SigmaCamp's *Problem of the Month* Contest

SEPTEMBER 2024

Starting from September 2024, we are requiring all submissions to be `.pdf` files (except for CS, which requires `.py` or `.java` files). If you are using Word, you may export to PDF by clicking File > Export > Create PDF/XPS Document.

Mathematics

For all mathematics problems, please provide full justification. **Do not include any code** in your submission – all code submissions will be awarded no points.

5 points:

Show that it is possible to find 2000 *distinct* natural numbers $n_1, n_2, \ldots, n_{2000}$ such that

$$\frac{1}{n_1} + \frac{1}{n_2} + \ldots + \frac{1}{n_{2000}} = 1.$$

Show that the same thing is possible for 2025 natural numbers.

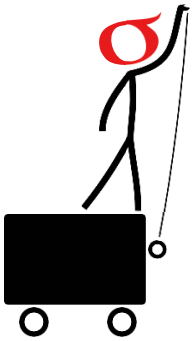
10 points:

Find all integer solutions of

$$\frac{x}{\sqrt{2x+3y-5z+6}} + \frac{y}{\sqrt{6x-y+z-8}} + \frac{z}{\sqrt{4z-8x-2y+5}} = 23.$$

Show your work in detail and prove that you have found *all* the solutions.

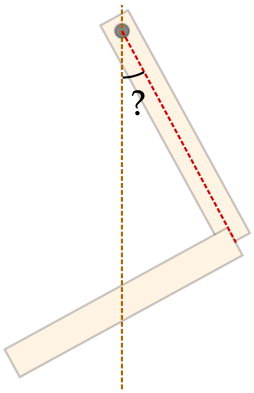
Physics



5 points:

During a workshop on double pendulums[1], everyone attaches one end of a ruler to a rod so that the ruler can rotate freely and a second ruler to the first ruler. The second ruler is also supposed to rotate freely, but by mistake Andrey glues it to the first ruler at a 90° angle. What will the angle be between the first ruler and the vertical when the rulers are allowed to hang freely?

Assume that both rulers are of the same length and mass, the rod is a negligible length from the start of the first ruler, and the mass of the glue is negligible. Note that for full points, justification must be provided in addition to the answer.

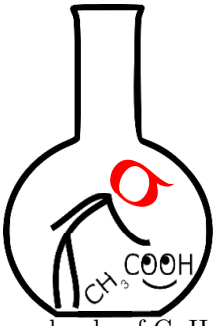


10 points:

A pitcher throws a baseball forward from a moving train while leaning out the window (very unsafe!). A physicist sits in the locomotive next to the spectacle and notices that the ball travels at a speed V relative to the train. Without much thought, she computes the kinetic energy of the ball and assumes that the pitcher just exerted the amount of energy equal to $m\,V^2/2$, where m is the mass of the baseball. At the same time, a second physicist happens to be on the ground and sees that the baseball is traveling in the same direction as the train with a speed of $2\,V$. He also, without much hesitation, computes the kinetic energy of the ball and assumes that the pitcher spent an amount of energy equal to $2\,m\,V^2$. Thus, we arrive at a paradox. Assuming that the mass of the train with the pitcher, the first physicist, and everyone else on it is M, compute the exact amount of energy that the pitcher spent throwing the ball in two different inertial reference frames: one moving with speed of V in the same direction as the train and another one at rest. Are these amounts different or the same?

[1] Sigma Camp actually did have a workshop on double pendulums as part of JIC. Consider applying to teach your own workshop next year!

Chemistry



5 points:

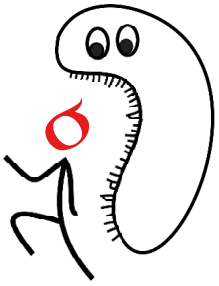
When one organic compound can be obtained from another one by addition of one or several CH_2 fragments, these compounds are *holomogous*. Thus, ethane (CH_3CH_3) and propane ($CH_3CH_2CH_3$) are homologous.[2]

Two compounds A and B are homologous that differ by a single CH_2, and they are composed only of C, H, and probably O. The amount of oxygen required for the complete combustion of compound B is twice that for compound A. Calculate the formulas for A and B.

10 points:

By mixing the acid A with the base B, followed by evaporation and crystallization of the mixture, Alice prepared a salt C1. She asked Bob to reproduce her results, but she didn't explain all details of her experiment. Bob took the same acid and the same base, mixed them, but his experiment yielded another salt, C2. Subsequent analysis demonstrated that C1 and C2 were pure compounds (not mixtures), and their chemical composition was somewhat different. To verify Alice and Bob's results, Cynthia took the same acid A and the same base B, mixed them according to her own procedure, and obtained a new compound: the salt C3. All three compounds, C1, C2, and C3, were pure compounds with different physical properties. All of them were salts, they were soluble in water, and the C1 solution was moderately acidic, C2 solution was slightly basic, and C3 solution was strongly basic. Can you explain how that can be possible? Provide at least one example of C1, C2, and C3.

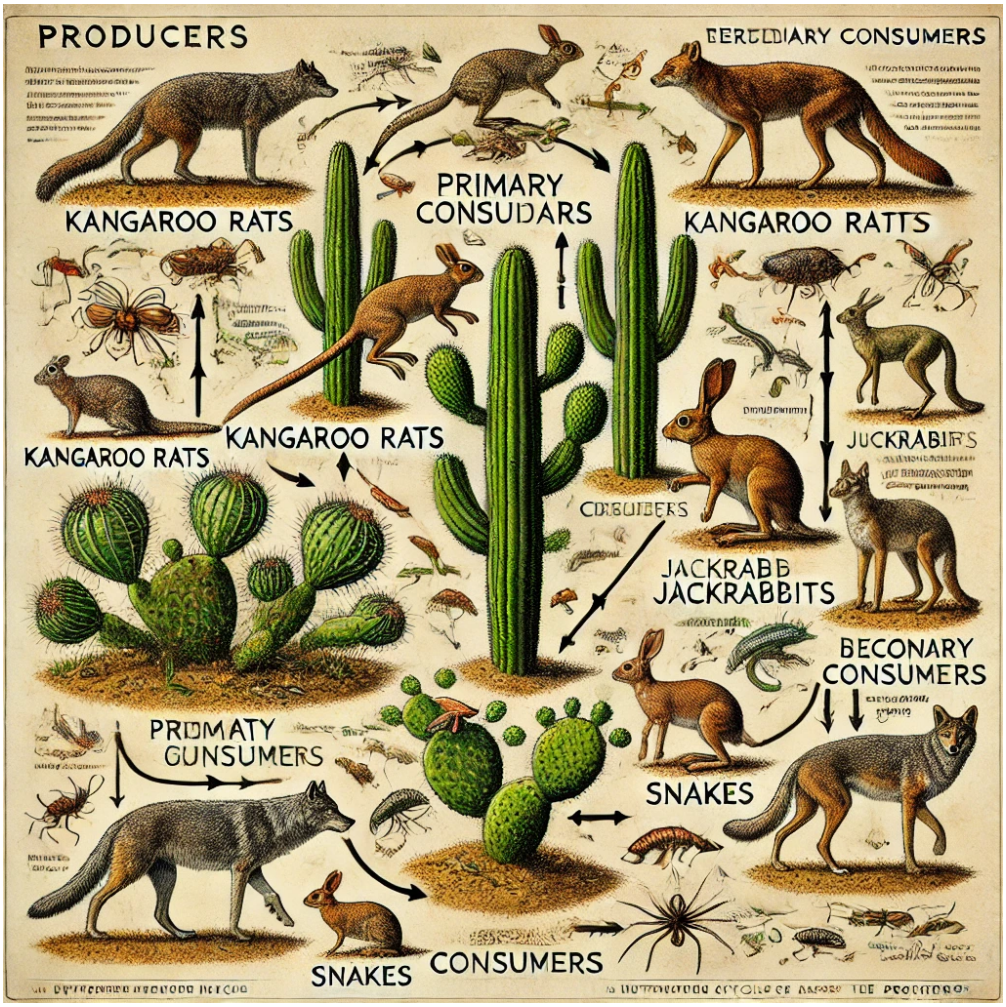
[2] Usually, addition of CH_2 cannot be done in one step. What is important, this transformation is theoretically possible.



Biology

5 points:

While preparing a biology homework assignment, a student asked ChatGPT to draw a diagram showing the food web of the ecosystem of some desert populated by cacti, kangaroo rats, coyotes, grasses, jackrabbits, snakes, bacteria, lizards, birds of prey, shrubs, and insects. The AI created the picture shown below.

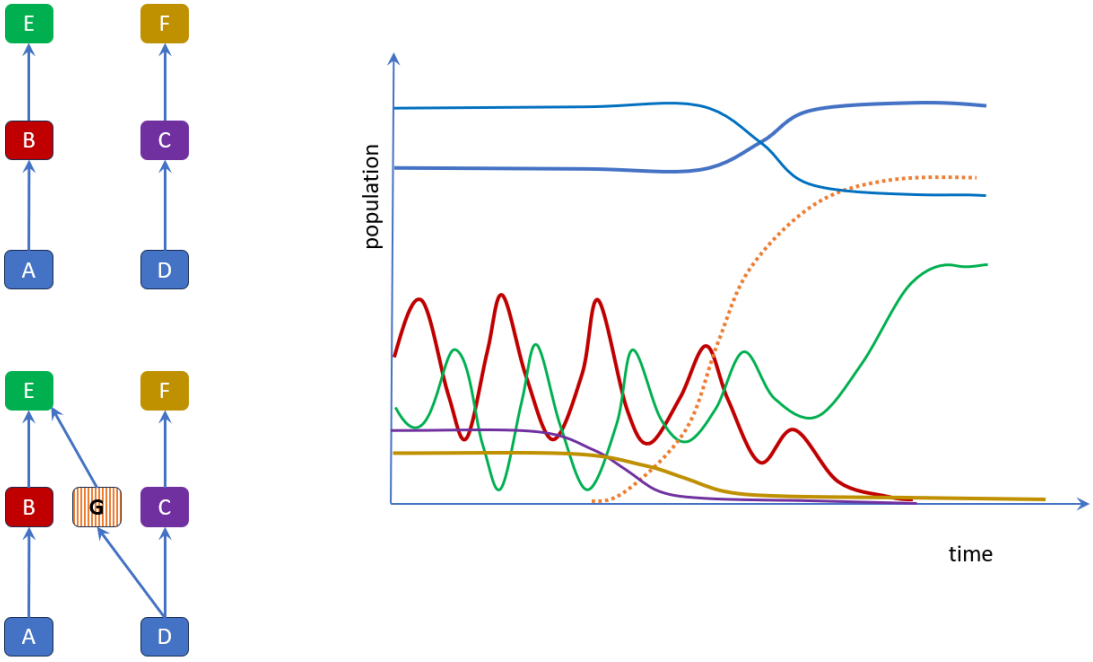


There are a few errors in the drawing, but some segments of the web seem reasonable. Please find as many errors as you can and explain what is correct in the diagram. Try to focus on general and fundamental errors, not just on minor typos. Explain your answer.

Draw a more realistic food web that includes these species.

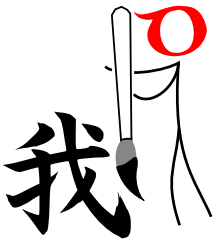
10 points:

Some stable ecosystem had the food web shown in the top left panel of the figure below. After an invasive species was introduced, the new food web was as shown in the bottom left panel.



The introduction of the invasive species G led to a significant change of the population dynamics (right panel). Explain the observed changes and propose possible candidates for the species A - G.

Linguistics & Applied Sciences



5 points:

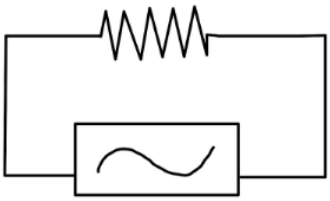


Figure 1

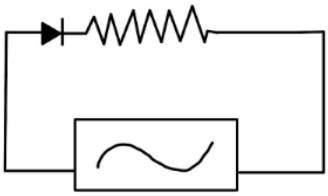


Figure 2

Consider an alternating voltage source which produces a sinusoidal voltage connected to a resistor, as shown on the circuit in **Figure 1**:

1. Plot the current through the resistor as a function of time.

2. A diode is a device that only allows current to flow in one direction (indicated by the arrow). Consider adding a single diode to the circuit as shown in **Figure 2** (left). Plot the current through the resistor as a function of time.

3. Now, imagine that you are given some electric wires, a battery, a *direct current* (DC) motor and some diodes. Unfortunately, you can't tell which side of the battery is positive and which side is negative. First, design a circuit that uses diodes to make sure that the DC motor will always rotate in the same direction, regardless of the battery's polarity. Then, plot two diagrams: (a) a circuit diagram that indicates how current would flow through the circuit for both battery positions, and (b) a graph that plots current through the motor as a function of time.

10 points:

The list of English sentences on the left was translated into an unknown Oceanian language and scrambled to produce the list on the right. The letter 'ə' (which looks like an upside-down <e>, called 'schwa') represents a neutral sounding, mid-central vowel such as 'o' in 'awesome' or in 'lemon'.

Match the English sentences to the Oceanian sentences. Make sure to **show your work** by explaining the meaning of words and grammatical features you find. Feel free to explain parts of a word in your explanation if you find it necessary. The more you show your work or reasoning, the more partial credit you can get for this problem. Even if you think you matched each sentence - explain how you did it!

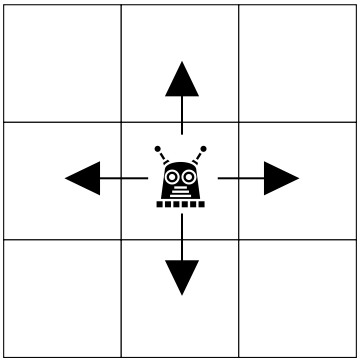
1. I shall kill deer with this magic I saw
2. after much thought, they won't kill the witch
3. a witch who's doing magic won't eat deer
4. they came after seeing you
5. what are you doing?
6. they shall come to see you
7. a witch will get an axe after seeing a deer
8. I am thinking
9. get this axe, then come
10. he ate the deer after doing good magic

A. koyb kuj gəsp, kəmən maningəngabo
B. kuj tep gi, kəmən ningəpak
C. etəp gəspan?
D. yad ak kuj nəngi, kəmən pakəng gəyin
E. gos nəng tep gi, koyb mapakəngabay
F. koyb kəmən nəngi, tuw dəngabo
G. nəp nəngi, opay
H. ak tuw di, owan
I. nəp nəngəng ospay
J. yad gos nəngəspin

Computer Science

- Your program should be written in Java or Python-3.
- No GUI should be used in your program (e.g. `easygui` in Python).
- All the input and output should be done through files named as specified in the problem statement.
- Java programs should be submitted in a file with extension `.java`; Python-3 programs should be submitted in a file with extension `.py`. **No `.txt`, `.dat`, `.pdf`, etc. Programs submitted in the incorrect format will not receive any points!**

Shurik is running a robotics semilab, where he is designing a moving robot with pathfinding capabilities. The robot moves on an infinite board, where, on each square of the board, it can decide to move in four directions: left, right, up, or down.



5 points:

To test the robot's basic movement capabilities, Shurik wants to ensure that the robot can get from point A to point B. Shurik gives the robot its starting position and the desired position to which it should move, and wants the robot to come up with a sequence of moves, and verify that its moves will get itself to the desired position before making them.

Write a program that receives the robot's starting position, desired position, and sequence of moves, and determines whether the robot will reach its desired position after the move sequence.

Your program should read the input file `input.txt`, which contains four lines:

- The first line contains a single nonnegative integer n, denoting the number of moves the robot makes.
- The second line contains two space separated integers denoting the x- and y-coordinates of the robot's **starting** position on the infinite board.
- The third line contains two space separated integers denoting the x- and y-coordinates of the **desired** position on the infinite board.
- The fourth line contains a list of n space separated characters "`L`", "`R`", "`U`", or "`D`", indicating that the robot moves left, right, up, or down, respectively.

Your program should produce the file `output.txt`, which contains either "`YES`" if the robot ended at its desired position after following the sequence of moves from its starting position, or "`NO`" otherwise.

Sample Input 1:

```
6
0 -1
5 -2
R R R R R D
```

Sample Output 1:

```
YES
```

Sample Input 2:

```
7
-1 5
2 3
R D R R D L U
```

Sample Output 2:

```
NO
```

Sample Explanation 2:

Although the robot reached the desired position $(2, 3)$, it did not end at that position.

10 points:

Next, Shurik programs the robot to solve a maze by finding the shortest path from the start to the finish, and record its movements along the way. The maze consists of obstructions that are placed on squares of the board. The robot will always avoid any square that has an obstruction on it. One day, the campers designed a maze and the robot navigated it. However, the next day, a camper discovers a bug in Shurik's code: the robot may not have taken the shortest route in the maze! Unfortunately, the maze was disassembled and no one remembers what it looked like.

Write a program that takes as input the movements of the robot, and determines whether there exists at least one maze such that the path the robot took in that maze was the shortest one.

Your program should read the input file `input.txt`, which contains two lines:

- The first line contains a single nonnegative integer n, denoting the number of moves the robot made.
- The second line contains a list of n space separated characters "`L`", "`R`", "`U`", or "`D`", indicating that the robot went left, right, up, or down, respectively.

Your program should produce the file `output.txt`, which contains either "`YES`" if there exists at least one maze arrangement such that the path the robot took was the shortest one to get from the robot's starting position to its ending position, or "`NO`" otherwise.

Sample Input 1:

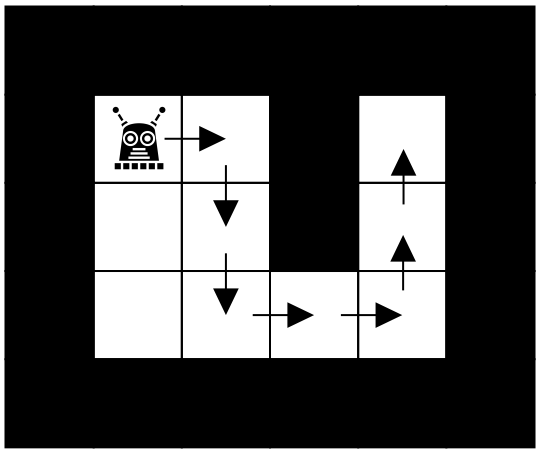
```
7
R D D R R U U
```

Sample Output 1:

```
YES
```

Sample Explanation 1:

Below is one possible maze that the robot could have navigated where its path is a shortest one (black squares indicate obstructions):



Sample Input 2:

```
14
R D L D D R R D R R U U U L
```

Sample Output 2:

```
NO
```