SigmaCamp's *Problem of the Month* Contest

NOVEMBER 2023

Mathematics

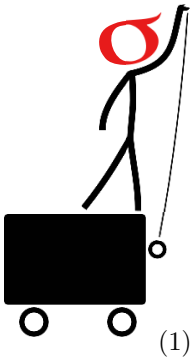
5 points:

You're a famous detective and you're trying to solve a second murder. You know that the murderer was one of exactly 6 suspects. Among the suspects, one was actually a witness of the crime, but you don't know who this is. The witness is afraid of the murderer and will not say anything if the murderer will be able to hear it. What you can do is repeatedly select a group of people to go to a separate room. There you can ask them if anyone knows who the murderer is. If the witness is in the room and the murderer is not, they will speak up. You could take all 6 people aside one-by-one, but that will take a long time. How small of a number of groups can you take aside to guarantee that the witness speaks up? (The smaller the number of groups you need in your strategy and the cleaner your solution, the more points you'll get!)

10 points:

You're a famous detective and you're trying to solve a murder. You know that the murderer was one of exactly 4 suspects: A, B, C, or D. You have 4 witnesses, P, Q, R, and S, who all know who the murderer is. However, exactly one of the witnesses always lies (the other witnesses always tell the truth) and you don't know which one! You are allowed to ask *each* of your witnesses *exactly one question* of the form "Is one of [some improper subset of A,B,C,D] the murderer?". The sets of suspects can be the same or different in your questions, but your questions must be submitted in writing in advance, so you do not get to find out the answer to one question before deciding what to ask the next witness. Your brilliant detective mind realizes that there is a guaranteed way to establish who the murderer was as well as which witness is lying! What sets of people do you ask about and why does that always let you find the murderer?

Physics



5 points:

Two friends, Simplico and Sagredo, are discussing a container of water on their table (pictured in Fig. 1). The container is peculiarly shaped – its base is perfectly square, with side length L_1, but at height H_0 its neck contracts to a smaller square of side length L_0.

Simplicio brags to Sagredo about an interesting fact he recently learned from their mutual friend Salviati. "Sagredo," he says, "did you know that the pressure of stationary water in a container is precisely determined by its depth below the surface? At depth h, the water pressure P is given by the formula

$$P = \rho g h, \tag{1}$$

where g is the gravitational constant, and ρ is the density of the water.

"For example," he goes on, "in this container, the water at the base is at height H below the surface. The pressure at the base is $\rho g H$, and so I can immediately deduce that the total force exerted by the container on the table must be $L_1^2 \rho g H$. How neat is that?"

Sagredo ponders this for a second. "But Simplicio," he asks, "the force the container exterts on the table should be equal to its weight. How is it that you've managed to deduce the weight without considering at all the weight or volume?"

"That's a great question," agrees Simplicio, confused. "But I am one hundred percent sure I understood Salviati correctly. What am I doing wrong?"

Can you help resolve Simplicio and Sagredo's confusion? What is the correct value for the force of the container on the table? What did Simplicio do wrong when calculating the force, and how can he correct his mistake? Ignore the weight of the container itself, and assume that only the water's weight matters.

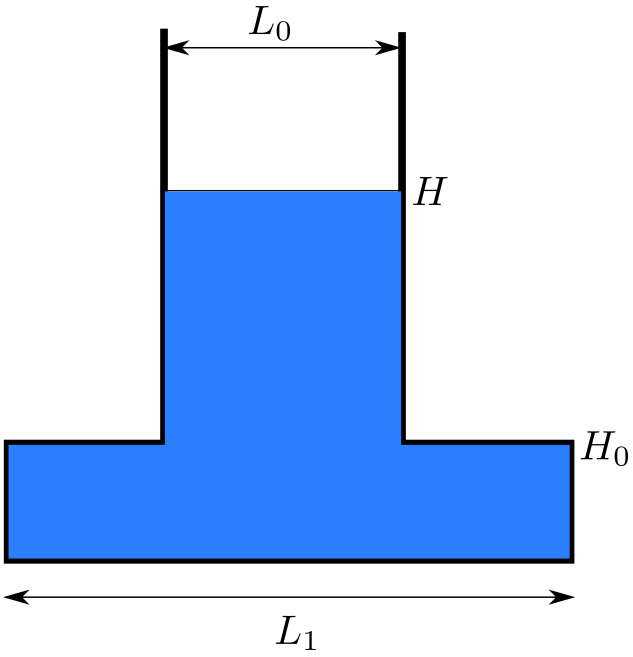
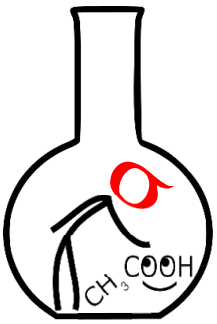


Figure 1: The shape of the water container Simplicio and Sagredo are discussing, as viewed from the side. The container is perfectly square when viewed from the top, and is a perfect square with side length L_1 narrowing to another prefect square of side length L_0 at height H_0.

10 points:

When you cover the mouth of a running hose with your finger, the water sprays. A typical garden hose has about a 1cm diameter, and a water pressure of about 50 N/cm^2. If you point the hose perfectly horizontal at a height of 1m, how far away from you will the water hit the ground? What percentage (by area) of the mouth of the hose will you have to plug with your finger in order to spray the water a distance of 10m? Ignore air resistance in your calculations.

Chemistry



5 points:

Circle all the explosive molecules, and justify your answers.

10 points:

Using materials from you kitchen or garage, find the best chemical that accelerates hardening of superglue. Propose the experiment to measure the acceleration/deceleration effect of additives, and do it. Provide a description of your experimental protocol (including photos of your experimental setup, if possible), your results and your conclusion.

If you want, you may follow a standard scientific article format, i.e. Part one: "Introduction" (a general idea), Part two: "Experiment" (this part describes technical details of your experiment), Part three: "Results" (this part describes what you observed during your experiment), Part four: "Discussion" (this part includes your interpretations of the results obtained and your conclusion). However, this format is not mandatory, you can write your solution in any form.

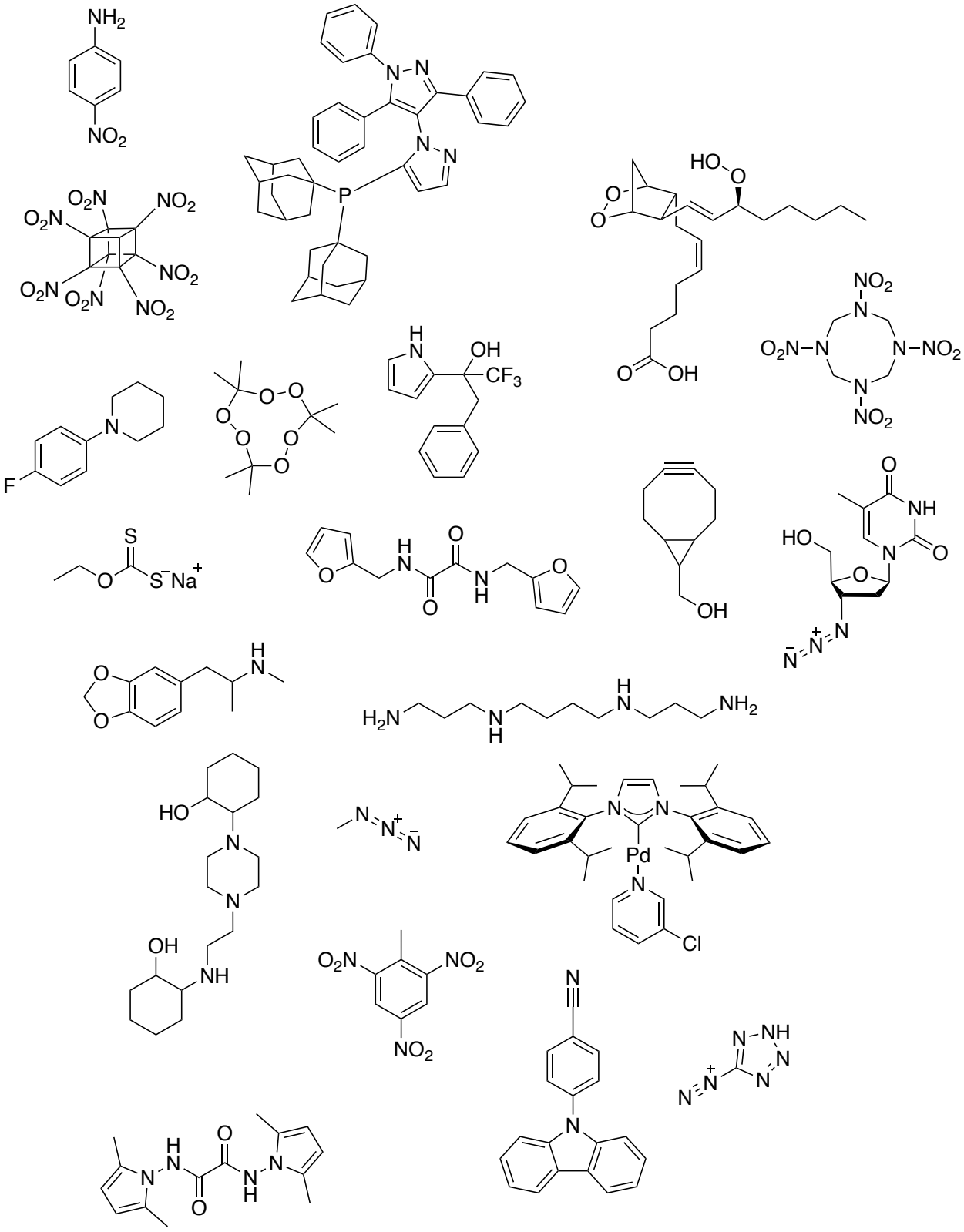
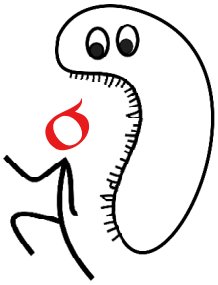


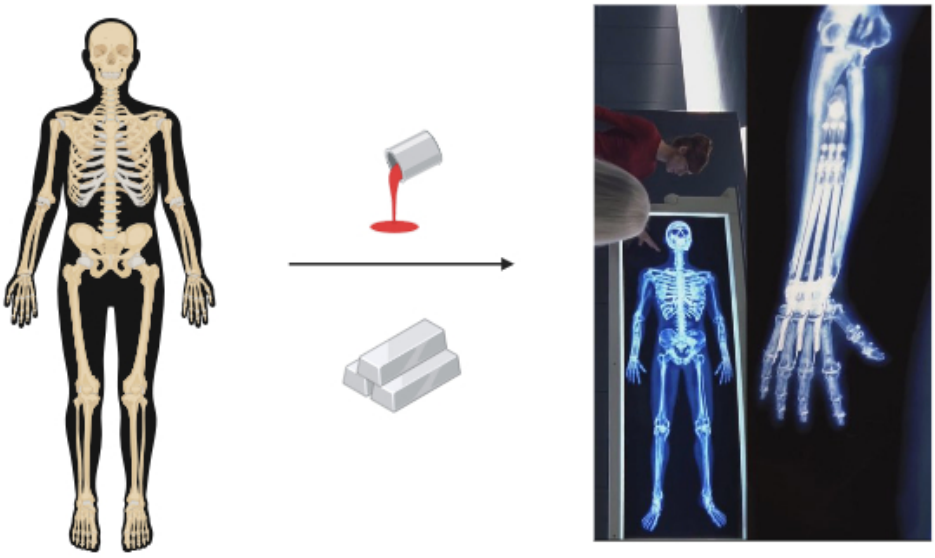
Figure 2: Possibly explosive compounds

Biology



5 points:

The following question was asked to a popular generative AI chat-bot, and the answer it returned is shown below. AI generated answers are not always entirely accurate, and often fail to successfully answer the question. Which aspects of the AI's answer shown below are accurate, and which are not? Justify your choices and provide the correct answers for the parts of the AI's answer that were incorrect. Keep in mind that in some cases the AI's answer may be right but its reasoning is wrong, or vice versa!



A character from a popular movie franchise undergoes the procedure shown below in order to acquire an 'adamantium' (hypo-allergenic, non-reactive, same density as bone) plated skeleton. Assuming he retains his superhuman ability to heal any wound, why is the procedure still likely to end in his death? What new dietary cravings is he likely to have before dying?

AI Answer:

Reasons for his probable death:

1. *Toxicity: Most metals, including hypothetical ones like adamantium, could be highly toxic to the body. Even if the character can heal tissue damage, the body might not be able to handle the systemic toxicity caused by the metal, leading to organ failure.*

2. *Immune Response: The body's immune system might recognize the metal as a foreign object and launch an aggressive immune response. While the healing factor could repair any physical damage, it might not be able to fully counteract a systemic immune reaction.*

3. *Thermal Conductivity and Electrical Conductivity: Metals conduct heat and electricity. The introduction of a large amount of metal into the body could disrupt the body's natural thermal regulation and electrical signals, potentially causing fatal disturbances in cardiac and neurological functions.*

4. *Weight and Density: The weight and density of the metal could physically burden the body's structure and organs, possibly leading to skeletal and muscular issues that even a superhuman healing ability might not fully rectify.*

Regarding dietary cravings, the character might experience the following:

1. *High Iron Intake: Given the metabolic demands of healing and incorporating a metal into the skeleton, there might be an increased need for iron, leading to cravings for iron-rich foods like red meat and leafy greens.*

2. *Calcium and Vitamin D: To support skeletal health, there might be cravings for calcium and vitamin D-rich foods, such as dairy products, fish, and eggs.*

3. *Electrolytes: The presence of metal in the body might disrupt electrolyte balance, causing cravings for foods high in potassium, sodium, and magnesium.*

It's important to note that this analysis is purely speculative and based on a fictional scenario, as the concept of an adamantium skeleton and superhuman healing abilities does not exist in real life.

10 points:

Ten pots are filled with clean quartz sand with the following seeds planted:

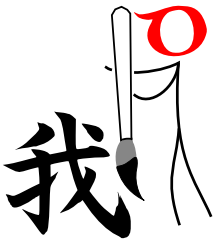
- Pots 1 and 6: Arabidopsis thaliana
- Pots 2 and 7: Brássica olerácea
- Pots 3 and 8: Phaseolus vulgaris
- Pots 4 and 9: Triticum aestivum
- Pots 5 and 10: Salix alba

The pots were then supplemented with the following solutions in water:

- Pots 1-5: Calcium nitrate (1 g/L), sodium hydrogen phosphate, magnesium chloride (0.5 g/L each).
- Pots 6-10: Calcium chloride (1 g/L), potassium hydrogen phosphate, magnesium sulfate (0.5 g/L each).

In most pots, plant growth was significantly suppressed. However in one or more of the pots, plant growth was normal. Name those pot(s) and explain why that happened.

Linguistics & Applied Sciences



5 points:

This month, Applied Sciences is diving into experimental design! It is an important skill that scientists use all the time to test hypotheses, no matter what discipline they are in.

People are always looking for ways to efficiently memorize information. For this problem, we'd like you to test out a method of memorization.

Find or create a technique that is supposed to help with memory. Some examples you can use:

1. Mnemonics
2. Chunking
3. Peg words
4. Imagery

Recruit a participant. This can be a parent, sibling, friend, etc.

Conduct the following experiment on them:

1. Show them the below list of 10 words and ask them to memorize it.
    - goofy
    - far
    - whirl
    - deliver
    - waves
    - sidewalk
    - gullible
    - value
    - connection
    - scarecrow
2. After 1 minute, take the list of words away and ask them to list as many as they can. Write down which words they remembered.
3. Ask them what technique they used to memorize the words and write down their answer.
4. Explain your chosen technique to the participant and ask them to use it. Make sure it is different from the technique they used.
5. Show them the below 10 list of words and ask them to memorize it.
    - teeth
    - dress
    - exotic
    - building

- stingy
- party
- tumble
- coherent
- beneficial
- ignore

6. After 1 minute, take the list of words away and ask them to list as many as they can. Write down which words they remembered.

For your submission, define the technique you chose to research with a scientific justification for why you believe it may work. Submit a list of which words they recalled both times, with a description of what technique they used each time.

Based on the results of the experiment, write about if your chosen technique worked. Explain the results of both the first and second trial. Some concepts you can cite include the recency effect, primacy effect, self reference effect, and of course the two techniques used. It is expected that you cite any external sources.

Grading: 1 point for explaining the chosen technique, 1 point for writing trial results, 2 points for explaining the results of the first and second trial, 1 point for citing sources.

10 points:

For this problem, we would like you to design and conduct an experiment on memory. The question you will be researching is "Do you retain information better if you learn it before you go to sleep or when you wake up?"

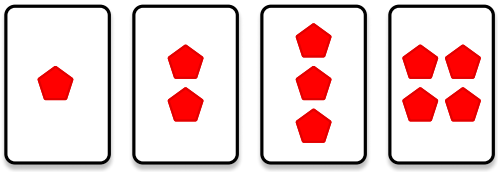
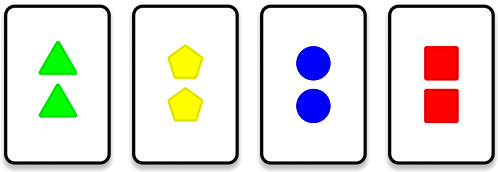
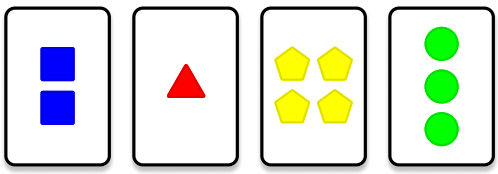
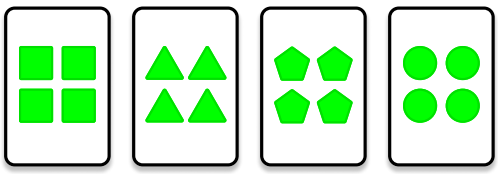
Write a hypothesis answering the question with a justification for said hypothesis. Design and write up an experimental procedure that you will conduct on yourself. Then run this procedure and record the results. Expected study length is anywhere between 3-10 days, but you may go longer. Write a conclusion explaining if your results support your hypothesis or not, with an explanation that is justified by scientific ideas. It is expected that you cite any external sources.

Grading: 2 points for the hypothesis, 3 points for a good/detailed experimental design, 2 points for the results, 2 points for the conclusion, 1 point for citing sources.

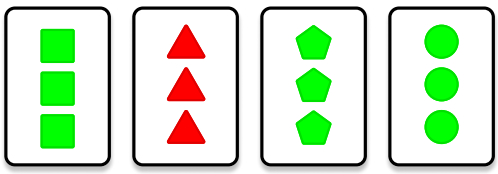
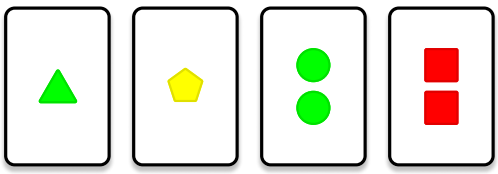
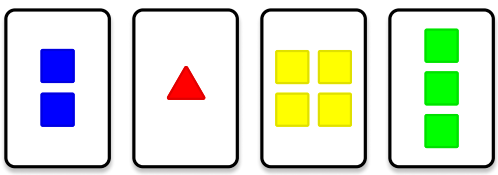
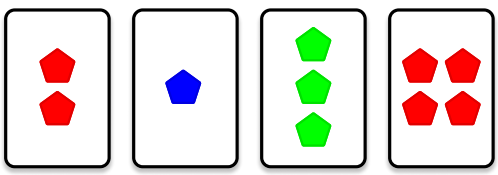
Computer Science

- Your program should be written in Java or Python-3.
- No GUI should be used in your program (e.g. `easygui` in Python).
- All the input and output should be done through files named as specified in the problem statement.
- Java programs should be submitted in a file with extension `.java`; Python-3 programs should be submitted in a file with extension `.py`. **No `.txt`, `.dat`, `.pdf`, `.doc`, `.docx`, etc. Programs submitted in the incorrect format will not receive any points!**

The card game "Batch" consists of many cards varying in three features: the number of shapes (1, 2, 3, 4), type of shape (square, circle, triangle, pentagon), and the color of the shapes (red, blue, yellow, green). A batch is a set of four different cards such that for every feature, each card in the batch shares the feature (e.g. all cards have the same number of shapes), or all cards in the batch differ on that feature (e.g. all cards are of a different color). Below are four examples of Batches:

The following are not Batches:

Boris isn't very good at playing Batch (he is colorblind), so he wants to write a program that will read in a list of cards and find Batches among the cards.

5 points:

Given a list of Batch cards, determine the number of different batches that could be formed using cards from the list.

Note: A card can be used in more than one batch.

10 points:

Given a list of Batch cards, determine the maximum number of *non-intersecting* batches. Two batches are non-intersecting if there is no overlap in the cards used (each card in the list is used for at most one batch).

Input specifications:

The 5pt and 10pt questions will use the same input format: Your program should read the file `input.txt`, with the lines of the file specified as below:

- The first line of the input will contain a single integer n denoting the number of Batch cards in the list.
- The next n lines will consist of three space-separated characters in the format `N S C`, representing each Batch card and its features. **Each line is guaranteed to be unique**, i.e., there are no duplicate cards.
  - The first character `N` represents the number of shapes on the card, and is either "`1`", "`2`", "`3`", or "`4`".
  - The second character `S` represents the type of shape on the card, and is either "`S`" (square), "`C`" (circle), "`T`" (triangle), or "`P`" (pentagon).
  - The third character `C` represents the color of the shapes, and is either "`R`" (red), "`G`" (green), "`B`" (blue), or "`Y`" (yellow).

Output specifications:

(5pt) Your program should produce the file `output.txt`, which contains a single integer representing the total number of Batches in the list of Batch cards.

(10pt) Your program should produce the file `output.txt`, which contains a single integer representing the maximum number of *non-intersecting* Batches in the list of Batch cards.

Examples:

Sample Input:

```
7
4 S R
2 C B
3 C B
1 C B
4 P G
4 T Y
4 C B
```

Sample Outputs:

(5pt)

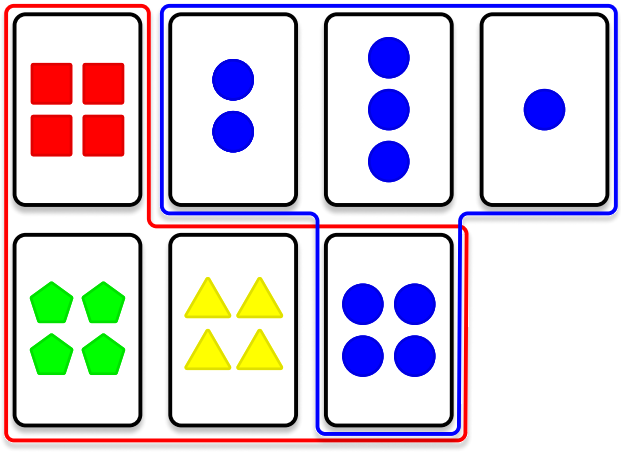
```
2
```

(10pt)

```
1
```

Explanation:

The set of cards are below with the two Batches circled:



Note that the card `4 C B` is used in both Batches, so the maximum number of non-intersecting Batches is 1.