SigmaCamp's *Problem of the Month* Contest

January 2025

Starting from September 2024, we are requiring all submissions to be `.pdf` files (except for CS, which requires `.py` or `.java` files). If you are using Word, you may export to PDF by clicking File > Export > Create PDF/XPS Document.

Mathematics

For all mathematics problems, please provide full justification. **Do not include any code** in your submission — all code submissions will be awarded no points.

5 points:

Two tangents AT and BT touch a circle at A and B, respectively; AT is perpendicular to BT. Q is on AT, S is on BT, and R is on the circle, so that $QRST$ is a rectangle with $QT = 8$ and $ST = 9$. Determine the radius of the circle.

Hint:

Draw a diagram and extend QR.

10 points:

Bob has unlimited supply of beads of 20 different colors. He needs to place some beads in a row, so that beads of any two different colors will be next to each other somewhere in this row. What is the minimal amount of beads Bob needs for his row?

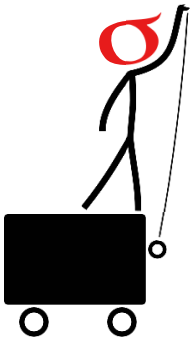
Hint:

Some pairs will have to appear more than once (why?).

Make sure to show that your answer is both a lower bound (you must have at least this many beads) and an upper bound (with this many beads Bob's task is doable).

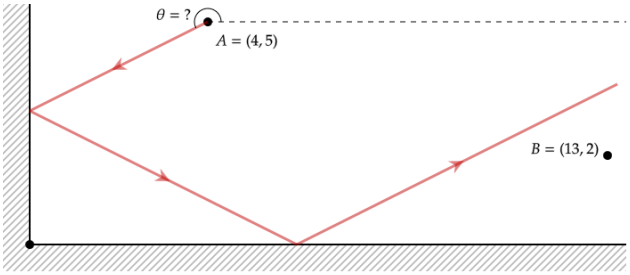
What is a mathematical structure that might be well suited to represent colors and pairs of colors?

Physics



5 points:

A mirror that lies along the positive x-axis and a mirror that lies along the positive y-axis are connected at a 90° angle as shown in the diagram. How many angles are there such that light shone from A will pass through B? The angles at A are measured from the positive horizontal direction as shown in the diagram.



Hint:

Try plotting B's images in the mirrors.

10 points:

A strange-looking funnel, shaped like a pyramid with three segments, is placed upside-down on a horizontal rubber surface. Each segment is a rectangular prism (see Figure 1 on the left). All segments have the same height of 5 cm and different horizontal dimensions (length × width): 20 cm × 20 cm at the bottom, 10 cm × 10 cm in the middle and 5 cm × 5 cm at the top. There are no internal walls, and the funnel can be filled with water through the thin tube attached to the top of the pyramid (the cross-sectional area of this tube is negligible). When the water level reaches a total height of $H = 20$ cm (15 cm inside the funnel and an additional 5 cm in the tube), the water lifts the funnel and starts leaking at the base. Find the mass M_p of this pyramid-shaped funnel without the water. What would be the mass M_c if we took a standard cone-shaped funnel with the same base area 20 cm × 20 cm and the same height of 15 cm (see Figure 1 on the right)?

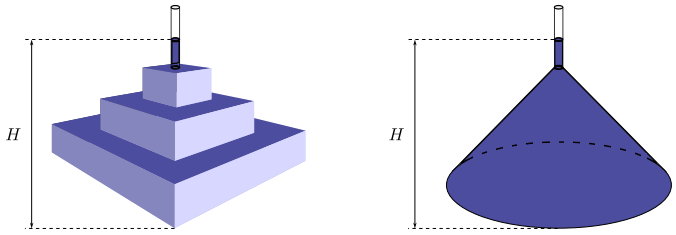
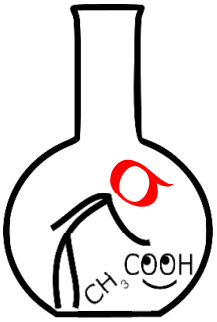


Figure 1: Pyramid-shaped and cone-shaped funnels

Hint:

The funnel lifts when the water pressure creates enough upward force to balance the weight of the funnel. Start by finding an expression for this lifting force.

Chemistry



5 points:

A chemist set out on an expedition to a remote location, bringing along various supplies, including a 1 M solution of NaCl and a 1 M solution of potassium sulfate for his experiments. He had prepared these solutions beforehand, distributing the first solution into two bottles and the second into one bottle. Unfortunately, he forgot to label them. Since all the bottles are identical and he has no additional solutions or equipment other than test tubes, suggest a simple method for determining which solution is in each bottle.

Hint:

No hint this month.

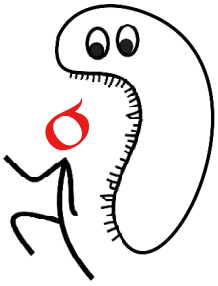
10 points:

Iodized salt is an important way to introduce iodine in diets, especially for populations that do not have access to sea food. Propose a way to prove that table salt actually contains iodine (in any form), and check it experimentally.

Hint:

It is hard to obtain chemicals that allow easy detection of I^-, but it is very easy to detect I_2.

Biology



5 points:

When Tarika was a young child, she cut planaria worms and sent them into space aboard a spaceship mission. What might have been the scientific objectives behind this project? Additionally, how would you propose measuring the rate of regeneration in these worms under microgravity conditions? Do you think the regeneration rate in space would be faster, slower, or unchanged compared to Earth's gravity?

Hint:

We won't limit your imagination, so there won't be any hints for this problem.

10 points:

Numerous films and books feature undead humans, commonly known as zombies. Often, these portrayals are extreme and stretch the bounds of biological plausibility. Recalling your knowledge of zombies from movies or books, identify and describe an example of the most biologically plausible and the most biologically implausible zombies. Explain what features contribute to their plausibility or implausibility. Some movies or books you could explore are *The Walking Dead*, *Angel Heart*, *Zombieland*, *The Serpent*, and *The Dead Don't Die*.

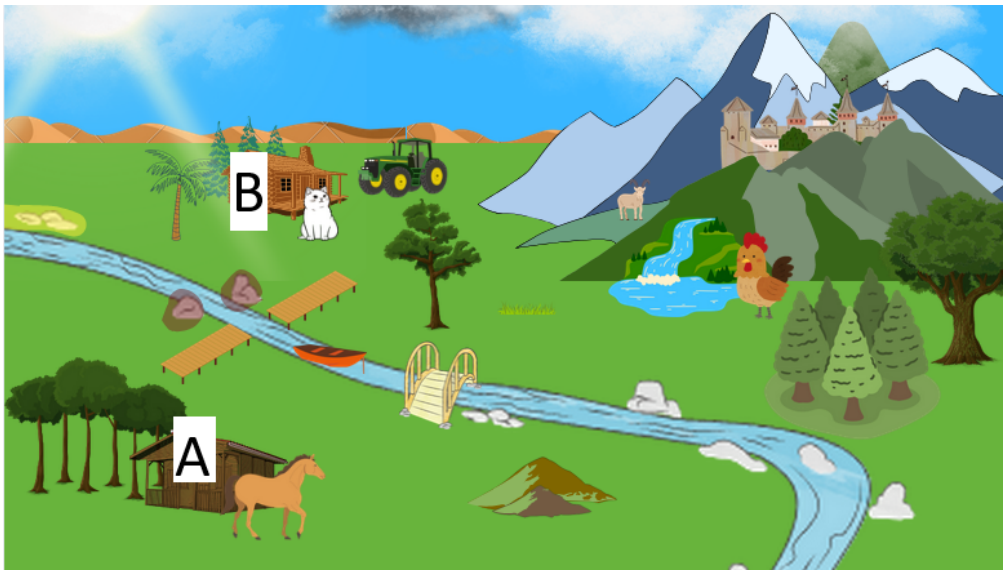
Hint:

Consider zombies in the context of fundamental principles like the laws of energy and matter conservation.

Linguistics & Applied Sciences

5 points:

Two sisters, Anabelle and Beatrice live in Sunny Valley, (depicted in the image below).



Anabelle and Beatrice are the last remaining speakers of an endangered language that used to be spoken in the valley. This language contains many *demonstrative words* (such as 'this', 'that', 'there') that change depending on what place or thing a person is talking about. Here are some demonstratives Anabelle and Beatrice used yesterday:

Anabelle and Beatrice are talking on the phone from their homes. Anabelle lives in house A and Beatrice lives in House B.

Anabelle is talking about the desert	aighkash
Beatrice is talking about the desert	aukash
Anabelle is talking about the river dock	eis
Beatrice sees the stretch of river between the colored rocks	eikash
Anabelle is talking about a cat near house B	aus
Anabelle is talking about a lake near house B	auzna
Beatrice is talking about the fields around house A	aukarnayo
Anabelle is talking about a horse near her house	hus
Beatrice is talking about the forests near house A	aukarna
Beatrice is talking about a tractor she is sitting in	tus

A and B go fishing at the dock.

Beatrice talks about all the fish she hopes to catch	hukash
Beatrice refers to a fish she caught on her fishing rod	tus
Anabelle refers to Beatrice's boat near the docks	hukarna
Anabelle mentions a fish she sees in the water	hus
Beatrice talks about a fishing rod back at her house	aus
Beatrice talks about a bridge downstream	eikarna

Anabelle mentions a different fishing spot upstream	eizna
Anabelle mentions Mount Lashka	aighs
Beatrice recalls a goat on the path to Lashka	aighzna
Anabelle talks about the mountains surrounding Lashka	aighznayo

Today, Anabelle and Beatrice decided to go treasure hunting at Mount Lashka. They took a boat down the river, hiked for an hour and sat down near the waterfall. **Decide what demonstrative term should be used for each object / place that Anabelle and Beatrice are talking about.**

1. Anabelle sees a chicken on the other side of the waterfall
2. Beatrice sees a goat on a hill across the valley
3. Beatrice talks about a faraway castle behind the mountains
4. Anabelle mentions the mountain flowers under her feet
5. Anabelle mentions glaciers visible from the path
6. Beatrice discusses other distant lands to see after Lashka
7. Anabelle mentions her house
8. Beatrice refers to trees that were left near the boat
9. Beatrice refers to the tractor near her house
10. Anabelle finds a coin on the side of the path

Hint:

Consider how the demonstrative word changes depending on where Anabelle and Beatrice are located with respect to each other, their environment, etc.

10 points:

The following problem asks you to design a catapult using **only** printer paper, scotch tape, paper clips and / or rubber bands. The catapult must be able to propel an object between 10 and 20 grams (such as: a battery, a ball of foil, etc) through the air in one direction. Only clear scotch tape is allowed (no duct, electrical, or masking tape). Please submit the following items:

1. A **link to a video** of your catapult in action completing the longest desired projectile travel length. Please upload your video to Google Drive, YouTube (unlisted) or Dropbox and copy-paste the link as part of your PDF submission. Make sure that the link is shared with 'everyone' so that graders can watch the video.
2. A **1-2 page writeup** explaining the catapult and documenting evidence pertaining to: a) the weight of the projectile, b) the distance travelled, c) the fact that only permitted materials are used in the design. Please include at least one drawing of your design in the writeup.

Student performance will be curved based on the best / worst performing designs. Roughly 5 points will be awarded for a good writeup with no major design flaws, and roughly 5 points will be awarded for projectile performance. Students will be **disqualified** (0 points for performance) if they provide no evidence for the weight of the object or distance travelled. Good luck!

Hint:

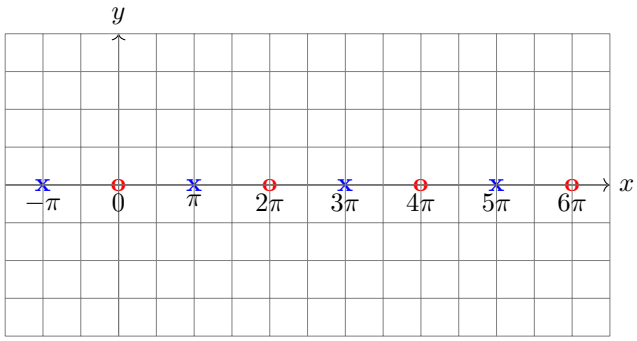
Once the ball touches the ground for the first time, distance is **no longer counted**. You can measure distance by leaving evenly spaced marks of tape on the floor or by laying a meterstick along the direction in which the projectile travels.
If you're having trouble uploading your video onto the SigmaCamp website, upload your file to Google Drive or DropBox first, then **make sure to change the sharing settings of your video** so that everyone could view / edit the file, then post a link as part of your PDF submission.

Computer Science

5 points:

A Support Vector Machine (SVM) is a machine learning tool that classifies data by finding the best dividing line (or boundary) between different groups. They aim to keep this boundary as far as possible from all data points to ensure accurate separation.

Anar gives Tarika the following set of points of two classes (**x** and **o**) that he wants to separate with a straight line using a Support Vector Machine. This set of points extends infinitely along the x-axis.



Tarika tells Anar, "How do I separate these two classes with a single straight line? All the points of both classes lie on the same line!" Anar then tells Tarika that she can transform her data in a way that allows it to be separable by a straight line. What should Tarika do?

Hint:

No hints this month.

10 points:

Hangman is a game in which a player tries to guess a hidden word by guessing each letter with a limited number of guesses. Implement your own variation of Hangman in Python or Java by using the following dataset of English words:

https://www.dropbox.com/scl/fi/joeoyuhc3iat7e8hlyiy4/words.txt?rlkey=bamyosttqp8wafh9zv8ogw3c0&e=1&dl=0

Include a PDF outlining your program's design, and explain your code structure thoroughly. Include your code either by pasting the code into your write-up or pasting a URL to your code on an external site (e.g. GitHub) in your write-up.

Be creative! Marks will be awarded for code structure, write-up structure and thoroughness, and the originality of your program.

Hint:

No hints this month.