SigmaCamp's *Problem of the Month* Contest

OCTOBER 2024

Starting from September 2024, we are requiring all submissions to be `.pdf` files (except for CS, which requires `.py` or `.java` files). If you are using Word, you may export to PDF by clicking File > Export > Create PDF/XPS Document.

Mathematics

For all mathematics problems, please provide full justification. **Do not include any code** in your submission — all code submissions will be awarded no points.

5 points:

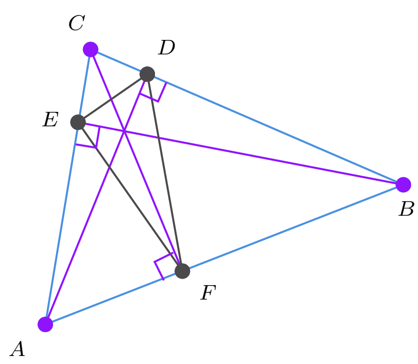
How many ways are there to fill a 17 by 78 table with digits 0 through 9 so that the sum of all the digits in each column and each row is even?

Hint:

No hint.

10 points:

In an acute $\triangle ABC$ construct altitudes AD, BE, CF. Find the ratio of the radii of circles circumscribed around $\triangle ABC$ and around $\triangle DEF$ and prove that this result holds for all acute triangles.



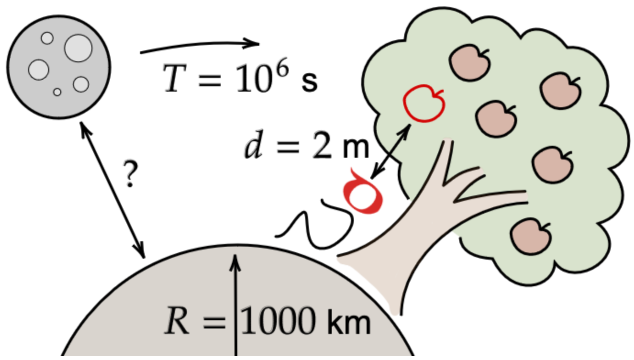
Hint:

1) Consider an easy triangle to figure out what the general answer must be.
2) Circumscribe a circle around $\triangle ABC$ and extend the altitudes.

Physics

5 points:

You've just moved to a new planet. You notice that at your new home, it takes an apple one second to fall from a tree onto your head. The planet's only moon, which travels in a circular orbit, makes a full rotation once every million seconds. (See the picture for relevant distances.) How far is it from the planet?



Hint:

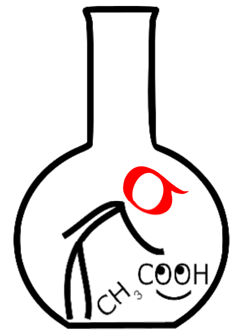
No hint this month.

10 points:

Our solar system has only one star, but many systems in the universe have two; they are called binary star systems. Imagine an ideal binary star system with two stars of identical mass orbiting on a circle around their collective center of mass. There are no planets in this system. What is the ratio of gravitational potential energy to kinetic energy in this system?

Hint:

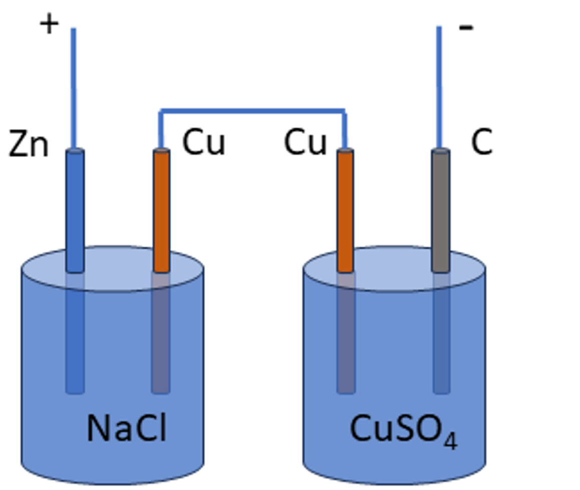
No hint this month.



Chemistry

5 points:

Two electrolytic cells are connected as shown in the figure below. The cell on the left has a zinc anode (the mass of the part immersed into the liquid is 3.25 g) and a copper cathode, the cell on the right has a copper anode and a graphite cathode. For both copper electrodes, the mass of the part immersed into the liquid is 4.25 g). The right cell contains 200 mL of 1 M $CuSO_4$, the left cell contains 200 mL of NaCl.



The cells were connected to the power source according to the polarity shown at the figure, and electric current was passed through the cells until the current could no longer flow though the cells. Calculate the mass of the copper electrode in the right cell by the end of the experiment.

Hint:

The electric current stops when there circuit is broken. Why did this happen in our case?

10 points:

Compound X is a gas with a density of 2.158 (relative to air). When X burns in a flow of oxygen, the combustion products are gaseous. These new gases were collected and bubbled sequentially through a flask containing a silver nitrate solution (flask A) and then through a flask containing a barium hydroxide solution (flask B). The formation of a white precipitate was observed in both flasks. When the same experiment was repeated, but the gas passed through flask B and then through flask A, a white precipitate was observed only in flask B.

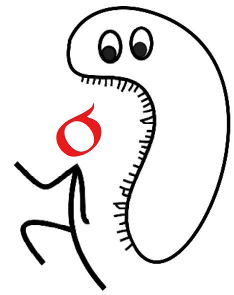
What is the name and structure of gas X, and why is it important for industry?

Hint:

At standard atmospheric pressure and 0° C, air density is 1.2922 g/L. At the same temperature and pressure, one mol of any gas occupies 22.414 L, which means 22.414 L of air weigh 28.96 g.

From that, you can calculate the exact molecular weight of the gas X.

Biology



5 points:

On the planet Pandora, astrobiologist Elena discovered an unknown brightly colored organism, which she called Tsvetick. But to her astonishment, an AI-based wide-spectrum biochemical analyzer demonstrated that this organism contains no pigments! “How can Tsvetick be colored, then?” wondered Elena. What possibilities do you think she should consider? Most importantly, how can these hypotheses be tested? What experiments should be conducted to test these hypotheses?

Hint:

No hint this month.

10 points:

Ph.D. student Belova is studying the functions of the proteins Alaphin and Bethin. Belova’s supervisor, Prof. Harold Varmus, suspected that these proteins were key players in cancer. Belova analyzed 100 samples of tumors collected from patients with advanced skin cancer. Sixty of these samples carried mutations in the Alaphin gene. These mutations increased the activity of the Alaphin protein (“gain of function”). The remaining 40 samples had mutations in other oncogenes, but no mutations in Bethin were found among them. Belova hypothesized that Alaphin, but not Bethin, is an oncogene (a gene whose mutations lead to cancer). To test her hypothesis, Belova used the cell line derived from human keratinocytes. Using genetic engineering methods, she constructed variants of TL1 that produced hyperactive forms of either Alaphin or Bethin. It turned out that both variants exhibited signs of oncogenic transformation: unlike the original, they formed growing multilayer cultures in a Petri dish in an incubator and, when injected into immunodeficient mice, formed noticeable tumors.

Belova brought her results to Prof. Varmus and asked him: “Professor, I really don’t understand this. How can it be possible that, although both proteins Alaphin and Bethin are equally favorable for cancer progression, only Alaphin seems to be involved in actual skin cancers?”

Harold Varmus looked at Belova’s data and replied: “I see no paradox here. Actually, you made one logical error in the interpretation of your data. In reality, if you organized your experiment differently, it would be more informative. I suggest you do the following ...” What modification to the experiment did Prof. Varmus propose?

Hint:

Cancer is a complex phenomenon, so tumor formation can be caused by several parallel processes that are essentially independent of each other. In our case, it could be a mutation in either alphaine or betaine. Usually, a single mutation in just one protein is not enough to turn a normal cell into a tumor cell, but it is only the first step in a chain of events that eventually lead to tumor formation. Since tumor formation initiated by different mutations occurs in different ways, its speed and efficiency can be different. Therefore...

Linguistics & Applied Sciences

5 points:

Failure modes and effects analysis (FMEA) is a method that system engineers use to evaluate potential problems in a system and implement solutions or mitigation techniques before the system is made. The system itself can be an item, a building, a process, or anything else that you can design.

The main step of FMEA is to create a risk analysis chart. This chart considers the process, potential failure modes, potential failure effects, potential causes, and current controls (how to handle the potential failure). With this in mind, we can assign scores to the severity of the failure, the occurrence of potential causes, and the detection possible with current controls. From there, the Risk Priority Number (RPN) is calculated by multiplying the severity, occurrence, and detection values.

Imagine you are taking over as a quality engineer at a deli manufacturing plant. Your predecessor has left an incomplete FMEA chart, and you need to fill in the blanks by the end of the month and justify the scores you assigned. Find the chart here. Please make a copy of it (electronic or not) to edit for submission.

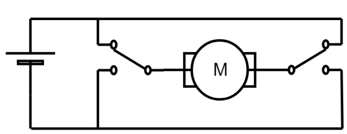
Recently, Boar's Head was in the news for multiple failures that occurred at their deli meat plant that led to listeria infections in 59 people, including 10 deaths. Based on the RPNs you have calculated, what failure do you think is the most likely to have contributed to the outbreak?

Hint:

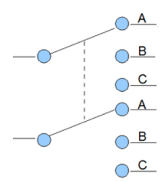
No hint this month.

10 points:

In a modern car window, you normally operate the windows by pressing a switch or a button located on the car door. Pushing the switch down opens the window by sliding it downwards, while pushing the switch up closes the window by sliding it upwards. A passenger's car window can usually be opened using two switches: one switch is attached to the passenger's door, while another switch is located by the driver. The window is actuated by a DC motor, which rotates in different directions depending on the polarity of the applied voltage.



Model A



Model B

(a) Model A above shows a potential model for the car window circuit. Depending on the position of the two switches, the motor will either stay off, move in one direction, or move in the opposite direction.

Explain why Model A *should not* be used as a base model for the car window circuit.

(b) Model B above shows a *double pole triple throw* (DPTT) "on-on-on" switch. Draw a car window circuit diagram that incorporates two DPTT switches and achieves the usual effect of operating the window independently from either switch.

Make sure that your solution will not create a dead short under any circumstances.

Hint:

No hint this month.

Computer Science

- Your program should be written in Java or Python-3.
- No GUI should be used in your program (e.g. `easygui` in Python).
- All the input and output should be done through files named as specified in the problem statement.
- Java programs should be submitted in a file with extension `.java`; Python-3 programs should be submitted in a file with extension `.py`. **No `.txt`, `.dat`, `.pdf`, `.doc`, `.docx`, etc. Programs submitted in the incorrect format will not receive any points!**

Adam has just returned from a long vacation far away, which involved several layovers. However, the airport baggage handling system misplaced his suitcase, placing it on a different flight! After several weeks of long calls with different airlines, Adam finally recovered his suitcase. However, he noticed that it took a very different route from him: the suitcase visited a variety of different airports throughout its journey, and each airport except the final destination left a tag on the suitcase indicating that it passed through there.

Out of sheer curiosity, Adam wants to determine the possible airport routes that his suitcase travelled around the world.

5 points:

For this problem, assume that each airport tag on Adam's suitcase has both the origin and destination airport code. The tags are not necessarily listed in order. Furthermore, you can assume that no airport is visited more than once, and that there is a unique path the suitcase took from the initial airport to the final airport.

Write a program that determines the path of airports that the suitcase took to get back to Adam.

Your program should read the input file `input.txt`, which has the following format:

- The first line contains a positive integer n denoting the number of tags on Adam's suitcase, as well as two space-separated airport codes representing the initial and final airports.
- The next n lines contain two space-separated 3-letter airport codes in the format `ABC XYZ`, representing the source and destination airports.

Your program should produce the file `output.txt`, which contains the path of airports that the suitcase took, represented as a space-separated list of airport codes.

IMPORTANT NOTE: There is always one less tag than the number of airports. The final airport does **not** leave a tag on the suitcase.

Sample Input 1:

```
4 BOS LAX
YUL YYZ
BOS YUL
YYZ JFK
JFK LAX
```

Sample Output 1:

```
BOS YUL YYZ JFK LAX
```

Sample Input 2:

```
6 EWR TPE
CDG KIX
FRA CDG
DTW FRA
KIX TPE
CLE DTW
EWR CLE
```

Sample Output 2:

```
EWR CLE DTW FRA CDG KIX TPE
```

Hint:

No hint this month.

10 points:

In spite of various standardization attempts, airports don't always follow the same luggage tag conventions. For this problem, some airports left both the source and destination airports on their tag (type A tag), some airports only left their own airport code (type B tag), and some airports left a blank tag without any information on it (type C tag).

Type A tags contain both the source and destination airports: For example, JFK may leave a type A tag "`JFK BOS`" indicating a flight from JFK to Boston. Type B tags contain only the source airport: For example, JFK's type B tag will always be "`JFK`". Type C tags do not identify the airport, and only indicate that the luggage visited some other airport.

This time, Adam wants to *count* the number of paths that his suitcase could have taken to get from the initial airport to the final airport.

As there may be more than one route that the suitcase took, you are given a file consisting of 37,500+ routes between 3,400+ airports across the world, which you may download below as a `.csv` file:

https://github.com/SigmaCode/POM-QQ/blob/main/CS/2024-2025/October/routes-10pt.csv

Each row of this `.csv` file (aside from the header row) represents a single route between two airports separated by a comma: the first is the source airport code, and the second is the destination airport code. Below are the first 6 lines of the file:

```
src,dest
SNN,PMI
TRF,LPL
NOS,TNR
IAH,TUS
BDJ,BDO
...
```

If you are reading from this file in your code, it should be referenced in your code as "`routes-10pt.csv`" and it should be located in the same directory as your `input.txt` and `output.txt` files.

You can assume that each airport visited left one tag on the suitcase, and that each airport is visited no more than once. Furthermore, the initial and final airports will always be different.

Your program should read the input file `input.txt`, which has the following format:

- The first line contains a positive integer n denoting the number of tags on Adam's suitcase, as well as two space-separated airport codes representing the initial and final airports.
- The next n lines will be one of the following three formats:
  - "`A ABC DEF`", representing a type A tag left by airport ABC, indicating a flight from airport ABC to airport DEF.
  - "`B ABC`", representing a type B tag left by airport ABC, indicating that the suitcase visited airport ABC.
  - "`C`", representing a type C tag left by some airport.

Your program should produce the file `output.txt`, consisting of a single line containing the total number of paths that the suitcase may have taken to get from the initial to the final airport.

IMPORTANT NOTE: There is always one less tag than the number of airports. The final airport does **not** leave a tag on the suitcase.

Sample Input 1:

```
5 BOS LAX
B YUL
B EWR
B YOW
A BOS YUL
B YTZ
```

Sample Output 1:

```
2
```

Sample Explanation 1:

The two possible flights are:

BOS → YUL → YTZ → YOW → EWR → LAX

BOS → YUL → YOW → YTZ → EWR → LAX

Note that among the airports given by the tags, YTZ only flies to and from YUL, YOW, and EWR.

Sample Input 2:

```
2 JFK FRA
C
B JFK
```

Sample Output 2:

```
85
```

Sample Explanation 2:

Since the initial and final airports are JFK and FRA, we know that the type C tag belongs to some airport that is a layover between JFK and FRA. There are precisely 85 possible one-stop layovers from JFK to FRA.

Hint:

View the airports as a graph, with edges given by the routes. Consider doing a breadth first search (BFS) or a depth first search (DFS) on this graph to count the total number of paths, restricted by the tags.