SigmaCamp's *Problem of the Month* Contest

OCTOBER 2024

Starting from September 2024, we are requiring all submissions to be `.pdf` files (except for CS, which requires `.py` or `.java` files). If you are using Word, you may export to PDF by clicking File > Export > Create PDF/XPS Document.

Mathematics

For all mathematics problems, please provide full justification. **Do not include any code** in your submission — all code submissions will be awarded no points.

5 points:

How many ways are there to fill a 17 by 78 table with digits 0 through 9 so that the sum of all the digits in each column and each row is even?

Hint:

No hint.

Solution:

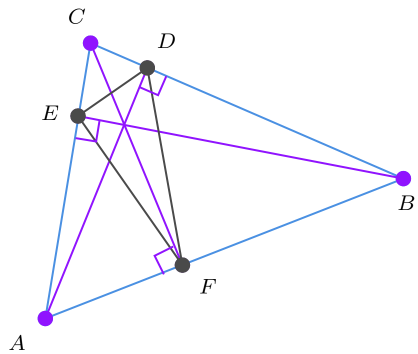
You can fill the first 16 rows and 77 columns arbitrarily, and there are $10^{16\times 77}$ options to do so. Then the first 16 rows of the last column and the first 77 columns of the last row need to be filled with either an odd or an even digit, to make the sum of all the digits in each column and each row even, and there are 5^{16+77} options to do this.

This leaves only the bottom-right corner, and the sum of the digits in both the last column and the last row must be made even. Thankfully, both of these condition depend only on the sum of all the digits in the upper-left 16×77 rectangle, and cannot contradict each other. This creates another 5 options.

Answer: $10^{16\times 77} \times 5^{16+78}$.

10 points:

In an acute $\triangle ABC$ construct altitudes AD, BE, CF. Find the ratio of the radii of circles circumscribed around $\triangle ABC$ and around $\triangle DEF$ and prove that this result holds for all acute triangles.



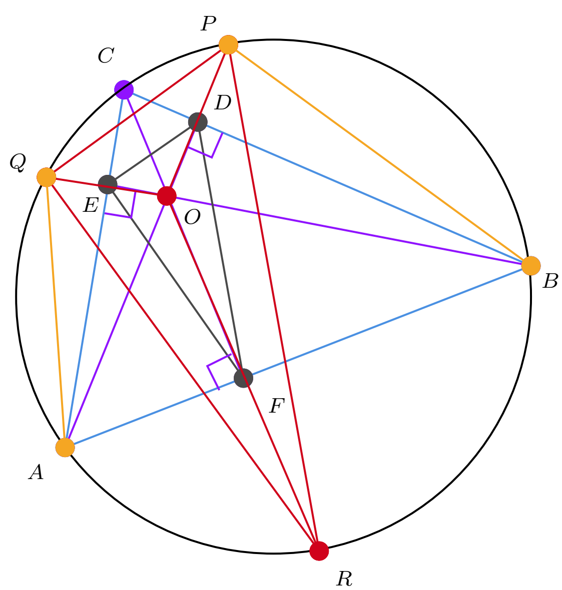
Hint:

1) Consider an easy triangle to figure out what the general answer must be.
2) Circumscribe a circle around $\triangle ABC$ and extend the altitudes.

Solution:

Extend lines OD, OE, OF to construct points P, Q, R where these lines intersect the circle circumscribed around $\triangle ABC$. Note that $\angle CAD = \angle CBE = 90^\circ - \angle C$ and $\angle CAD = \angle DBP$ as inscribed angles intercepting arc CP, thus $\angle CBE = \angle DBP$, so right triangles $\triangle DBO = \triangle DBP$ are congruent and $OD = DP$. For similar reasons, $OE = EQ$.

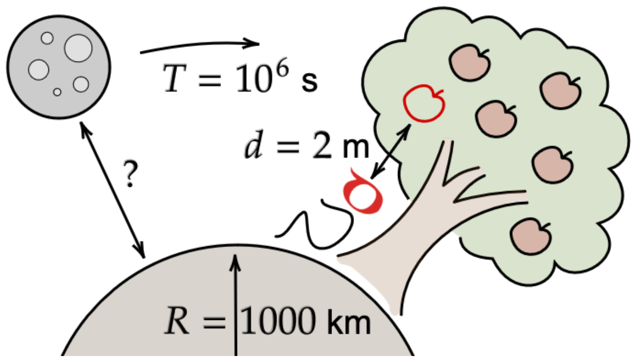
Then from similar triangles $\triangle OED \sim \triangle OQP$ we have $PQ = 2 \cdot ED$. Similarly, $PR = 2 \cdot DF$ and $QR = 2 \cdot EF$, and triangles $\triangle DEF \sim \triangle PQR$ are similar with ratio of similarity $2:1$. Since triangles $\triangle PQR$ and $\triangle ABC$ share the circumscribed circle, the ratio in question is $2:1$.



Physics

5 points:

You've just moved to a new planet. You notice that at your new home, it takes an apple one second to fall from a tree onto your head. The planet's only moon, which travels in a circular orbit, makes a full rotation once every million seconds. (See the picture for relevant distances.) How far is it from the planet?



Hint:

No hint this month.

Solution:

Let g be the acceleration due to gravity on the surface of your new home. Then $\frac{g}{2}t^2 = d$. Rearranging, $g = \frac{2d}{t^2} = \frac{2 \cdot 2\text{ m}}{(1\text{ s})^2} = 4\text{ m/s}^2$.

$g = GM/R^2$, so the mass of the planet multiplied by the gravitational constant is $GM = gR^2 = 4\text{ m/s}^2 \cdot (10^6\text{ m})^2 = 4 \cdot 10^{12}\text{ m}^3/\text{s}^2$.

Call the distance from the moon to the planet r. Then $\frac{GM}{r^2} = a_c = \omega^2 r$, so

$$r = \sqrt[3]{\frac{GM}{\omega^2}} = \sqrt[3]{\frac{4 \cdot 10^{12}\text{ m}^3/\text{s}^2}{\left(2\pi/\left(10^6\text{ s}\right)\right)^2}} = \sqrt[3]{\frac{1}{\pi^2}} \cdot 10^8\text{ m} \approx 4.66 \times 10^7\text{ m}$$

Answer: 4.66×10^7 **m**

10 points:

Our solar system has only one star, but many systems in the universe have two; they are called binary star systems. Imagine an ideal binary star system with two stars of identical mass orbiting on a circle around their collective center of mass. There are no planets in this system. What is the ratio of gravitational potential energy to kinetic energy in this system?

Hint:

No hint this month.

Solution:

Call the mass of each star M and the distance between them $2R$. Then the gravitational potential energy is

$$U = -\frac{GM^2}{2R}.$$

If the stars are moving with speed v, the kinetic energy is

$$K = 2 \cdot \frac{1}{2} M v^2 = M v^2.$$

The centripetal force on the stars is gravity, so

$$G \frac{M^2}{4R^2} = F_g = F_c = \frac{M v^2}{R}.$$

Then

$$v^2 = G \frac{M}{4R},$$

so

$$K = M v^2 = G \frac{M^2}{4R}.$$

The ratio of gravitational to potential energy is therefore

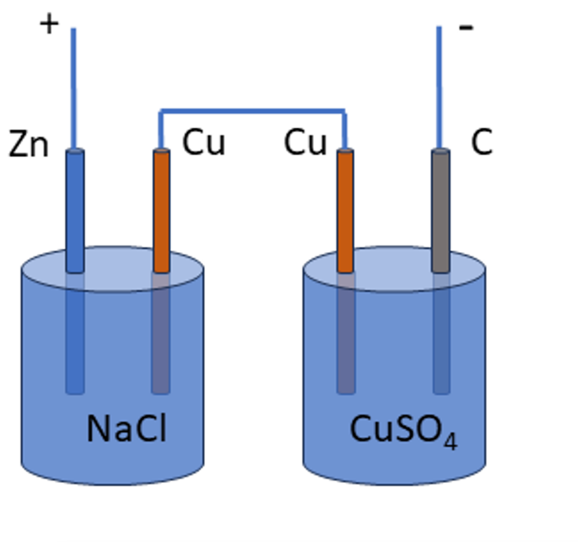
$$U/K = -2.$$

Answer: -2

Chemistry

5 points:

Two electrolytic cells are connected as shown in the figure below. The cell on the left has a zinc anode (the mass of the part immersed into the liquid is 3.25 g) and a copper cathode, the cell on the right has a copper anode and a graphite cathode. For both copper electrodes, the mass of the part immersed into the liquid is 4.25 g). The right cell contains 200 mL of 1 M $CuSO_4$, the left cell contains 200 mL of NaCl.



The cells were connected to the power source according to the polarity shown at the figure, and electric current was passed through the cells until the current could no longer flow though the cells. Calculate the mass of the copper electrode in the right cell by the end of the experiment.

Hint:

The electric current stops when there circuit is broken. Why did this happen in our case?

Solution:

The first question to answer is: why could the current no longer flow through the cells? The reason is simple: in the left cell, the anode was made of zinc, which releases Zn^{2+} ions when current passes through the cell. During this process, the electrode dissolves, and once it has completely dissolved, the electrical contact is lost and no current can flow.

Dissolution of zinc occurs because electrons are being withdrawn from it, so withdrawal of two electrons results in the formation of one zinc cathion, as described by the equation:

$Zn^0 - 2\,e^- \longrightarrow Zn^{2+}$

What happens at the other three electrodes? At the cathode of the left cell, where the electrons *come in*[1] hydrogen gas is formed:

[1] It is easy to see that as the *positive* ions flow from left to right, the electrons flow in the opposite direction, from right to left. The power supply pumps electrons into the carbon rod, then from the copper rod (cathode) in the right cell to the copper rod (anode) in the left cell, and finally the electrons are extracted by the power supply from the zinc electrode.

$2\,H^+ + 2\,e^- \longrightarrow H_2$,

therefore, for every dissolved zinc atom, one H_2 molecule is formed.

In the right cell, we observe the *dissolution* of the anode (copper rod), since electrons are pulled away from it, so the metal ions on the metal surface are no longer held by electrostatic forces and pass into solution:

$Cu^0 - 2\,e^- \longrightarrow Cu^{2+}$.

> *It is easy to see that for every one dissolved zinc atom in the left cell, exactly one copper atom passes into the solution in the right cell, and, in fact, this is the only fact we need to know to solve this problem.*

Let's finish our story, though. At the cathode in the right cell, electrons can interact with hydrogen cations or copper cations. However, unlike the right cell, where Na^+ (and, toward the end of the process, some Zn^{2+}) are present, Cu^{2+} are more active electron acceptors, so copper is formed instead of hydrogen gas:

$Cu^{2+} + 2\,e^- \longrightarrow Cu^0$.

In other words, the process in the right cell can be described as *transfer* of copper from the left rod (where copper dissolves) to the carbon rod (where it deposits), with the dissolution of one copper atom from the copper rod being accompanied by the deposition of one copper atom on the carbon rod. But let's get back to the problem.

We know that the total amount of zinc dissolved is 3.25 g, and we know that the dissolution of one atom of zinc in the left cell is accompanied by the dissolution of one atom of copper in the right cell.

Therefore, to calculate the amount of dissolved copper, we need to solve a simple proportion:

$$\frac{m_{Zn}}{M_{Zn}} = \frac{x}{M_{Cu}}$$

where m_{Zn} is the mass of dissolved zinc, M_{Zn} and M_{Zn} are the molar masses of Zn and Cu, respectively, and x is the mass of dissolved copper. It is easy to calculate that $x = 3.159$. We know that the mass of the copper anode (the part immersed in the liquid) was initially 4.24 g, so by the end of the process it will be $4.25 - 3.159 = 1.09$.

10 points:

Compound X is a gas with a density of 2.158 (relative to air). When X burns in a flow of oxygen, the combustion products are gaseous. These new gases were collected and bubbled sequentially through a flask containing a silver nitrate solution (flask A) and then through a flask containing a barium hydroxide solution (flask B). The formation of a white precipitate was observed in both flasks. When the same experiment was repeated, but the gas passed through flask B and then through flask A, a white precipitate was observed only in flask B.

What is the name and structure of gas X, and why is it important for industry?

Hint:

At standard atmospheric pressure and 0° C, air density is 1.2922 g/L. At the same temperature and pressure, one mol of any gas occupies 22.414 L, which means 22.414 L of air weigh 28.96 g.

From that, you can calculate the exact molecular weight of the gas X.

Solution:

In this problem, we have two different sources of information. First, we know the density of gas X relative to air. One mole of air weighs 28.96 g, and given that one mole of any gas occupies 22.414 L under standard conditions, we can easily calculate the molecular mass of gas X:

$$M_X = D_{air} \cdot M_{air} = 2.158 \cdot 28.96 = 62.496$$

The second source of information is the analysis of its combustion products. We know that the combustion products of X react with silver nitrate solution, and the first idea that comes to mind is that the gas causing the precipitation is HCl, HBr, or HI. The last two gases can be ruled out because X can only contain one chlorine atom, and even one bromine is heavier than X.

Therefore, most likely, X burns as follows:

$X \longrightarrow HCl + nCO_2 + mH_2O$

In the first flask, the reaction is as follows:

$HCl + AgNO_3 \longrightarrow AgCl + HCl$

Can CO_2 react with $AgNO_3$? Actually yes, since silver carbonate is insoluble, so the process would be:

$CO_2 + H_2O \longrightarrow H_2CO_3$, and

$H_2CO_3 + 2\,AgNO_3 \longrightarrow Ag_2CO_3 + 2\,HNO_3$

However, when we have a mixture of CO_2 with HCl, the first reaction is suppressed, Ag_2CO_3 is never formed, and CO_2 passes into the second flask where it reacts with $Ba(OH)_2$

$CO_2 + Ba(OH)_2 \longrightarrow BaCO_3 + H_2O$

In the second experiment, when the gases pass trough the second flask, both HCl and CO_2 react with $Ba(OH)_2$:

$2\,HCl + Ba(OH)_2 \longrightarrow BaCl_2 + 2\,H_2O$

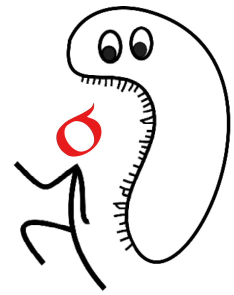
and nothing comes to the first bottle (hence no precipitation in it).

Now we know that gas X contains hydrogen, carbon, and chlorine[2]. What could it be? The only compound with a general formula C_nH_mCl that has a mass of 62.496 is C_2H_3Cl, and that is vinyl chloride.

This gas is the source of the production of polyvinyl chloride (PVC), the second most popular plastic, so it is really important.

[2] It contains hydrogen because gases that contain only C and Cl are not flammable. Theoretically, it could contain other elements, but it would be hard to squeeze them into 62.496 Da

Biology



5 points:

On the planet Pandora, astrobiologist Elena discovered an unknown brightly colored organism, which she called Tsvetick. But to her astonishment, an AI-based wide-spectrum biochemical analyzer demonstrated that this organism contains no pigments! "How can Tsvetick be colored, then?" wondered Elena. What possibilities do you think she should consider? Most importantly, how can these hypotheses be tested? What experiments should be conducted to test these hypotheses?

Hint:

No hint this month.

Solution:

Answer:

For some objects to be colored, they must differently reflect light of different wavelengths. That can be achieved if the object contains some pigment (i.e. the molecules that specifically absorb photons with some concrete energy or wavelength). However, a similar effect may be achieved even if the object contains no pigment. Thus, the object may be colored if it reflects the incident light at different angles depending on its wavelength. That may happen when the surface has tiny structural elements with a characteristic size comparable to the wavelength of light (several hundreds of nanometers). In that case, the object behaves like a diffraction grating (a typical example is a CD). Another example is an object covered with a thin reflecting film (thus, anodized aluminum or iron can be red, blue, or green, but that is not a result of painting: in reality, a thin (about half a micron) film of a colorless metal oxide forms at the metal surface, and the surface is colored due to light interference.

In the above-described case, the object is colored despite the fact that it contains no pigments or other light absorbing molecules.[3] These are examples of "structural coloration".

Therefore, the first thing that is necessary to do with Tsvetik is to study (using, e.g. an electron microscope) a fine structure of its outer surface to reveal a possible fine structure. Most likely, Tsvetik is an example of an organism that, like butterflies, is colored due to structural coloration.

10 points:

Ph.D. student Belova is studying the functions of the proteins Alaphin and Bethin. Belova's supervisor, Prof. Harold Varmus, suspected that these proteins were key players in cancer. Belova analyzed 100 samples of tumors collected from patients with advanced skin cancer. Sixty of these samples carried mutations in the Alaphin gene. These mutations increased the activity of the Alaphin protein ("gain of function"). The remaining 40 samples had mutations in other oncogenes, but no mutations in Bethin were found among them. Belova hypothesized that Alaphin, but not Bethin, is an oncogene (a gene whose mutations lead to cancer). To test her hypothesis, Belova used the cell line derived from human keratinocytes. Using genetic engineering methods, she constructed variants of TL1 that produced hyperactive forms of either Alaphin or Bethin. It turned out that both variants exhibited signs of oncogenic transformation: unlike the original, they formed growing multilayer cultures in a Petri dish in an incubator and, when injected into immunodeficient mice, formed noticeable tumors.

[3] That refers not only to ordinary pigments but to fluorescent or chemiluminescent dyes: they are just specific instances of dyes. Fluorescent molecules are the dyes that, instead of converting the absorbed light into heat (like ordinary dyes do), emit it back, although its wavelength becomes longer. Fluorescent dyes may be excited not by light but as a result of some chemical reaction. In that case, we observe chemiluminescence. In both cases, some dye must be present, so if Tsvetik were fluorescent or chemiluminescent, chemical analysis would reveal the presence of some dye.

Belova brought her results to Prof. Varmus and asked him: "Professor, I really don't understand this. How can it be possible that, although both proteins Alaphin and Bethin are equally favorable for cancer progression, only Alaphin seems to be involved in actual skin cancers?"

Harold Varmus looked at Belova's data and replied: "I see no paradox here. Actually, you made one logical error in the interpretation of your data. In reality, if you organized your experiment differently, it would be more informative. I suggest you do the following ..." What modification to the experiment did Prof. Varmus propose?

Hint:

Cancer is a complex phenomenon, so tumor formation can be caused by several parallel processes that are essentially independent of each other. In our case, it could be a mutation in either Alaphin or Bethin. Usually, a single mutation in just one protein is not enough to turn a normal cell into a tumor cell, but it is only the first step in a chain of events that eventually leads to tumor formation. Since tumor formation initiated by different mutations occurs in different ways, its speed and efficiency can be different. Therefore...

Solution:

Answer: All cancer cells must have three properties:

1. They must be immortal (divide an infinite number of times);
2. They must be able to form a multilayered structure:
3. Their connection to the neighbor cells should not be strong, and they must be able to travel freely.

The first property is achieved by activating telomerase, a specialized enzyme that adds a short oligonucleotide sequence to the ends of each chromosome after each DNA replication event. This sequence consists of repeating motifs rich in guanines (for example, most vertebrates have TTAGGG repeats). Instead of forming a double helix, these motifs fold into a fancy structure that does not resemble DNA, so the ends of the chromosomes become protected. In normal somatic cells, telomeres become shorter after each cell division, and after 50 divisions they disappear. After this, the free (unprotected) ends of the chromosomes become exposed, which is recognized by tumor suppressor proteins as broken DNA, i.e., as an alarm signal that triggers the cell's self-destruction process.

In cancer cells, telomerase is active, it lengthens telomeres after each division, cancer cells can divide an infinite number of times. However, telomerase activation alone is not enough for a cell to become cancerous.

The second condition is the ability to form 3D structures. Most cancers are the result of the transformation of epithelial cells, which is not surprising because any epithelium has many immortal cells: our skin is constantly growing (the outer layer peels off, and new cells are formed underneath); the epithelium in our intestines is renewed every day: the outer cells die and peel off, and new cells are formed in their place. In other words, epithelial cells are more prepared to become cancerous, since condition number 1 is already partially fulfilled for many of them. However, all epithelial cells form only 2D structures and quickly stop growing when the tissue forms a thick layer. To turn into a tumor, they must overcome this obstacle (acquire some mutation or similar change). However, even if a cell can divide an infinite number of times and form a 3D structure, the tumor will not be malignant (it does not form metastases). For this, the third type of transformation is necessary.

This transformation makes the cells capable of detaching from their neighbors and crawling (like macrophages). Only then is the transformation into full-fledged cancer complete.

All this together means that cancerous transformation is a complex and multi-stage process, and it can begin with the mutation of various genes. This is usually accompanied by an increase in the rate of mutations in precancerous cells, which gives rise to their accelerated evolution.

In other words, each cancer is the result of a mini-evolution of some somatic cell, which can happen in multiple steps and in different ways. Some mutations lead to a rapid increase in the rate of cell growth and division, while others are less efficient. However, regardless of the specific type of mutation, each cancer cell exhibits exponential growth, although some cells grow faster and others grow more slowly. And during this process, new mutations appear in cancer cells, which further affects their growth rate and aggressiveness, so each cancer tumor can be considered a mini-evolutionary system, where different versions of cancer cells can coexist, and they behave like different species in a real ecosystem (i.e. they compete for the same resource).

Theoretical biologists have shown that if several exponentially growing species compete for the same resource, the principle of "survival of the fittest" applies. What does this mean for us?

Imagine that two cells in the same tissue acquire different mutations. In one cell, the Alaphin gene mutates, and in the other, the Bethin gene. These mutations trigger a cascade of malignant transformations, but the cells with the mutated Alaphin seem to grow more aggressively and accumulate new mutations faster, and eventually, the cells with the Bethin mutation are displaced and, in full accordance with Darwin's principle of "survival of the fittest," are completely replaced by cells carrying the Alaphin mutation. That is why, despite the fact that both Alaphin and Bethin mutations lead to cancer transformation, Alaphin-mutated cells outcompete Bethin cells in real tumors.

To prove this, Belova could, for example, grow a mixture of mutated Alaphin and Bethin cultures over a long period of time. She would likely observe the complete disappearance of the latter.

Linguistics & Applied Sciences

5 points:

Failure modes and effects analysis (FMEA) is a method that system engineers use to evaluate potential problems in a system and implement solutions or mitigation techniques before the system is made. The system itself can be an item, a building, a process, or anything else that you can design.

The main step of FMEA is to create a risk analysis chart. This chart considers the process, potential failure modes, potential failure effects, potential causes, and current controls (how to handle the potential failure). With this in mind, we can assign scores to the severity of the failure, the occurrence of potential causes, and the detection possible with current controls. From there, the Risk Priority Number (RPN) is calculated by multiplying the severity, occurrence, and detection values.

Imagine you are taking over as a quality engineer at a deli manufacturing plant. Your predecessor has left an incomplete FMEA chart, and you need to fill in the blanks by the end of the month and justify the scores you assigned. Find the chart here. Please make a copy of it (electronic or not) to edit for submission.

Recently, Boar's Head was in the news for multiple failures that occurred at their deli meat plant that led to listeria infections in 59 people, including 10 deaths. Based on the RPNs you have calculated, what failure do you think is the most likely to have contributed to the outbreak?

Hint:

No hint this month.

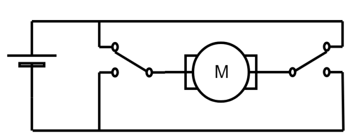
Solution:

A sample filled-out table is provided here.

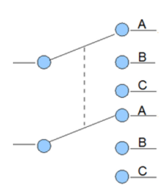
An important aspect of the problem was to identify which component resulted in the highest RPN and, therefore, is the most likely cause of failure. Students who used outside sources, rather than the RPN, to justify their component selection received points off.

10 points:

In a modern car window, you normally operate the windows by pressing a switch or a button located on the car door. Pushing the switch down opens the window by sliding it downwards, while pushing the switch up closes the window by sliding it upwards. A passenger's car window can usually be opened using two switches: one switch is attached to the passenger's door, while another switch is located by the driver. The window is actuated by a DC motor, which rotates in different directions depending on the polarity of the applied voltage.



Model A



Model B

(a) Model A above shows a potential model for the car window circuit. Depending on the position of the two switches, the motor will either stay off, move in one direction, or move in the opposite direction.

Explain why Model A *should not* be used as a base model for the car window circuit.

(b) Model B above shows a *double pole triple throw* (DPTT) “on-on-on” switch. Draw a car window circuit diagram that incorporates two DPTT switches and achieves the usual effect of operating the window independently from either switch.

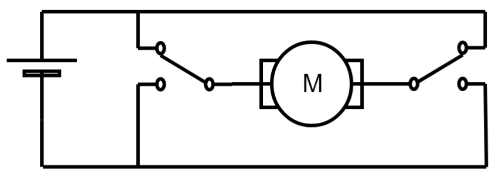
Make sure that your solution will not create a dead short under any circumstances.

Hint:

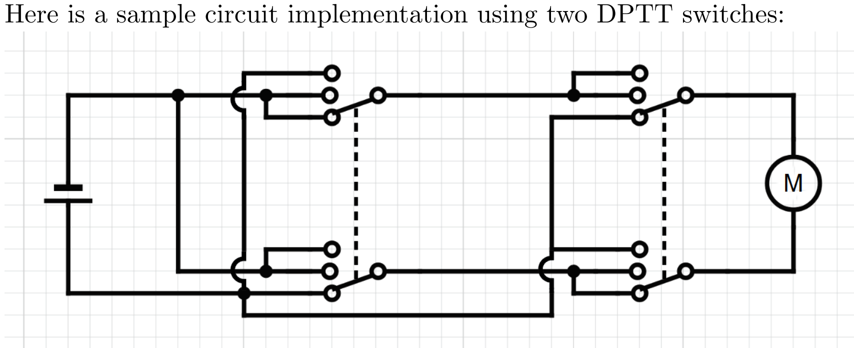
No hint this month.

Solution:

(a) In the diagram below, there is no way for two users to individually control the opening and closing of the window. The problem either enables one user to control both motor directions, or the circuit can be interpreted as each user only controlling one of the directions. Either way, the circuit doesn't work.



(b) Here is a sample circuit implementation using two DPTT switches:



By gaining control of an individual DPTT, each user can independently control opening and closing of the window by switching from neutral position to either one motor direction or the opposite motor direction.

Computer Science

- Your program should be written in Java or Python-3.
- No GUI should be used in your program (e.g. `easygui` in Python).
- All the input and output should be done through files named as specified in the problem statement.
- Java programs should be submitted in a file with extension `.java`; Python-3 programs should be submitted in a file with extension `.py`. **No `.txt`, `.dat`, `.pdf`, `.doc`, `.docx`, etc. Programs submitted in the incorrect format will not receive any points!**

Adam has just returned from a long vacation far away, which involved several layovers. However, the airport baggage handling system misplaced his suitcase, placing it on a different flight! After several weeks of long calls with different airlines, Adam finally recovered his suitcase. However, he noticed that it took a very different route from him: the suitcase visited a variety of different airports throughout its journey, and each airport except the final destination left a tag on the suitcase indicating that it passed through there.

Out of sheer curiosity, Adam wants to determine the possible airport routes that his suitcase travelled around the world.

5 points:

For this problem, assume that each airport tag on Adam's suitcase has both the origin and destination airport code. The tags are not necessarily listed in order. Furthermore, you can assume that no airport is visited more than once, and that there is a unique path the suitcase took from the initial airport to the final airport.

Write a program that determines the path of airports that the suitcase took to get back to Adam.

Your program should read the input file `input.txt`, which has the following format:

- The first line contains a positive integer n denoting the number of tags on Adam's suitcase, as well as two space-separated airport codes representing the initial and final airports.
- The next n lines contain two space-separated 3-letter airport codes in the format `ABC XYZ`, representing the source and destination airports.

Your program should produce the file `output.txt`, which contains the path of airports that the suitcase took, represented as a space-separated list of airport codes.

IMPORTANT NOTE: There is always one less tag than the number of airports. The final airport does **not** leave a tag on the suitcase.

Sample Input 1:

```
4 BOS LAX
YUL YYZ
BOS YUL
YYZ JFK
JFK LAX
```

Sample Output 1:

```
BOS YUL YYZ JFK LAX
```

Sample Input 2:

```
6 EWR TPE
CDG KIX
FRA CDG
DTW FRA
KIX TPE
CLE DTW
EWR CLE
```

Sample Output 2:

```
EWR CLE DTW FRA CDG KIX TPE
```

Hint:

No hint this month.

Solution:

The 5pt solution is available on the SigmaCamp GitHub repository here:

https://github.com/SigmaCode/POM-QQ/tree/main/CS/2024-2025/October

10 points:

In spite of various standardization attempts, airports don't always follow the same luggage tag conventions. For this problem, some airports left both the source and destination airports on their tag (type A tag), some airports only left their own airport code (type B tag), and some airports left a blank tag without any information on it (type C tag).

Type A tags contain both the source and destination airports: For example, JFK may leave a type A tag "`JFK BOS`" indicating a flight from JFK to Boston. Type B tags contain only the source airport: For example, JFK's type B tag will always be "`JFK`". Type C tags do not identify the airport, and only indicate that the luggage visited some other airport.

This time, Adam wants to *count* the number of paths that his suitcase could have taken to get from the initial airport to the final airport.

As there may be more than one route that the suitcase took, you are given a file consisting of 37,500+ routes between 3,400+ airports across the world, which you may download below as a `.csv` file:

https://github.com/SigmaCode/POM-QQ/blob/main/CS/2024-2025/October/routes-10pt.csv

Each row of this `.csv` file (aside from the header row) represents a single route between two airports separated by a comma: the first is the source airport code, and the second is the destination airport code. Below are the first 6 lines of the file:

```
src,dest
SNN,PMI
TRF,LPL
NOS,TNR
IAH,TUS
BDJ,BDO
...
```

If you are reading from this file in your code, it should be referenced in your code as "`routes-10pt.csv`" and it should be located in the same directory as your `input.txt` and `output.txt` files.

You can assume that each airport visited left one tag on the suitcase, and that each airport is visited no more than once. Furthermore, the initial and final airports will always be different.

Your program should read the input file `input.txt`, which has the following format:

- The first line contains a positive integer n denoting the number of tags on Adam's suitcase, as well as two space-separated airport codes representing the initial and final airports.
- The next n lines will be one of the following three formats:
  - "`A ABC DEF`", representing a type A tag left by airport ABC, indicating a flight from airport ABC to airport DEF.
  - "`B ABC`", representing a type B tag left by airport ABC, indicating that the suitcase visited airport ABC.
  - "`C`", representing a type C tag left by some airport.

Your program should produce the file `output.txt`, consisting of a single line containing the total number of paths that the suitcase may have taken to get from the initial to the final airport.

IMPORTANT NOTE: There is always one less tag than the number of airports. The final airport does **not** leave a tag on the suitcase.

Sample Input 1:

```
5 BOS LAX
B YUL
B EWR
B YOW
A BOS YUL
B YTZ
```

Sample Output 1:

```
2
```

Sample Explanation 1:

The two possible flights are:

$$\texttt{BOS} \to \texttt{YUL} \to \texttt{YTZ} \to \texttt{YOW} \to \texttt{EWR} \to \texttt{LAX}$$

$$\texttt{BOS} \to \texttt{YUL} \to \texttt{YOW} \to \texttt{YTZ} \to \texttt{EWR} \to \texttt{LAX}$$

Note that among the airports given by the tags, YTZ only flies to and from YUL, YOW, and EWR.

Sample Input 2:

```
2 JFK FRA
C
B JFK
```

Sample Output 2:

```
85
```

Sample Explanation 2:

Since the initial and final airports are JFK and FRA, we know that the type C tag belongs to some airport that is a layover between JFK and FRA. There are precisely 85 possible one-stop layovers from JFK to FRA.

Hint:

View the airports as a graph, with edges given by the routes. Consider doing a breadth first search (BFS) or a depth first search (DFS) on this graph to count the total number of paths, restricted by the tags.

Solution:

The 10pt solution is available on the SigmaCamp GitHub repository here:

https://github.com/SigmaCode/POM-QQ/tree/main/CS/2024-2025/October