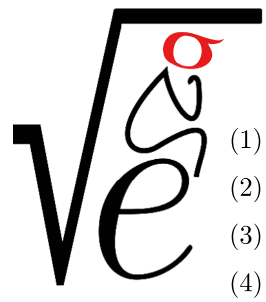
SigmaCamp's *Problem of the Month* Contest

OCTOBER 2023

Mathematics



5 points:

Suppose that

$$O + N + E = 1 \quad (1)$$

$$T + H + R + E + E = 3, \quad (2)$$

$$N + I + N + E = 9, \quad (3)$$

$$T + E + N = 10, \quad (4)$$

$$T + H + I + R + T + E + E + N = 13. \quad (5)$$

What is the value of O?

Hint:

Solution:

By subtracting Eq. 1 from Eq. 4, we see that $T - O = 9$, and by subtracting Eq. 4 from Eq. 5, we see that $I + T + N = 10$. Combining this with Eq. 4, $I = E$. Plugging this into Eq. 3, we get $2N + 2E = 9$, so $N + E = 9/2$. By Eq. 4, $T = 10 - (N + E) = 11/2$, so $O = T - 9 = 11/2 - 18/2 = -7/2$.

Answer: $O = -7/2$

10 points:

A grandfather clock has three hands: for hours, minutes, and seconds. Three flies are sitting on the hands of a grandfather clock, which is showing precisely noon. Alice the fly is sitting on the hour hand, and Bob and Charlie are on the minute and second hands respectively. The clock is set in motion and the flies ride the hands around, but with an important rule: any time one of the clock's hands passes another on its way around, the flies riding those hands switch places. By the time the clock strikes midnight, how many times will each of Alice, Bob, and Charlie have gone around the face of the clock?

Hint:

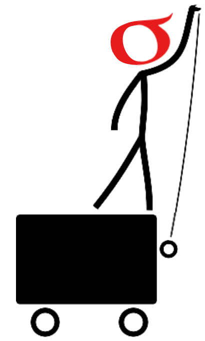
No hint this month.

Solution:

The key realization is that the flies never “lap” each other, as each time a fly would pass another, it hops off onto the slower hand and the fly from the slower hand hops onto the faster one. This means that the total number of laps traveled by the flies will be almost the same. The total number of laps made by the hands is 12*60+12+1=733. 733/3 = 244.33... This means that Bob and Charlie will travel 244 laps and Alice will travel 245 laps.

Physics

5 points:



Your friend hands you a sealed aluminum box and tells you that it's entirely filled with water. The total mass of the box and the water is 6.5 kg, but your friend doesn't tell you the individual masses of the aluminum box or the water.

Luckily, you happen to have a perfectly efficient heater that can take some amount of energy and directly transfer it as heat to any object you choose. Using this device you transfer 24 kJ of energy into the box and its contents, raising its temperature by precisely 1°C.

What is the mass of water contained inside the aluminum box? Use 0.9 J/g°C for the heat capacity of aluminum and 4.2 J/g°C for the heat capacity of water.

Hint:

No hint this month.

Solution:

Using the equation

$$Q_T = Q_w + Q_a = m_w c_w \Delta T + m_a c_a \Delta T,$$

we find that

$$24 \text{ kJ} = (m_w \cdot 4.2 \text{ J/g}^\circ\text{C} + m_a \cdot 0.9 \text{ J/g}^\circ\text{C})\, 1^\circ\text{C}.$$

Additionally, we know that

$$m_w + m_a = 6.5 \text{ kg}.$$

Simplifying the units and substituting $m_a = 6.5 \text{ kg} - m_w$, produces

$$24 \text{ g} \cdot \text{kJ/J} = m_w \cdot 4.2 + (6.5 \text{ kg} - m_w) \cdot 0.9.$$

Multiplying out,

$$24 \text{ kg} = 4.2 m_w + 6.5 \cdot 0.9 \text{ kg} - 0.9 m_w,$$

so

$$24 \text{ kg} - 5.85 \text{ kg} = 4.2 m_w - 0.9 m_w$$

and

$$m_w = \frac{24 - 5.85}{4.2 - 0.9} \text{ kg} = 5.5 \text{ kg}.$$

Answer: $m_w = 5.5$ kg

10 points:

The Silver Lake kitchen has a bowl containing solid chunks of ice floating in liquid water. It is in thermal equilibrium, meaning both the ice and the water are the same, constant temperature. Calculate its heat capacity.

Hint:

No hint this month.

Solution:

Water and ice behave a bit differently than typical materials because when you add or subtract a small quantity of heat, they won't heat up or cool down. Instead, some of the ice melts or some of the water freezes.

Despite what some websites may tell you, heat capacity is *not* the amount of energy it takes to raise or lower an object's temperature by 1°C. Because the specific heat capacities of many materials (ex: water) are different at different temperatures, the amount of heat you need to add to increase the temperature by 1°C is (slightly) different from the amount of energy you need to take away from the system for the temperature to decrease by 1°C. Usually, this difference is small enough that we can ignore it, pretend the heat capacity is constant, and use the definition given above.

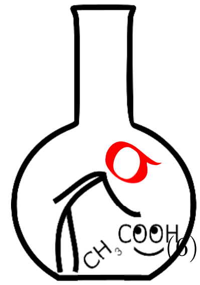
In this problem, however, we cannot make such an approximation. Heat capacity is defined as the limit as $\Delta T \to 0$ of $Q/\Delta T$, where $Q = \Delta U =$ the amount of heat needed to produce a change in temperature of ΔT (you may also see this presented as $\frac{\partial U}{\partial T}$).

As $\Delta T \to 0$ from the positive side, Q goes to some positive value. That is, $Q/\Delta T$ just gets larger and larger — it goes to ∞. If ΔT instead approaches 0 from the negative side, Q goes to some negative value. This means that $Q/\Delta T$ still goes to infinity. Therefore, the heat capacity is infinite. (It's important to check both sides because if we got $-\infty$ from one side and $+\infty$ from the other, then the heat capacity wouldn't be infinite — it would just be plain old undefined.)

Another way of seeing this is to graph Q vs ΔT. The slope of the graph at 0 is infinite.

If you want to read more about heat capacity, this is a good reference. However, it does assume knowledge of calculus.

Chemistry



5 points:

Potassium thiocyanate (KSCN) solution is colorless, and dilute iron (III) chloride ($FeCl_3$) solution is pale brown. When these two solutions are mixed, the color changes to dark red.

$$Fe^{3+} + 3\,SCN^- \longrightarrow Fe(SCN)_3 \tag{6}$$

This reaction is a convenient method to determine small traces of trivalent iron in water. During an expedition, a chemist needed to determine the iron content in a sample of water that was contaminated with small amount of iron salts. Unfortunately, no specialized equipment was available. The chemist had only a set of pre-prepared solutions of $FeCl_3$ solutions of with concentrations of 1 uM, 10 uM, 100 uM, 1 mM, 10 mM, and 3% solution of KSCN. The chemist decided to add an excess of KSCN solution to each $FeCl_3$ solution, thereby creating a color scale. The idea was to compare the color of the solution with unknown concentration of iron chloride with the color of the solutions where the concentration is known.

Unfortunately, the glassware available to the chemist was very limited. He only had a few flasks and bottles, each of which was cylindrical in shape, but of different size and height, and none of which had volume markings on them. In that situation, reliable comparison of color intensity is very problematic.

However, the chemists found the solution to this problem. How did he do that?

Hint:

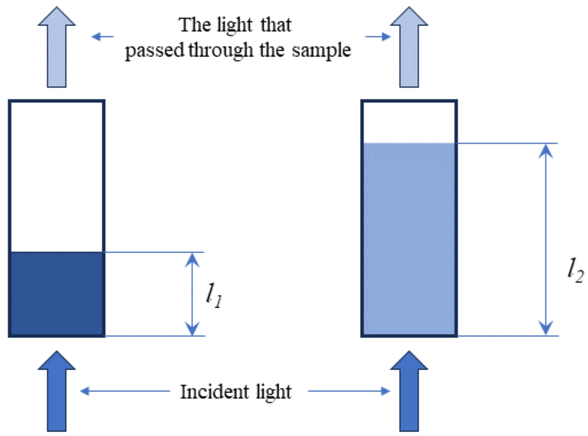
Absorbance of a colored solution is proportional to the concentration of a dye AND to the optical path, i.e. the thickness of the solution layer the incident light is crossing.

Solution:

Answer: The Bouguer-Beer-Lambert law or merely the extinction law says that absorbance of light by a solution of some substance **B** in a transparent solution is described by the equation:

$$A = \epsilon l c$$

In this equation, A is absorption of light. It is calculated by comparison of intensity of the incident light (the light that comes into the sample) and the intensity of the light that passes through the sample. We will not discuss how absorbance is calculated; for us, it is important that absorbance of some colored solution is roughly proportional to the intensity of the color as our eye sees it. The parameter l is just an optical path, i.e. the distance the light travels (see the figure below), c is a concentration of the light absorbing material in the solution, and ϵ, or an extinction coefficient, is a parameter that characterizes the ability of a substance X to absorb light.



Let's consider the solution of some colored substance **B**, as shown on the figure, left panel. The amount of **B** is x grams, and the volume of the solution is V. The extinction coefficient of **B** is ϵ. If the vessel is a cylinder, or a cube, a prism, etc, the optical path of the light that passes the vessel from the bottom to the top equals the height of the liquid layer, and can be calculated as $l = \frac{V}{S}$, where S is a surface of the vessel's bottom. From that, we can easily calculate the absorbance of the light that passes a solution:

$$A = \epsilon \frac{x}{V} \frac{V}{S} = \epsilon \frac{x}{S}$$

That is a very interesting observation: the light observed by the solution poured into a cylindrical vessel does not depend on dilution *when we look from the top*, and the source of light is placed *under* the vessel: it depends only on the amount of dissolved material. Thus, if we place a solution of some substance into a cylindrical vessel and dilute it two times with water, the concentration of the solution becomes two times lower, but the optical path becomes two times bigger, so we will see no change in color intensity *if we place the light source under the vessel and will look from the top.*

From that, we immediately see what should we do in our case. Since we have solutions of $FeCl_3$ with known concentrations, we just pour them into separate vessel in such a way that the height of liquid is the same in each vessel. Then we add an excess of KSCN solution to each cylinder. As a result, we have a color scale, which we can use for analysis of our sample.

then we take a sample of water that we want to analyze and add an excess of KSCN to it. after that, we put a light source *under* each cylinder and compare color intensity in the cylinder with our water sample and each cylinder from our color scale by looking *from the top*. When we find the cylinder whose color approximately matches that of our sample, we are done.

10 points:

Chemical reactions are often reversible. Given a reversible chemical reaction:

$$\mathrm{M} + \mathrm{L} \rightleftharpoons \mathrm{ML}$$

We can define an equilibrium constant K:

$$K = \frac{[\mathrm{ML}]}{[\mathrm{M}][\mathrm{L}]}$$

The brackets around the different chemical species represent the concentration of that species in a reaction. Reactions with a large equilibrium constant (greater than unity) are pushed to the right: more of the products are formed. Reactions with small equilibrium constant (less than unity) are instead pushed to the left: mostly reactants remain. Consider the reaction in Figure 1. The equilibrium constant for reaction A is 4.0×10^{10}, while the equilibrium constant for reaction B is 3.3×10^{6}. Equilibrium constants for these reactions are written:

$$K_A = \frac{[\mathbf{A}]}{[\mathbf{R}][\mathbf{a}]}$$
$$K_B = \frac{[\mathbf{B}]}{[\mathbf{R}][\mathbf{b}]}$$

The concentration of water is ignored, because the reactions are done in water as solvent, so it is practically constant.

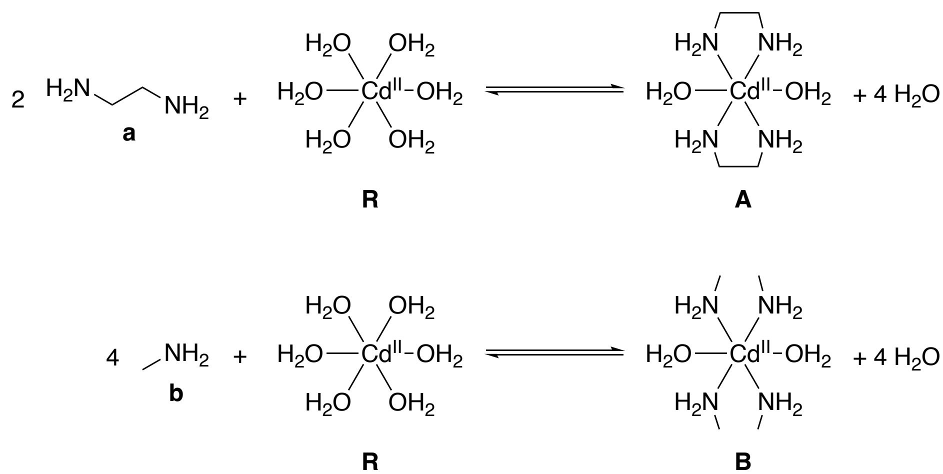


Figure 1: Complexation of cadmium by different ligands in aqueous solution

The difference in the strength of metal-nitrogen bonds between **A** and **B** is negligible.

Explain why there is such a large difference in equilibrium constants between the two reactions. We are looking for an explanation that covers the physical mechanism for the increased stability of complex **A** compared to complex **B**.

Hint:

No hint this month.

Solution:

Answer: Briefly, in contrast to methylamine, which is a monodentate ligand, ethylenediamine is a bidentate ligand. Bidentate ligands from two bonds with a metal ion, which make their complexes more stable.

Explanation 1

The first explanation is purely formal. It is based on the fact that, whereas only two ethylenediamine molecules interact with Cd^{2+} in the second reaction, the number of methylamine molecules is four. Therefore, the rate of the first reaction depends on the methylamine concentration raised to the power of four, and the rate of the second reaction depends on the square of the ethylenediamine concentration. Keeping in mind that the equilibrium constant equation is derived from kinetic equations of a direct and reverse reaction[1], the association constant equations for ethylenediamine and methylamine complexes formation should be rewritten as:

$$K_A = \frac{[\mathbf{A}]}{[\mathbf{R}][\mathbf{a}]^2}$$
$$K_B = \frac{[\mathbf{B}]}{[\mathbf{R}][\mathbf{b}]^4}$$

accordingly.

Why it is important? Let's assume that the energy of each chemical bond between one nitrogen and a cadmium ion is the same in ethylenediamine and methylamine complexes.[2] Therefore, it would be correct to conclude that $K_A = K_B$, and, therefore, $[\mathbf{b}] = [\mathbf{a}]^2$. That means that the equilibrium concentration of **b** must be much higher: if $[\mathbf{b}] = 10^{-4}$, $[\mathbf{a}]$ is as low as 10^{-8}! However if the concentration of a free ligand **b** in a presence of **R** is much higher than the concentration of a free ligand **a**, that means **b** is much poorer ligand than **a**.

Let's reiterate: even if the total energy of four cadmium - nitrogen bonds in **A** and **B** is the same, the complex **B** requires much higher concentration of the ligand **b**, and, therefore, it is less stable. How can it be understood?

To understand that, we must keep in mind that the equilibrium constant equation is derived from kinetic equations of a direct and reverse reaction (see the footnote above). Such an equation is a product of the rate constant, which reflects the likelihood that one molecule (or one mole of molecules) undergoes a conversion in one minute interval, and the effective concentration of reactants. If the reaction is monomolecular its rate equals the rate constant times the actual concentration of the reactant molecule: if the concentration becomes 10 times smaller, the rate is 10 times slower. If a reaction is bimolecular, the rate is a product of the rate constant and concentrations of each reactant: if each concentration becomes 10 times smaller, the rate is 100 times slower. Indeed, for the bimolecular reaction to occur, two molecules must meet each other, and the probability of this event is proportional to the concentration of each reactant.

Obviously, the reactions where three, four or even five molecules participate are even more sensitive to dilution, which dramatically slows down its rate.

Therefore, a correct (although an incomplete) answer to the question about lower stability of the methylamine complex is as follows: that happens because a probability that *four* methylamine molecules meet one cadmium ion is much lower than the probability that just *two* ethylenediamine complexes meet cadmium.

[1] For example, in a reversible reaction where the reactants A and B form C the rate of the direct reaction is

$$rate_d = k_d[A][B]$$

and the rate of the reverse reaction is

$$rate_r = k_r[C]$$

where square brackets denote the actual concentration of A, B, and C, and k_d is so called *rate constant*, which is equal to the rate reaction when concentration of each reactant in 1 M.

Now let's remember that our solution is at equilibrium, and that equilibrium is dynamic. That means that the rate of the direct and reverse reactions are the same, and, therefore, right parts of the above equations are equal to each other:

$$k_d[A][B] = k_r[C]$$

We can regroup this equation as follows:

$$\frac{[A][B]}{[C]} = \frac{k_r}{k_d} = K$$

The right part is called an *association constant*, and it is denoted as a capital K.

[2] That assumption is not totally unreasonable, because electronic structures of methylamine and ethylenediamine nitrogens are pretty much the same.

That explanation seems to contradict to two popular answers one can obtain by googling. One answer is "chelating effect", and another is "entropy". Actually, there is no contradiction here: sometimes, the same idea may be explained in different terms. To see that, let's take a look at two other explanations.

Explanation 2

The above explanation uses the term "probability", and that immediately gives us a link to the chelating effect. To better understand that, let's introduce the concept of "effective concentration". What does it mean? The process of addition of the ethylenediamine to Cd^{2+} is a two step process: initially, one nitrogen atom binds to Cd^{2+}, and the remaining ethylenediamine molecule is exposed to a solvent. That is essentially the same as addition of methylamine: one methylamine molecule binds Cd^{2+} first, followed by the second methylamine, etc. However, there is one significant difference between addition of ethylenediamine and methylamine, and the difference is as follows.

Imagine that in one test tube we mixed 1 uM Cd^{2+} and 1 mM ethylenediamine, and in another test tube we mixed 1 uM Cd^{2+} and 1 mM methylamine. Let's consider only a first half of the reaction, i.e. the precess of formation of just two Cd - N bonds. The rate of the first step (first Cd - N bond formation) will be the same in both cases. It is:

$$rate_1 = k[\mathrm{Cd}^{2+}][L]$$

In this equation, [L] is a concentration of the ligand, which is the same in both test tubes. Since the equation is the same in both cases, and the parameters are the same, it would be correct to conclude that the rate of both reactions will be the same. However, when we start to consider the second step, the difference will be dramatic. Indeed, for methylamine, the rate of the second step will be:

$$rate_1 = k[\mathrm{Cd}^{2+}L][L]$$

In this equation, [Cd^{2+}L] is a concentration of [Cd^{2+}] bound to one methylamine, and [L] is a concentration of free methylamine (since the initial concentration of cadmium was very low, we can assume [L] didn't change, and it is still 1 mM). Therefore, the rate of the second step (addition of the second methylamine) is expected to be roughly the same as the rate of the first step. For ethylenediamine, the situation is totally different. Indeed, in this case, the second reactant is the opposite side of the same ethylenediamine molecule (its second nitrogen). Unlike methylamine molecules, which freely travel in the solution, the position of the second ethylenediamine nitrogen is constrained, it is dangling in a close proximity to the cadmium ion, and the average distance between them is as little as few angstroms. It is easy to calculate that the concentration of a solution where the average distance between two dissolved particles is just few angstroms is about 10 M. In other words, we can say that the reaction between cadmium and the second ethylenediamine nitrogen occurs in conditions where the *effective concentration* of the nitrogen is several orders of magnitude higher that for the first step of this process. If we substitute [L] in the above equation with the effective concentration (e.g. 10 M), we immediately get a 10,000 fold acceleration of the reaction rate.

In other words, the difference between methylamine (a monodentate ligand) and ethylenediamine (a bidentate ligand) is that the formation of the first bond between the latter and the the metal ion dramatically accelerates the formation of the second bond. That happens because the second active atom in the bidentante ligand is brought to a close proximity to the ion, so the reaction is driven not by some energetic effect, but by an increased probability of an encounter between the ion and the second active atom.

Explanation 3

Many textbooks and web sites explain the same using the term "entropy". Although that is correct, they rarely explain what does it mean. They also frequently use the term "free energy", without providing an adequate explanation of the meaning of that term.

The free energy formula is

$$\Delta G = \Delta H - T\Delta S$$

In this equation, ΔH is a *total energy*[3] released or consumed in some process. For example, if one stable bond forms in some reaction, the total energy of the reaction product becomes *lower* (ΔH is negative), and the excess of energy is released in a form of heat (increased ΔU) or work ($p\Delta V$) or a combination thereof. From that, we can make an intuitively clear conclusion: if energy is released in some process (the total energy becomes lower), this process is spontaneous, if no energy is released, or if the energy is consumed, then such a process is not spontaneous. However, that is not always the case. The parameter that allows us to predict if some process is spontaneous, is not a total energy change, but a free energy change. As we can see from the formula above, free energy is the total energy minus the hidden energy. The term "hidden energy" means the energy that can never be transformed to a work or to a heat.

To demonstrate the hidden energy concept, let's consider some gas that was placed in a cylinder with a piston. The gas temperature is 0°. When we are heating the gas, its total energy increases, it warms up, the pressure goes up, and the gas expands. Due to the expansion, the temperature goes down to 0°, and the net result of this process can be understood as conversion of all heat transferred to the gas into the work. That is an example of a complete conversion of energy into work. There is no hidden energy in this process. This process is reversible: if we start to cool the cylinder, the piston goes down, and the gas starts to shrink, this process leads to a release of a heat, and finally all the heat that was initially transferred to the cylinder will be recovered.

Now consider the same experiment, but let's put a piece of ice into the cylinder. Like in the previous experiment, the temperature is 0°. When we start heating the cylinder, no gas expansion occurs, and no work is made, because all the heat is consumed by ice, which starts to melt. In this experiment, the energy (in a form of heat) is consumed, but no work is performed. But the heat is not lost: is you decide to cool the cylinder down, the molten ice crystallizes back, and all the heat transferred to it will be recovered. As in the previous case, the process is reversible, but no work is preformed by the system: the energy is stored in a form of a *hidden energy.*

What happens when the ice melt? Where is the hidden energy stored? To answer this question, let's take a look at the structures of ice and water. In ice, all molecule are arranged in such a way that they form a regular crystal lattice, so each of them has a quite concrete orientation is space and a strictly defined position. The only freedom they possess is a freedom to oscillate in three dimensions: up-down, left-right, forward-backward[4]. In water, there is no periodic structure, and all molecules may rotate in three independent directions, they may move in any direction, so the number of additional degrees of freedom is immense. This highly disordered state consumes a huge amount of energy, which is not transformed to a heat or work. This energy becomes *hidden* inside a molten ice.

It is literally hidden: it is trapped in multiple degrees of freedom (internal rotations, switching positions, etc.), which are not seen from the outside world in a form of a heat or work, but which serve as a good storage for an external energy transferred to the system.

What relation all of that has to our story? A relation is as follows. The driving force of some process is a decrease of a free energy, which can be achieved by a transition to the state with lower total energy (a more stable state), or by a transition to a state with higher hidden energy (a more disordered state), or by a combination thereof. Sometimes, the transition is possible to the state with a slightly higher total energy, which is significantly more disordered than the initial state. A typical example of a transition to a state with a high hidden energy is a ice to water transition.

[3] Strictly speaking, "total energy" is an internal energy (ΔU), or the total energy of all atoms in the system. However, we prefer to use enthalpy ($\Delta H = \Delta U + p\Delta V$) instead, because we perform our experiments not in closed vessels, but at a constant pressure: our experimental setup is connected to the atmosphere, and our reaction mixtures may expand or shrink, which consumes or releases some energy in a form of work.

[4] Please, don't understand it literally. Obviously, a molecule may oscillate in any direction, but each of them can be represented as a combination of those three "pure modes", e.g. "30% of up-down + 25% of left-right + 45% of forward-backward". These pure forms of movement all other forms of movement can be derived from are called "degrees of freedom".

There is a direct analogy with our case here. As you probably remember, initially, the cadmium ion is bound to six molecules of water, which exists in this complex in a highly ordered state. We can say that they are kind of "frozen". Moreover, the molecules that make a contact with those six molecules are also more ordered than the molecules in a bulk water. When four methylamine molecule substituted one water molecule in the complex, those four water molecules are released, and they become much more disordered. However, the incoming methylamine molecules become more ordered (initially, they were freely traveling in the solution, and now they are sitting in strict positions). Therefore, the whole system becomes neither more ordered nor more disordered in that case.

However, when *two* ethylenediamine molecules come to $(Cd(H_2O)_6{}^{2+}$, they substitute *four* water molecules. That means that a position of two molecules became constrained, but the position of four molecules became non-constrained as a result of this process.

Going back to the free energy formula, that can be interpreted as follows:

The total energy change during the reaction equals to the energy of formation of four cadmium-nitrogen bonds, and this energy (ΔH) is essentially the same for ethylenediamine and methylamine[5] The hidden energy change ($T\Delta S$) is significantly higher for the ethyelnediamine complex. Therefore, the free energy decrease will be more significant for ethylenediamine than for methylamine.

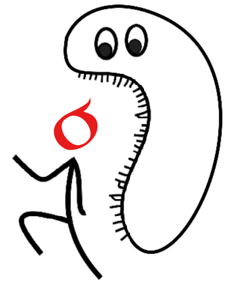
Conclusion

As you can see, these three explanations look totally different. But this difference is only apparent. As you can see, all three explanations use the terms "probability" and "disorder", which means all three explain the observed phenomenon in terms of entropy change, although they are doing it differently.

Unfortunately, we have not enough space to provide a more detailed explanation, and that is why some aspects seem not completely clear in the above text. If you didn't understand something from it, that is normal. We are considering to devote a separate lecture at SigmaCamp 2024 to explain this topic in more detail.

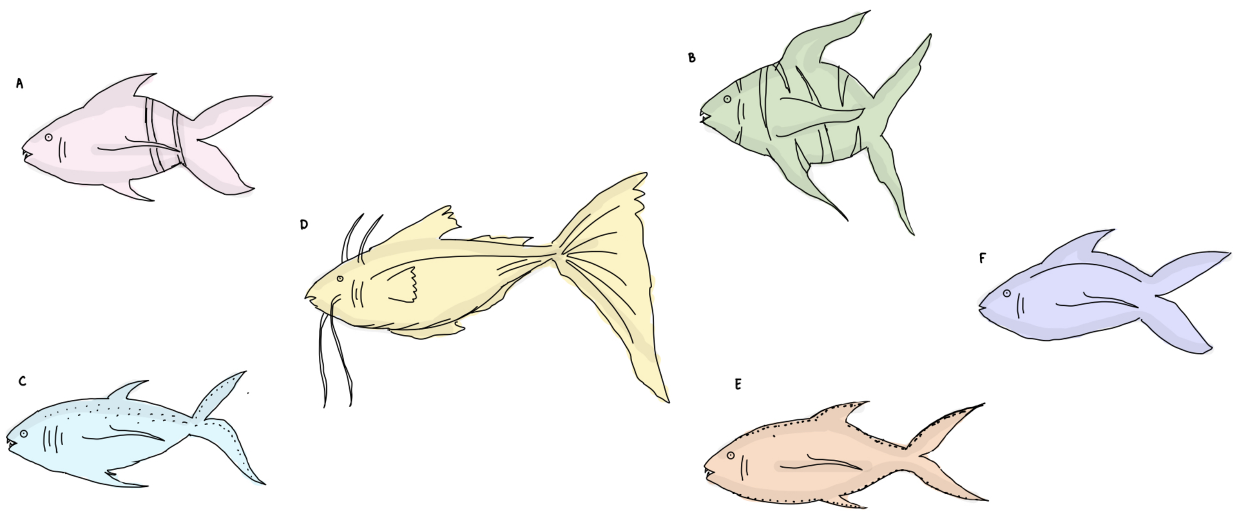
[5] Actually, it is somewhat higher for methylamine, but we have no space to discuss it here, because this nuance is not too significant.

Biology



5 points:

Marine Biology researchers from SigmaCamp went back in time to see prehistoric fish and witness evolution in action. They brought back their notes and the six fish seen below, but unfortunately, their notes were destroyed during the travel through time. Draw a phylogenetic tree to explain the evolutionary relationships between five of the specimens purely based on morphological observations, and justify your choices. Also identify which specimen does not belong to this evolutionary tree.



The six specimens collected over time during SigmaCamp's Time Travel Semilab

Hint:

No hint this month.

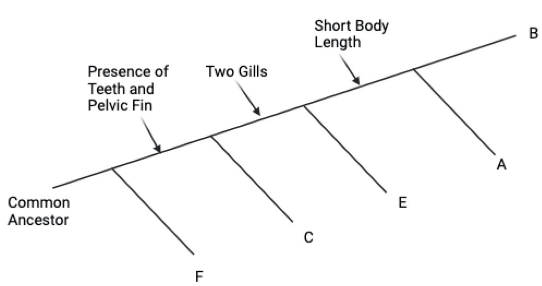
Solution:

Answer: Fish (D) does not belong to the same evolutionary line, or likely diverged from the other five fish a very long time ago. We can come to this conclusion from the lack of shared features, such as the forked tail that we see in every other fish, and the presence of multiple other morphological characteristics such as whiskers, three gills, vestigial fins, and a corrugated pectoral fin that are not present in the other five.

We can then make a table as seen below, demarcating the presence or morphology of each observed feature in the five remaining fish.

Feature	A	B	C	~~D~~	E	F
Number of Gills	2	2	4	~~3~~	2	2
Presence of Teeth	√	√	√	~~x~~	√	x
Presence of Pelvic Fin	√	√	√	~~√~~	√	x
Pectoral Fin Morphology	Smooth	Smooth	Smooth	~~Corrugated~~	Smooth	Smooth
Presence of Whiskers	x	x	x	~~√~~	x	√
Color	Pink	Green	Blue	~~Yellow~~	Orange	Purple
Markings	Two Stripes	Zebra	Striation	~~Dots along lateral line~~	Dots along dorsal and ventral lines	Lateral line
Body Length	Short	Short	Mid	~~Long~~	Mid	Mid
Tail Morphology	Forked	Forked	Forked	~~Truncated~~	Forked	Forked

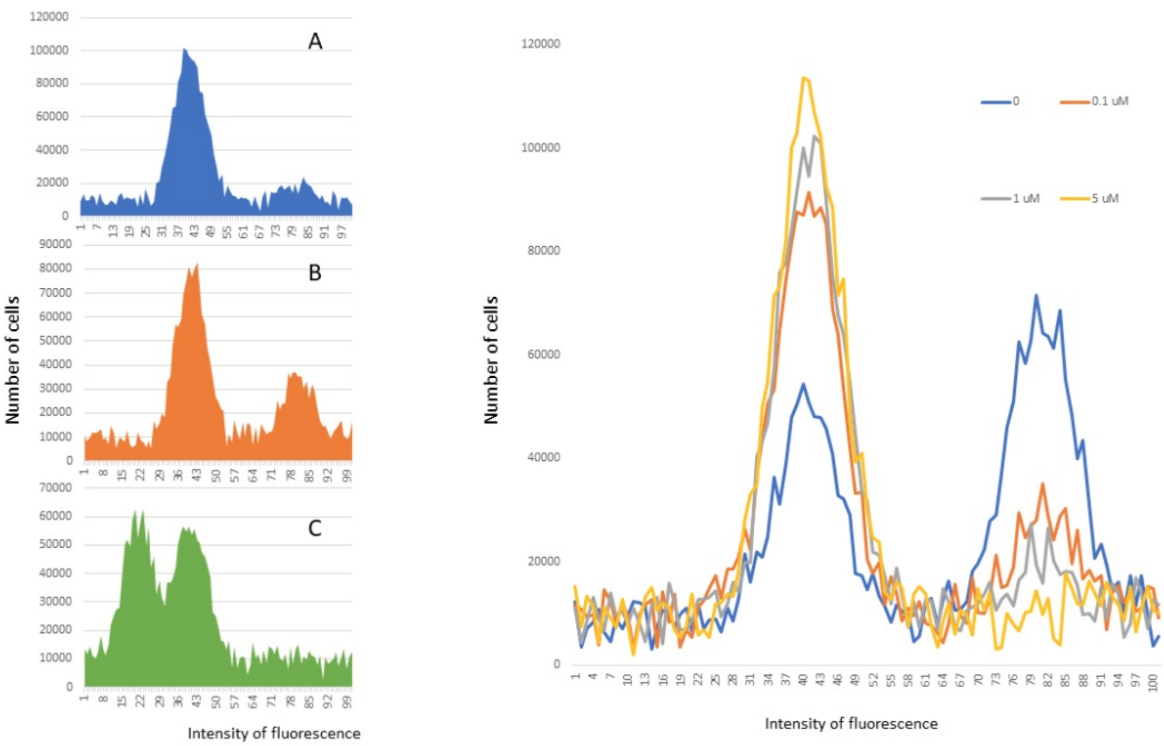
Tail and pectoral fin morphology is shared by all five remaining fish, while there are no similarities in color or markings, and as such, these characteristics will not help us to determine the order by which these fish evolved and diverged. The presence of the pelvic fin and the presence of teeth are seen only in four fish (A, B, C, and E), suggesting F diverged first. Next, the number of gills is two for all remaining fish except C, suggesting that it was the next to diverge. Lastly, A and B have shorter body lengths than their ancestors, suggesting they were the next to diverge, with B displaying a more exaggerated phenotype including hooked fins. As every characteristic of E is shared with at least one other of our five main fish, it likely resembles their common ancestor the most. This gives us the following phylogenetic tree.



Although Fish B looks 'pre-historic', based on this logic, it was one of the last fish to diverge. As this solution uses a non-exhaustive list of features in a hypothetical scenario, other potential solutions are also viable as long as the logic and explanations are sound.

10 points:

Three cell cultures (A, B, and C) obtained from three different mammalian tissues were treated with a dye that is essentially non-fluorescent, but becomes strongly fluorescent upon binding to DNA. After that, each of them were passed through a special instrument that measures the intensity of fluorescence of each individual cell. The results (histograms that show a distribution of cells depending on their fluorescence) are presented on the left panel. A right panel shows the results of the similar experiment with the culture D obtained from the same organism. The culture D was split into four portions, and each of them was treated with increasing concentrations of the compound called cisplatin 7 hours before the measurement.



Graphs depicting the result of the experiment, with the number of cells against the intensity of fluorescence

From which tissues were the cultures A-C obtained? Explain your answer, and keep in mind that this question may have more than one correct answer. Explain what happens on the right panel, and what practical application the observation made during this experiment may have.

Hint:

If we disregard mitochondria (which have their own DNA, and its amount may vary) all cells in our body have the same amount of DNA, unless ....

Solution:

Answer:
The cultures have been dyed with a fluorescent marker of DNA, such as ethidium bromide, DAPI, or acridine orange. The graphs show a histogram of cells emitting a certain amount of fluorescent signal during flow cytometry – this acts as a proxy for DNA in a cell. In graphs B and C, we notice the presence of 2 peaks, one of which has double the fluorescence value of the other, and therefore twice the amount of DNA. The lower fluorescence peaks represent cells that have not yet entered the S phase, while the higher fluorescence peaks represent cells in G2, having already replicated their DNA and preparing for cell division. In graph C, the two peaks have fluorescence values of roughly 20 and 40, while in graph B the two peaks are at 40 and 80 - exactly double. Since mammalian cells are typically either haploid (possess one copy of each chromosome) or diploid (possess two copies), this tells us that culture C is most likely haploid while culture B is most likely diploid. While polyploidy is present in some mammalian tissues and cancers, the most likely scenario is n and 2n: haploid and diploid. In mammals, only gamete cells are haploid. Thus, culture C is composed of germline cells. Culture B can be derived from any replicating somatic tissue: skin, lung, colon, fibroblasts, and certain immune cells are some possible examples. In addition, stem cells and rapidly dividing cancerous cells could also produce such a graph. Culture A has only one peak and is therefore post-mitotic: that is, these cells are no longer dividing. In mammals, only neurons, adipocytes, and mature muscles cells are post-mitotic, so culture A will be one of those.

- A: post-mitotic cells – either neurons, adipocytes, or mature muscle cells
- B: mitotic cells such as skin, breast, lung, colon, etc.; could also be stem cells or cancer cells
- C: germline cells

Cisplatin is a common chemotherapy drug that is a key component of the treatment for many cancers. It attacks rapidly proliferating cells by interfering with DNA replication – and therefore cell division – eventually triggering apoptosis during the G2/M phase of the cell cycle. In the experiment with culture D, cells were treated with 0, 0.1, 1, and 5 uM of cisplatin before being treated with ethidium bromide. Cells were then sorted based on their fluorescence using flow cytometry. The blue trace (0 uM) shows two peaks, indicating the normal process of cell division: the lower peak represents those cells that have not yet entered the S phase of the cell cycle to replicate their DNA, while the higher peak represents those cells that have already copied their DNA and are preparing for division. Once again, the higher peak has almost exactly double the fluorescence, indicating the duplication of DNA. The cultures treated with 0.1 and 1 uM of cisplatin have much fewer cells that have replicated their DNA, while those treated with 5 uM have none.

Linguistics & Applied Sciences

5 points:

In one African language, nouns fall into different categories generally based on the type of objects they describe, which affects the prefixes that form singular and plural nouns. Here are examples of words that belong to seven of those categories:

Category 1: **qũrhũ** - a person, **ono** - females

Category 2: **rcera** - a bird, **rcuna** - grass,

Category 3: **nĩore** - a quiver, **nĩluqfu** - a drum

Category 4: **vũũa** - a river, **‘lile** - wind

Category 5: **ũleve** - information, **ũke** - magic

Category 6: **nũkũvũ** - a foot, **qoxũ** - ears

Category 7: **nolĩĩ** - a small boy, **nolako** - a little stone

Below is a list of ten nouns. Place each one into the category it belongs to.

- **nĩvĩkũ**
- **ũxlaũ**
- **rmorkĩva**
- **qene**
- **kĩnure**
- **oxloqona**
- **‘lili**
- **nokũrho**
- **ũrhe**
- **rhzova**

Match the English words to the words from the first part of the question. Describe your reasoning behind word placement and what characteristic each of the noun categories is based on.

- Joy
- Sandal
- Rulers
- Arms
- Facial expression
- An illness
- Outcast
- Love
- Garden

Hint:

No hint this month.

Solution:

Answer:
Category 1 (people) : oxloqona (rulers) Common trait: o- prefix Category 2 (lesser living things) : rmorkĩva (outcast), rhzova (illness) Common trait: r- prefix Category 3 (inanimate objects): kĩnure (joy), nĩvĩkũ (sandal) Common trait: nĩ- (singluar) and kĩ- (plural) prefix Category 4 (undulation): ‘lili (bat - accidentally left off English word list) Common trait: ’- sound Category 5 (abstract objects): ũxlaũ (facial expression), ũrhe (love) Common trait: ũ- prefix Category 6 (body parts): qene (arms) Common trait: q- prefix (plural) Category 7 (diminutive) : nokũrho (garden - think of it as a little farm) Common trait: no- prefix
Note: The English words could only be matched to a category; there was not enough information to match them to specific words if more than one word was in a category.

10 points:

Match the sentences in the same unknown African language to their corresponding English translations.

1) Alice planted the spinach last year	A) Bob aarugira irigu tene
2) Bob cut the spinach already this year	B) Alice atemita mwaka giki nyeni Bob aratema riu nyeni
3) Alice will cut the grass this year	C) Alice arahandera mwaka giki turari nyeni
4) Bob will plant the corn tomorrow	D) Bob akatema mwaka tuguthii nyeki
5) Alice planted bananas yesterday	E) Alice ahandira ira irigu
6) Bob cooked the bananas long ago	F) Bob atemira mwaka giki nyeni
7) Alice had cut the spinach this year before Bob cut the spinach now	G) Alice akutema mwaka giki nyeki
8) Bob will cut the grass next year	H) Bob akuhanda ruciu mbembe

We know that at some point, these events happened in this order relative to each other: Bob planted the corn, Alice cut the corn, and then Bob cooked the corn. Write a sentence in the unknown language that describes two of these events in the correct order.

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Hint:

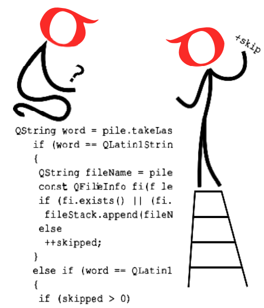
No hint this month.

Solution:

Answer:
1:C 2:F 3:G 4:H 5:E 6:A 7:B 8:D
Alice planted the spinach last year = Alice arahandera (8) mwaka giki turari nyeni = [alice planted during the previous unit of time, previous last year, the spinach]
Bob cut the spinach already this year = Bob atemira (2) mwaka giki nyeni = [bob has already within this unit of time cut this year the spinach]
Alice will cut the grass this year = Alice akutema (4) mwaka giki nyeki = [alice will in the future within this unit of time cut this year the grass]
Bob will plant the corn tomorrow = Bob akuhanda (4) ruciu mbembe = [bob will in the future within this unit of time tomorrow plant the corn]
Alice planted bananas yesterday = Alice ahandira (2) ira irigu = [alice has already within this unit of time yesterday planed the bananas]
Bob will cut the grass next year = Bob akatema (6) mwaka tuguthii nyeki = [bob will in the next unit of time cut next year the spinach]
Alice had cut the spinach this year before Bob cut the spinach now = Alice atemita (3) mwaka giki nyeni Bob aratema (1) riu nyeni
Bob cooked the bananas long ago = Bob aarugira (10) irigu tene
.
Verb tenses:

1. Present unit of time, at the moment of speaking = pronoun + ra + stem + ending
2. Present unit of time, has already happened within this unit of time = pronoun + stem + ir + ending
3. Present unit of time, a 'had' tense = pronoun + stem + ĩt + ending
4. Present unit of time, immediate future = pronoun + kũ /gũ + stem + ending
5. Present unit of time, immediate future = pronoun + kũ /gũ + stem + ending
6. Present unit of time, immediate future = pronoun + kũ /gũ + stem + ending
7. Present unit of time, immediate future = pronoun + kũ /gũ + stem + ending
8. Present unit of time, something was done while something was happening = pronoun + kũ /gũ + stem + ir + ending
9. Next unit of time, immediate following = pronoun + ka/ga + stem + ending
10. Future other than next unit of time, indefinite future = pronoun + rĩ + stem + ending
11. Preceding unit of time, immediate preceding = pronoun + ra + stem +ir + ending
12. Preceding unit of time, had tense (not he took, he had taken) = pronoun + ra + stem +ĩt + ending
13. Past other than previous unit, = pronoun + a + stem +ir + ending
14. Past other than previous unit, had tense = pronoun + a + stem +ĩt + ending

Computer Science



- Your program should be written in Java or Python-3.
- No GUI should be used in your program (e.g. `easygui` in Python).
- All the input and output should be done through files named as specified in the problem statement.
- Java programs should be submitted in a file with extension `.java`; Python-3 programs should be submitted in a file with extension `.py`.
  No `.txt`, `.dat`, `.pdf`, `.doc`, `.docx`, etc. Programs submitted in the incorrect format will not receive any points!
- Need some help? You can view our resource list, which has some great ways to learn about code.

Have you ever wanted to try your hand at writing your own POM problem? This month, we are continuing the **CS POM Participant Challenge**! Write and submit your own POM problem and solution by **November 10th** and it might just be published alongside our November problems.

- For inspiration, you can take a look at problems from previous months! We're looking for well written problems that are fun to solve.
- Submit your problem by emailing a PDF of the problem statement and the Python or Java file of your solution to pom@sigmacamp.org.
- If you have any questions, you can contact pom@sigmacamp.org.
- While no points will be awarded for the Participant Challenge, depending on interest participants who have their problems published may receive a small prize.

Consider the following game called "Chained Words". The board consists of four piles of three letters each (12 different letters in total). For example:

`TKZ  AMW  ELR  HSI`

To play the game, you form English words by using only these letters, with a few simple rules:

- You cannot use two letters from the same pile one after another within a word (that also means no double letters). In the example above, `SLATES`, `MIRTH`, `MIME` and `EARS` are valid words, but `RAM` and `MEEK` are not.
- A letter can only appear in one pile, for example "`TKZ AKZ TLR ASI`" would not be a valid board setup.
- Each word should be at least three letters long. In the example above, `TAT` is valid, but `AS` is not.
- A letter may be used multiple times within the sequence of words.
- The first word formed may start anywhere, but subsequent words must start with the last letter of the previous word. In the example above, the sequence `KEW-WHET-TEAK` is valid, but `KEW-TEAK` is not.

The goal of the game is to find a sequence with as few words as possible that uses all the 12 letters on the board.

5 points:

Write a program that receives a description of the board and a sequence of words, and determines if the sequence follows all the rules of Chained Words and uses all 12 of the letters on the board.

Your program should read the input file `input.txt`, which will contain two lines. The first line specifies the board position, given as a string of uppercase letters, with piles separated by spaces (e.g. "`TKZ ELR HSI AMW`"). The second line of `input.txt` consists of a list of words in the sequence, separated by spaces.

Your program should produce the file `output.txt`, which will contain either

- "`CORRECT`", in the case the sequence is valid and uses all 12 letters, or
- "`INCORRECT`", otherwise.

Example `input.txt`:

```
LNC DPO TRB IUA
BARD DUPLICATION
```

Example `output.txt`:

```
CORRECT
```

Hint:

No hint this month.

10 points:

Write a program that receives a description of the board and a list of English words, and finds the alphabetically-first sequence of English words that follows all the rules of Chained Words, uses all the 12 letters, and has as few words as possible.

Your program should read the input file `input.txt`, which will contain one line specifying the board position, given as a string of uppercase letters, with piles separated by spaces (e.g. "`TKZ ELR HSI AMW`"). However, there is a separate file called `words.txt` that your program should use as the list of valid English words, with one word per line. We will provide the file when grading (in the same directory where your program is run). You can download it yourself here for testing:

https://www.dropbox.com/scl/fi/joeoyuhc3iat7e8hlyiy4/words.txt?rlkey=bamyosttqp8wafh9zv8ogw3c0&dl=0

Your program should produce the file `output.txt`, which should consist of the alphabetically-first shortest sequence, with the words separated by spaces. In the case that there is no valid sequence that uses all the letters, `output.txt` should read "`IMPOSSIBLE`".

Example `input.txt`:

```
LNC DPO TRB IUA
```

Example `output.txt`:

```
ABDICATION NONPLUTOCRATIC
```

Hint:

No hint this month.

Solution:

The SigmaCamp GitHub repository contains the solutions for both the 5 point and 10 point questions:
https://github.com/SigmaCode/POM-QQ/tree/main/CS/2023-2024/October