SigmaCamp's *Problem of the Month* Contest

OCTOBER 2023

Mathematics

5 points:

Suppose that

$$\begin{aligned} O+N+E &= 1, \\ T+H+R+E+E &= 3, \\ N+I+N+E &= 9, \\ T+E+N &= 10, \\ T+H+I+R+T+E+E+N &= 13. \end{aligned}$$

What is the value of O?

Hint:

10 points:

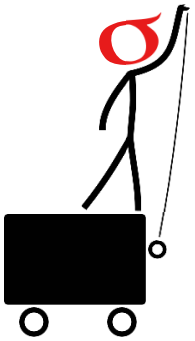
A grandfather clock has three hands: for hours, minutes, and seconds. Three flies are sitting on the hands of a grandfather clock, which is showing precisely noon. Alice the fly is sitting on the hour hand, and Bob and Charlie are on the minute and second hands respectively. The clock is set in motion and the flies ride the hands around, but with an important rule: any time one of the clock's hands passes another on its way around, the flies riding those hands switch places. By the time the clock strikes midnight, how many times will each of Alice, Bob, and Charlie have gone around the face of the clock?

Hint:

No hint this month.

Physics

5 points:



Your friend hands you a sealed aluminum box and tells you that it's entirely filled with water. The total mass of the box and the water is 6.5 kg, but your friend doesn't tell you the individual masses of the aluminum box or the water.

Luckily, you happen to have a perfectly efficient heater that can take some amount of energy and directly transfer it as heat to any object you choose. Using this device you transfer 24 kJ of energy into the box and its contents, raising its temperature by precisely 1°C.

What is the mass of water contained inside the aluminum box? Use 0.9 J/g°C for the heat capacity of aluminum and 4.2 J/g°C for the heat capacity of water.

Hint:

No hint this month.

10 points:

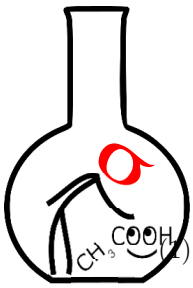
The Silver Lake kitchen has a bowl containing solid chunks of ice floating in liquid water. It is in thermal equilibrium, meaning both the ice and the water are the same, constant temperature. Calculate its heat capacity.

Hint:

No hint this month.

Chemistry

5 points:



Potassium thiocyanate (KSCN) solution is colorless, and dilute iron (III) chloride ($FeCl_3$) solution is pale brown. When these two solutions are mixed, the color changes to dark red.

$$Fe^{3+} + 3\,SCN^- \longrightarrow Fe(SCN)_3$$

This reaction is a convenient method to determine small traces of trivalent iron in water. During an expedition, a chemist needed to determine the iron content in a sample of water that was contaminated with small amount of iron salts. Unfortunately, no specialized equipment was available. The chemist had only a set of pre-prepared solutions of $FeCl_3$ solutions of with concentrations of 1 uM, 10 uM, 100 uM, 1 mM, 10 mM, and 3% solution of KSCN. The chemist decided to add an excess of KSCN solution to each $FeCl_3$ solution, thereby creating a color scale. The idea was to compare the color of the solution with unknown concentration of iron chloride with the color of the solutions where the concentration is known.

Unfortunately, the glassware available to the chemist was very limited. He only had a few flasks and bottles, each of which was cylindrical in shape, but of different size and height, and none of which had volume markings on them. In that situation, reliable comparison of color intensity is very problematic.

However, the chemists found the solution to this problem. How did he do that?

Hint:

Absorbance of a colored solution is proportional to the concentration of a dye AND to the optical path, i.e. the thickness of the solution layer the incident light is crossing.

10 points:

Chemical reactions are often reversible. Given a reversible chemical reaction:

$$M + L \rightleftharpoons ML$$

We can define an equilibrium constant K:

$$K = \frac{[ML]}{[M][L]}$$

The brackets around the different chemical species represent the concentration of that species in a reaction. Reactions with a large equilibrium constant (greater than unity) are pushed to the right: more of the products are formed. Reactions with small equilibrium constant (less than unity) are instead pushed to the left: mostly reactants remain. Consider the reaction in Figure 1. The equilibrium constant for reaction A is 4.0×10^{10}, while the equilibrium constant for reaction B is 3.3×10^6. Equilibrium constants for these reactions are written:

$$K_A = \frac{[\mathbf{A}]}{[\mathbf{R}][\mathbf{a}]}$$
$$K_B = \frac{[\mathbf{B}]}{[\mathbf{R}][\mathbf{b}]}$$

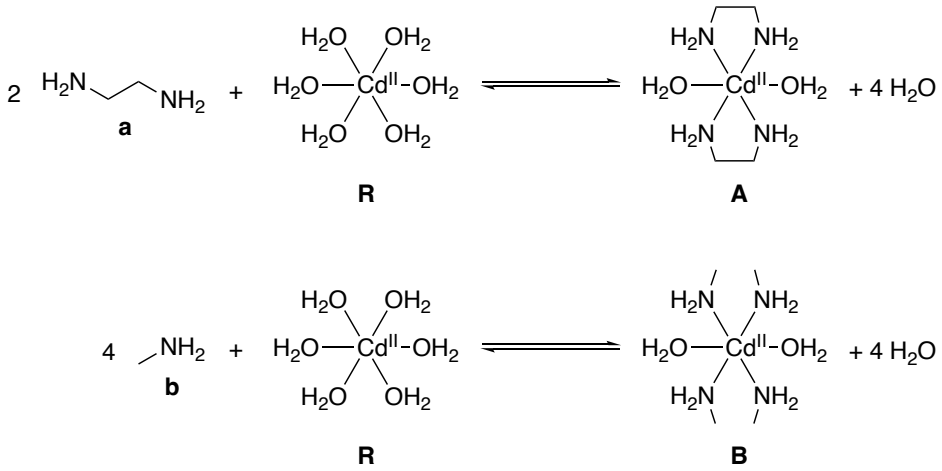


Figure 1: Complexation of cadmium by different ligands in aqueous solution

The concentration of water is ignored, because the reactions are done in water as solvent, so it is practically constant.

The difference in the strength of metal-nitrogen bonds between **A** and **B** is negligible.

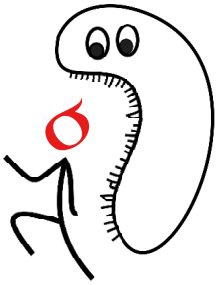
Explain why there is such a large difference in equilibrium constants between the two reactions. We are looking for an explanation that covers the physical mechanism for the increased stability of complex **A** compared to complex **B**.

Hint:

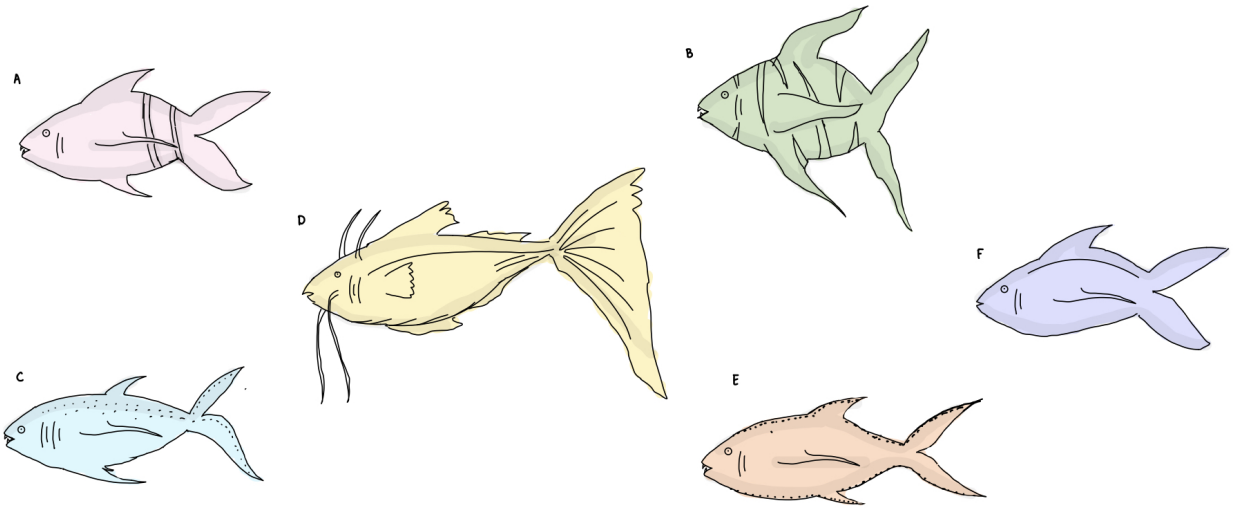
No hint this month.

Biology

5 points:



Marine Biology researchers from SigmaCamp went back in time to see prehistoric fish and witness evolution in action. They brought back their notes and the six fish seen below, but unfortunately, their notes were destroyed during the travel through time. Draw a phylogenetic tree to explain the evolutionary relationships between five of the specimens purely based on morphological observations, and justify your choices. Also identify which specimen does not belong to this evolutionary tree.



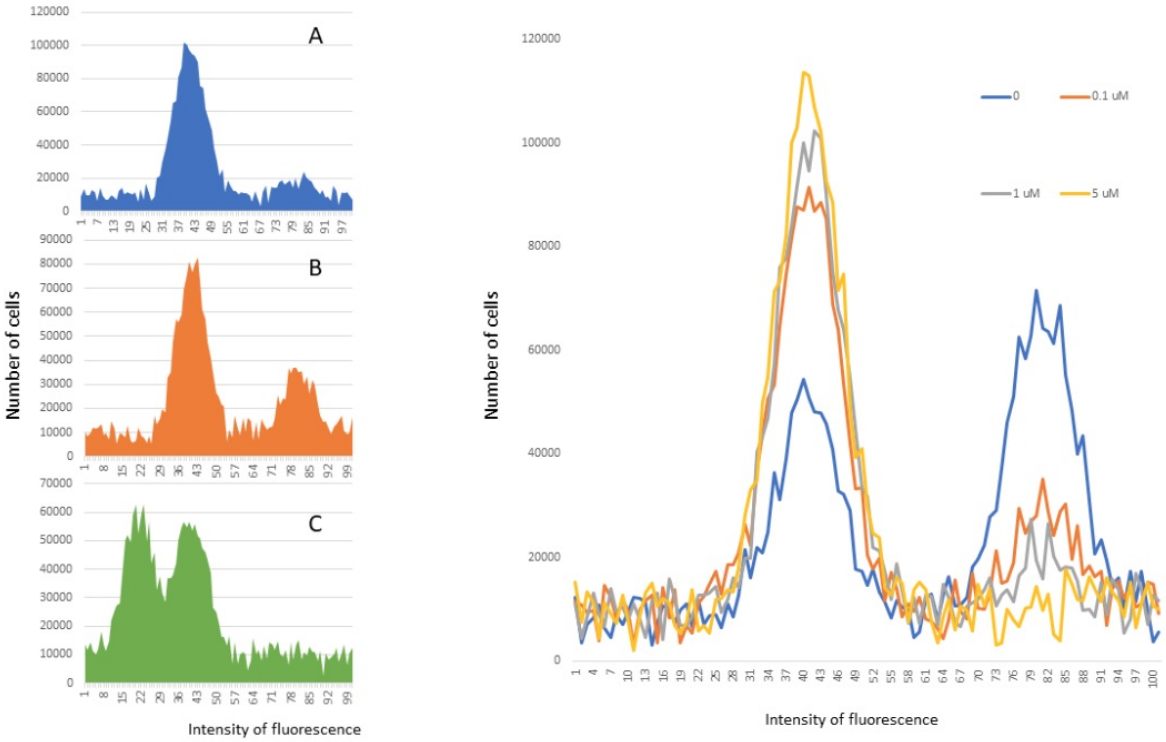
The six specimens collected over time during SigmaCamp's Time Travel Semilab

Hint:

No hint this month.

10 points:

Three cell cultures (A, B, and C) obtained from three different mammalian tissues were treated with a dye that is essentially non-fluorescent, but becomes strongly fluorescent upon binding to DNA. After that, each of them were passed through a special instrument that measures the intensity of fluorescence of each individual cell. The results (histograms that show a distribution of cells depending on their fluorescence) are presented on the left panel. A right panel shows the results of the similar experiment with the culture D obtained from the same organism. The culture D was split into four portions, and each of them was treated with increasing concentrations of the compound called cisplatin 7 hours before the measurement.



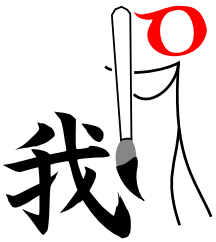
Graphs depicting the result of the experiment, with the number of cells against the intensity of fluorescence

From which tissues were the cultures A-C obtained? Explain your answer, and keep in mind that this question may have more than one correct answer. Explain what happens on the right panel, and what practical application the observation made during this experiment may have.

Hint:

If we disregard mitochondria (which have their own DNA, and its amount may vary) all cells in our body have the same amount of DNA, unless ....

Linguistics & Applied Sciences



5 points:

In one African language, nouns fall into different categories generally based on the type of objects they describe, which affects the prefixes that form singular and plural nouns. Here are examples of words that belong to seven of those categories:

Category 1: **qũrhũ** - a person, **ono** - females

Category 2: **rcera** - a bird, **rcuna** - grass,

Category 3: **nĩore** - a quiver, **nĩluqfu** - a drum

Category 4: **vũũa** - a river, **ʻlile** - wind

Category 5: **ũleve** - information, **ũke** - magic

Category 6: **nũkũvũ** - a foot, **qoxũ** - ears

Category 7: **nolĩĩ** - a small boy, **nolako** - a little stone

Below is a list of ten nouns. Place each one into the category it belongs to.

- **nĩvĩkũ**
- **ũxlaũ**
- **rmorkĩva**
- **qene**
- **kĩnure**
- **oxloqona**
- **ʻlili**
- **nokũrho**
- **ũrhe**
- **rhzova**

Match the English words to the words from the first part of the question. Describe your reasoning behind word placement and what characteristic each of the noun categories is based on.

- Joy
- Sandal
- Rulers
- Arms
- Facial expression
- An illness
- Outcast
- Love
- Garden

Hint:

No hint this month.

10 points:

Match the sentences in the same unknown African language to their corresponding English translations.

1) Alice planted the spinach last year	A) Bob aarugira irigu tene
2) Bob cut the spinach already this year	B) Alice atemita mwaka giki nyeni Bob aratema riu nyeni
3) Alice will cut the grass this year	C) Alice arahandera mwaka giki turari nyeni
4) Bob will plant the corn tomorrow	D) Bob akatema mwaka tuguthii nyeki
5) Alice planted bananas yesterday	E) Alice ahandira ira irigu
6) Bob cooked the bananas long ago	F) Bob atemira mwaka giki nyeni
7) Alice had cut the spinach this year before Bob cut the spinach now	G) Alice akutema mwaka giki nyeki
8) Bob will cut the grass next year	H) Bob akuhanda ruciu mbembe

We know that at some point, these events happened in this order relative to each other: Bob planted the corn, Alice cut the corn, and then Bob cooked the corn. Write a sentence in the unknown language that describes two of these events in the correct order.

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Hint:

No hint this month.

Computer Science

- Your program should be written in Java or Python-3.
- No GUI should be used in your program (e.g. `easygui` in Python).
- All the input and output should be done through files named as specified in the problem statement.
- Java programs should be submitted in a file with extension `.java`; Python-3 programs should be submitted in a file with extension `.py`.
  No `.txt`, `.dat`, `.pdf`, `.doc`, `.docx`, etc. Programs submitted in the incorrect format will not receive any points!
- Need some help? You can view our resource list, which has some great ways to learn about code.

Have you ever wanted to try your hand at writing your own POM problem? This month, we are continuing the **CS POM Participant Challenge**! Write and submit your own POM problem and solution by **November 10th** and it might just be published alongside our November problems.

- For inspiration, you can take a look at problems from previous months! We're looking for well written problems that are fun to solve.
- Submit your problem by emailing a PDF of the problem statement and the Python or Java file of your solution to pom@sigmacamp.org.
- If you have any questions, you can contact pom@sigmacamp.org.
- While no points will be awarded for the Participant Challenge, depending on interest participants who have their problems published may receive a small prize.

Consider the following game called "Chained Words". The board consists of four piles of three letters each (12 different letters in total). For example:

`TKZ AMW ELR HSI`

To play the game, you form English words by using only these letters, with a few simple rules:

- You cannot use two letters from the same pile one after another within a word (that also means no double letters). In the example above, `SLATES`, `MIRTH`, `MIME` and `EARS` are valid words, but `RAM` and `MEEK` are not.
- A letter can only appear in one pile, for example "`TKZ AKZ TLR ASI`" would not be a valid board setup.
- Each word should be at least three letters long. In the example above, `TAT` is valid, but `AS` is not.
- A letter may be used multiple times within the sequence of words.
- The first word formed may start anywhere, but subsequent words must start with the last letter of the previous word. In the example above, the sequence `KEW-WHET-TEAK` is valid, but `KEW-TEAK` is not.

The goal of the game is to find a sequence with as few words as possible that uses all the 12 letters on the board.

5 points:

Write a program that receives a description of the board and a sequence of words, and determines if the sequence follows all the rules of Chained Words and uses all 12 of the letters on the board.

Your program should read the input file `input.txt`, which will contain two lines. The first line specifies the board position, given as a string of uppercase letters, with piles separated by spaces (e.g. "`TKZ ELR HSI AMW`"). The second line of `input.txt` consists of a list of words in the sequence, separated by spaces.

Your program should produce the file `output.txt`, which will contain either

- "`CORRECT`", in the case the sequence is valid and uses all 12 letters, or
- "`INCORRECT`", otherwise.

Example `input.txt`:

```
LNC DPO TRB IUA
BARD DUPLICATION
```

Example `output.txt`:

```
CORRECT
```

Hint:

No hint this month.

10 points:

Write a program that receives a description of the board and a list of English words, and finds the alphabetically-first sequence of English words that follows all the rules of Chained Words, uses all the 12 letters, and has as few words as possible.

Your program should read the input file `input.txt`, which will contain one line specifying the board position, given as a string of uppercase letters, with piles separated by spaces (e.g. "`TKZ ELR HSI AMW`"). However, there is a separate file called `words.txt` that your program should use as the list of valid English words, with one word per line. We will provide the file when grading (in the same directory where your program is run). You can download it yourself here for testing:

https://www.dropbox.com/scl/fi/joeoyuhc3iat7e8hlyiy4/words.txt?rlkey=bamyosttqp8wafh9zv8ogw3c0&dl=0

Your program should produce the file `output.txt`, which should consist of the alphabetically-first shortest sequence, with the words separated by spaces. In the case that there is no valid sequence that uses all the letters, `output.txt` should read "`IMPOSSIBLE`".

Example `input.txt`:

```
LNC DPO TRB IUA
```

Example `output.txt`:

```
ABDICATION NONPLUTOCRATIC
```

Hint:

No hint this month.