SigmaCamp's *Problem of the Month* Contest

JANUARY 2024

Mathematics

5 points:

You are given the fraction 2/3. You are allowed to perform the following operations arbitrarily many times in any order:

- Add 2023 to the fraction's numerator
- Add 2024 to the fraction's denominator

Is it possible, using only these operations, to arrive at the fraction 3/4?

Hint:

Think about odd/even.

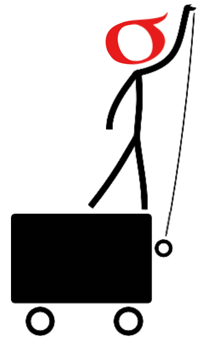
10 points:

On the first day of SigmaCamp, every camper gets a name tag with their name on it. As part of an icebreaker activity, the counselors of team Omega, which has 7 campers, decide to give every camper another camper's name tag. There are $7! = 5040$ ways to distribute name tags to all 7 campers, and 1854 ways to give every name tag to the incorrect camper. How many ways are there to give every name tag to the incorrect camper, such that no two campers can switch to both get their correct name tags?

Hint:

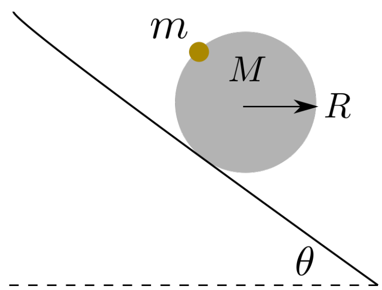
You may find the inclusion/exclusion principle helpful.

Physics



5 points:

A uniform disc of mass M and radius R is on an incline of angle θ (see Figure). The coefficient of static friction is large enough such that the disc never slips. A point mass of mass m is attached to the circumference of the disc. What is the minimum mass m can have so that the entire disc-mass system can rest in *some* stable configuration without rolling down the incline? Your configuration should include the position of the point mass relative to the center of the disc.

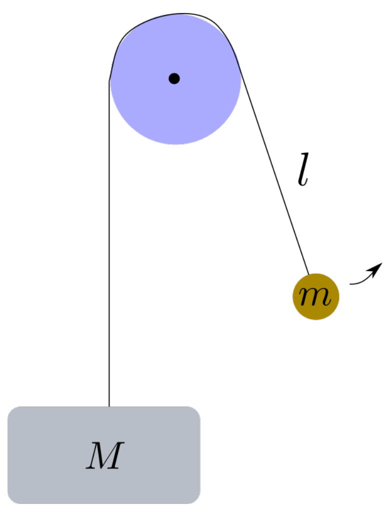


Hint:

No hint this month.

10 points:

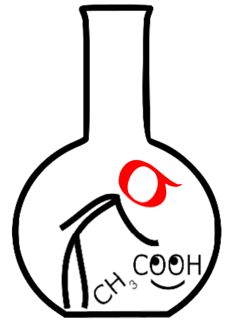
Around an ideal pulley, a weight of mass M is counterbalanced by a pendulum (see Figure). The pendulum has effective length l and mass m. At what frequency must the pendulum oscillate with so that on average (meaning averaged over time) neither the pendulum nor the mass descend?



Hint:

No hint this month.

Chemistry



5 points:

A lab technician accidentally added approximately 5 g of Li_2SO_4 into the bottle containing 100 g of $CuSO_4 \cdot H_2O$. Is it possible to obtain 50 grams of nearly pure (more than 99% purity) $CuSO_4 \cdot H_2O$ from this mixture using the equipment that you can find in a standard chemistry laboratory? If yes, please, describe the procedure.

Hint:

Solubility.

10 points:

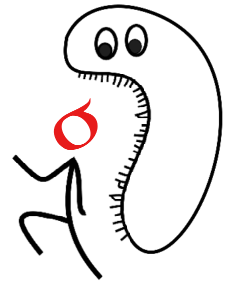
Water boils at 100°, and acetonitrile boils at 82°. Is it possible to obtain 1 L of 99% acetonitrile (less than 1% of water) by distilling 3 L of 80% aqueous acetonitrile (80% of acetonitrile, 20% of water) at the atmospheric pressure? If your answer is "yes", describe the procedure; if you answer is "no", explain the reason.

Hint:

Azeotrope.

Biology

5 points:



A SigmaCamp researcher is breeding flies - with pink/red eye color and with or without wings - for his semilab, but he's startled by the dining hall bell and accidentally opens a cage. The cage contained the parents of the following offspring, so he no longer knows their genotype. Help him figure out the genotype of each of the parents from the ratios of their offspring as shown below, and answer the question posed in his semilab: are the genes that code for eye color and wings linked? Which cross supports your answer?

Parents	Pink, wings	Pink, Wingless	Red, wings	Red, Wingless
Pink, wings x Pink, wings	84	27	0	0
Red, wings x Red, wingless	20	17	64	65
Red, wings x Red, wingless	0	0	74	80
Red, wings x Red, wings	29	12	92	35
Red, wings x Pink, wingless	14	13	17	15

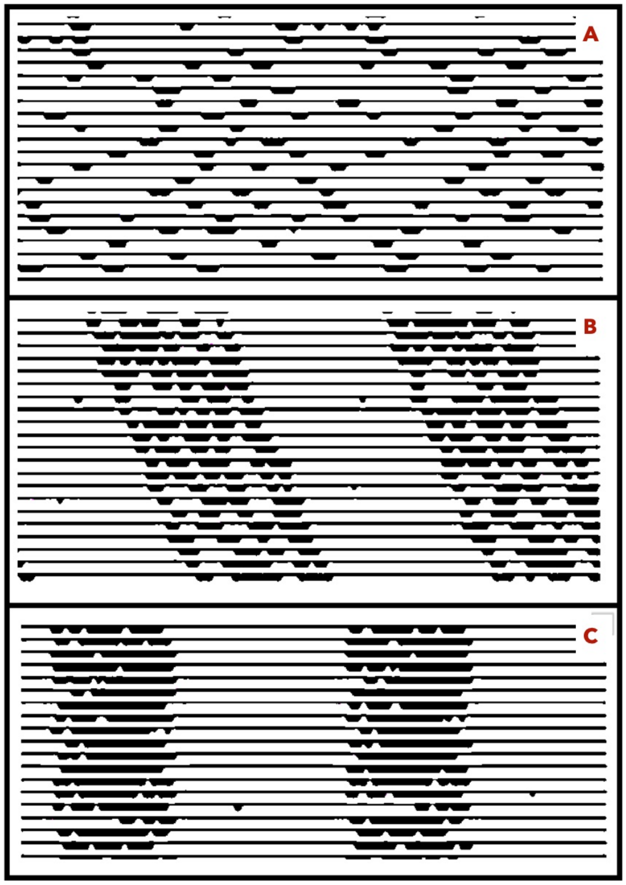
You should show your answers for the genotypes of all the parents, but only need to fully work out one cross to support your answer.

Hint:

No hint this month.

10 points:

Not wanting to risk his research on flies flying away, the SigmaCamp researcher switches to mice. He has one group raised in a normal 12 hour day/night cycle, one raised in total darkness, and one raised in total light. He was studying their activity on running wheels, and plotted his data on the graphs shown below. Unfortunately, the flies from the previous question returned to take their revenge on him and stole all his notes, so now he no longer knows which graph corresponds to which group. Match each graph to each experimental group, and explain your answer, clearly denoting which parts of each graph correspond to subjective night and day.



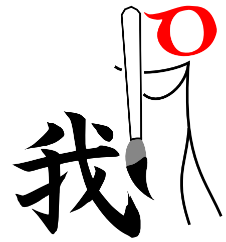
Is there a way he could mimic the effect seen in graph A using surgery? Explain.

Hint:

No hint this month.

Linguistics & Applied Sciences

5 points:



Some languages, such as Latin, change the endings of nouns and adjectives based on the role of the word in the sentence. For example, the ending on the word "boy" would change depending on if the boy was the subject of the sentence or the direct object of the sentence. These changes in endings, called declining, are fairly common among languages. A handful of languages also decline their articles, such as "the." In this ancient language, "the" is declined based on the role the noun it is paired with takes in the sentence. Based on the following sentences, describe the rules of declining "the." In other words, describe which versions of "the" correspond to what role in the sentence, which includes but is not limited to subject, direct object, indirect object, and prepositional phrase. If you would like to describe these rules using cases, you may, but it is not required.

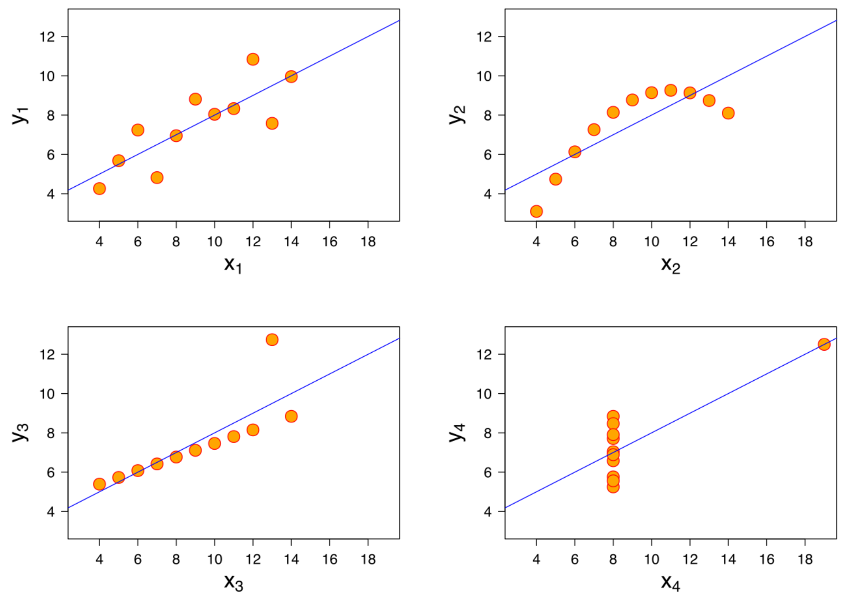
1. The people are coming from the plane. = **Ta ny sleih çheet voish yn etlan.**
2. It was up in the hills. = **Row eh heose ayns ny crink.**
3. The boy carried books. = **Y scollag dymmyrk eh lioaraghyn.**
4. They go back to the house. = **T'ad goll erash dys thie**.
5. Take the book. = **Gow o lioaragh.**
6. Bob drives the car around the cliffs. = **Ta Bob gimman y gleashtan mygeayrt ny finganyn.**
7. They park the car and eat their friend's sandwiches. = **T'ad pairkal y gleashtan as gee braghtanyn nyn garrey.**

Hint:

No hint this month.

10 points:

Anscombe's quartet is a collection of four different sets of 11 two-dimensional coordinates which all have the same simple descriptive statistics (mean, variance, etc), but look very different when plotted on a graph.



The four datasets above all have:

- Mean of x values of 9.0
- Mean of y values of 7.5
- Sample variance of $x = 11.0$
- Sample variance of $y = 4.1$
- Correlation coefficient of $r = 0.81$
- Linear regression (line of best fit): $y = 3.0 + 0.50x$
- A total of 11 data points

The values of the four data sets can be found in this Wikipedia article:

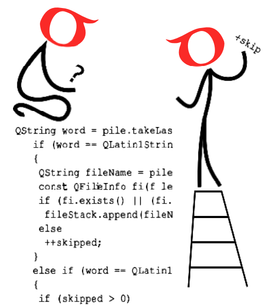
https://en.wikipedia.org/wiki/Anscombe's_quartet/

Create a fifth dataset that will match the quartet. Full credit will only be given for sets that have the same descriptive characteristics and also look **significantly** different than any of the 4 plots above, but partial credit will still be given for datasets with the same characteristics but similar-looking graphs.

Hint:

No hint this month.

Computer Science



- Your program should be written in Java or Python-3.
- No GUI should be used in your program (e.g. `easygui` in Python).
- All the input and output should be done through files named as specified in the problem statement.
- Java programs should be submitted in a file with extension `.java`; Python-3 programs should be submitted in a file with extension `.py`. **No `.txt`, `.dat`, `.pdf`, `.doc`, `.docx`, etc. Programs submitted in the incorrect format will not receive any points!**

SigmaCampers are building a tower in the Game Room out of wooden blocks. However, this year, instead of using wooden blocks of the same height, the campers find that the dimensions of the wooden blocks are $1 \times 1 \times b^n$, where $b \geq 2$ is a positive integer and n is a non-negative integer (n may differ from block to block). For example, a block may be $b^0 = 1$ centimeter tall, another may be b^2 centimeters, another b^5 centimeters, etc.

The campers wish to determine if it is possible to build a tower of a certain height h using these blocks by vertically stacking them, such that the tower's dimensions are $1 \times 1 \times h$.

5 points:

For this problem, assume that $b = 2$ and that each wooden block has a *unique height*.

Write a program that receives a list of the heights of the wooden blocks and the desired tower height, and determine if it possible to create a tower of *exactly* that height using the blocks provided, and if so, determine which blocks to use.

Your program should read the input file `input.txt`, which contains three lines:

- The first line contains a single integer m representing the total number of wooden blocks available.
- The next line contains m unique space-separated non-negative integers that are guaranteed to be of the form 2^n (where n is a non-negative integer), representing the heights of the blocks.
- The third line contains a positive integer h representing the desired height of the tower.

Your program should produce the file `output.txt`, which contains either:

- A space-separated list of integers, sorted in ascending order, of the heights of the wooden blocks used to construct the tower of height h, if it is possible to construct a tower of height exactly h using the provided blocks, or
- "`IMPOSSIBLE`" otherwise.

Sample Input 1:

```
5
1 4 16 2 32
51
```

Sample Output 1:

```
1 2 16 32
```

Sample Input 2:

```
4
4 2 16 128
5
```

Sample Output 2:

```
IMPOSSIBLE
```

Sample Explanation 2:

The height of the tower must be *exactly* 5, which cannot be attained with the given blocks.

Sample Output 3:

```
0

5
```

Sample Output 3:

```
IMPOSSIBLE
```

Hint:

Think about binary.

10 points:

For this problem, b can be arbitrary, and there can be multiple wooden blocks of the same height.

Write a program that receives a list of the heights of the wooden blocks and the desired tower height, and determine if it possible to create a tower of *exactly* that height using the blocks provided.

Your program should read the input file `input.txt`, which contains three lines:

- The first line contains two space-separated integers m and b, where m is the total number of wooden blocks available, and b is the base of the exponent for the blocks' lengths. It is guaranteed that $b \geq 2$.
- The next line contains m non-negative integers that are guaranteed to be of the form b^n (where n is a non-negative integer), representing the heights of the blocks.
- The third line contains a positive integer h representing the desired height of the tower.

Your program should produce the file `output.txt` which contains either:

- "`POSSIBLE`", if it is possible to construct a tower of height exactly h using the provided blocks, or
- "`IMPOSSIBLE`" otherwise.

Note: Do not use a "bruteforce" approach for the 10pt (such as checking all combinations of blocks). Any solution using such an approach will receive a maximum of 1 point.

Specifically, we require that the runtime of your algorithm is *below-exponential-time in m*, meaning that the running time of your algorithm does not grow exponentially with m. An appropriate solution should not take more than a minute to run on large m (such as $m = 25, \ldots, 30$).

Sample Input 1:

```
7 3
3 9 1 3 3 1 27
20
```

Sample Output 1:

```
POSSIBLE
```

Sample Explanation 1:

The tower can be attained by using the blocks of heights $1, 1, 3, 3, 3$, and 9.

Sample Input 2:

```
11 5
125 1 625 1 5 5 25 5 1 25 1
70
```

Sample Output 2:

```
IMPOSSIBLE
```

Sample Explanation 2:

The height of the tower must be *exactly* 70, which cannot be attained with the given blocks.

Sample Input 3:

```
0 2

10
```

Sample Output 3:

```
IMPOSSIBLE
```

Hint:

Numbers can be represented in other bases, not just binary and decimal!