.

1. He did not want rain from the sky.	A) El a mira e solo ayente den e laira.
2. The rain falls from the dark cloud.	B) E ta muhá pasobra e awacero.
3. The bird flies behind the dog.	C) E solo ta peor cu e awacero.
4. He sees clouds in the sky.	D) E cachó ta mira e para scur.
5. He saw the hot sun in the sky.	E) El no ker awacero fuera di e laira.
6. The dog sees the dark bird.	F) E para a bula patras e cachó
7. The sun is worse than the rain.	G) E awacero a bin' cai fuera di e nubia scur.
8. He is wet because of the rain.	H) E ta mira nubianan den e laira.

Languages have different phrases and figures of speech based on the culture and environment, but some concepts can be found in multiple languages with a twist for local context. Using what you learned above, the given vocabulary, and intuition of English cognates, roughly translate each of the phrases below into English and name which English phrase it corresponds to. Example: Si heru ta ayente, laga strika = "If iron is hot, let's iron" = Strike while the iron is hot

1. Sali for di boca di leon, bin' cai den boca di tribon.
    - Sali - to go out; Boca - mouth; Tribon - shark;
2. Cachó cu ta grita no ta morde.
    - Grita - to bark; Morde - to bite;
3. Awa no ta muha macacu pa di dos bes.

- Macacu - monkey; Dos bes - twice

4. E ke bula cu ala di manteca..
    - Ala - wings; Manteca - butter
5. Mas bal un para den man cu dies den laira.
    - Mas bal - more value; Man - hand; Dies - ten
6. Maske con scur un nubia ta, su delineacionan ta di plata.
    - Maske - however; plata - silver

Computer Science

- Your program should be written in Java or Python-3.
- No GUI should be used in your program (e.g. `easygui` in Python).
- All the input and output should be done through files named as specified in the problem statement.
- Java programs should be submitted in a file with extension `.java`; Python-3 programs should be submitted in a file with extension `.py`. **No `.txt`, `.dat`, `.pdf`, `.doc`, `.docx`, etc. Programs submitted in the incorrect format will not receive any points!**

A large network of gears is mounted to the wall of the Sigma Gift Factory, including a starting gear that rotates clockwise, and an ending gear.

The Factory wishes to determine which of the following situations occurs if the starting gear rotates clockwise:

- The ending gear rotates clockwise,
- The ending gear rotates counterclockwise,
- The ending gear is not connected to the starting gear, or
- The system of gears is locked and unable to rotate.

In addition, if the ending gear rotates, the Factory wishes to know how quickly it does so relative to the starting gear.

Recall that if gears are in a "cycle", they will only spin if there is an even number of gears in the cycle. Otherwise, the configuration will be locked. Furthermore, adjacent gears that rotate will do so in opposite directions. See the figures below.

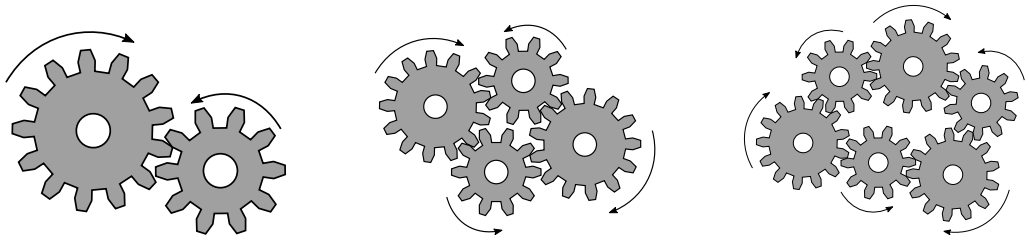


Figure 4: With an even cycle of gears, the system rotates.

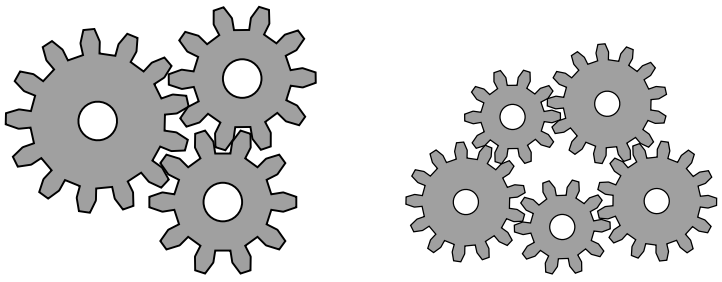


Figure 5: With an odd cycle of gears, the system is locked.

For both problems, assume that no gears overlap, and gears whose circumferences touch rotate without slipping. Furthermore, assume that the speed of rotation of the ending gear relative to the starting gear is a *rational number* (can be expressed as a fraction).

Lastly, instead of working with the radii of the gears, we give the squares of the radii. This is because the radii themselves may not be integers (especially for the 10 point question).

5 points:

For this problem, we assume that all the gears lie along a single line, so you will only be provided with the x-coordinate of each gear's center. Write a program that receives a list of the positions of the centers of the gears and the squares of their radii, and determine whether the ending gear rotates, and if so, how quickly it rotates relative to the starting gear.

Your program should read the input file `input.txt`. The first line specifies the total number n of gears on the Sigma Gift Factory wall. The next n lines contain two space separated integers in the format "`X R`", where `X` represents the x-coordinate of each gear (could be negative!), and `R` represents the *square* of its radius. The first line represents the starting gear, and the last line represents the ending gear.

Your program should produce the file `output.txt` which contains either

- "`CLOCKWISE` N/D" if the ending gear is rotating clockwise, where N/D denotes the *simplified fraction* of the number of revolutions the ending gear completes when the starting gear completes one revolution. If this number is an integer, output it as "$N/1$".
- "`COUNTERCLOCKWISE` N/D" if the ending gear is rotating counterclockwise, with N and D as above,
- "`NOT CONNECTED`" if the ending gear is not connected to the starting gear, or
- "`LOCKED`" if the system of gears is locked.

Sample Input 1:

```
2
1 9
8 16
```

Sample Output 1:

```
COUNTERCLOCKWISE 3/4
```

Sample Explanation 1:

The radii of the starting gear and the ending gear are 3 and 4 units respectively. As they are 7 units apart, the gears are connected, and the ending gear rotates 3/4 as fast as the starting gear counterclockwise (since the starting gear spins clockwise).

Sample Input 2:

```
5
0 9
8 25
19 36
33 49
41 1
```

Sample Output 2:

```
NOT CONNECTED
```

Sample Explanation 2:

The radii of the gears are $3, 5, 6, 7$, and 1. Gears 1-3 and 4-5 are connected. But gears 3 and 4 are not connected, so gears 4 and 5 do not spin.

10 points:

Now, we assume that the gears can be at any point on the wall, and their centers will be given both x- and y-coordinates. Write a program that receives a list of the positions of the centers of the gears and the squares of their radii, and determine whether the ending gear rotates, and if so, how quickly it rotates relative to the starting gear.

Your program should read the input file `input.txt`. The first line specifies the total number n of gears on the Sigma Gift Factory wall. The next n lines contain three space separated integers in the format "`X Y R`", where `X` represents the x-coordinate of each gear (could be negative!), `Y` represents its y-coordinate (could be negative!), and `R` representing the *square* of its radius. The first line represents the starting gear, and the last line represents the ending gear.

Your program should produce the file `output.txt` with the same specifications as in the 5 point question.

Sample Input:

```
3
0 0 16
8 0 16
4 -3 1
```

Sample Output:

```
LOCKED
```

Sample Explanation:

The three gears are all touching each other and form an odd cycle, and the system is consequently locked.