SigmaCamp's *Problem of the Month* Contest

SEPTEMBER 2024

Starting from September 2024, we are requiring all submissions to be `.pdf` files (except for CS, which requires `.py` or `.java` files). If you are using Word, you may export to PDF by clicking File > Export > Create PDF/XPS Document.

Mathematics

For all mathematics problems, please provide full justification. **Do not include any code** in your submission – all code submissions will be awarded no points.

5 points:

Show that it is possible to find 2000 *distinct* natural numbers $n_1, n_2, \ldots, n_{2000}$ such that

$$\frac{1}{n_1} + \frac{1}{n_2} + \ldots + \frac{1}{n_{2000}} = 1.$$

Show that the same thing is possible for 2025 natural numbers.

Hint:

Proving the statement for all integers $m (m > 3)$ might be easier than just showing the specific cases of 2000 and 2025. Also note that $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$.

Solution:

For any integer m greater than 2,

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^{m-2}} + \frac{1}{3 \cdot 2^{m-3}} + \frac{1}{3 \cdot 2^{m-2}} = 1.$$

(If you don't see why this is true, try plugging in a small number like $m = 4$.) Since the denominators of these fractions are all distinct, it is possible to choose distinct integers

$$\frac{1}{n_1} + \frac{1}{n_2} + \ldots + \frac{1}{n_m} = 1$$

for any integer $m > 2$, including 2000 and 2025.

10 points:

Find all integer solutions of

$$\frac{x}{\sqrt{2x+3y-5z+6}}+\frac{y}{\sqrt{6x-y+z-8}}+\frac{z}{\sqrt{4z-8x-2y+5}}=23.$$

Show your work in detail and prove that you have found *all* the solutions.

Hint:

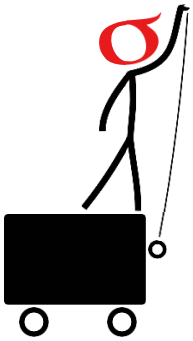
Consider the domain.

Solution:

Answer: $(2, 12, 9)$

Consider the 3 expressions in the denominators under the square roots, $2x+3y-5z+6$, $6x-y+z-8$, and $4z-8x-2y+5$. To satisfy the domain requirements of the fractions (not dividing by 0) and square roots (defined for non-negative numbers only) these have to be positive. Also, since we are looking for integer solutions, they are integers. Also notice, that the sum of the 3 expressions is 3. The only option for 3 positive integers to have a sum of 3 is if each of them equals 1. Thus, the original equation becomes $x+y+z=23$, and we also know that $2x+3y-5z+6=1$ and $6x-y+z-8=1$. The only solution of this 3×3 system is $(2, 12, 9)$.

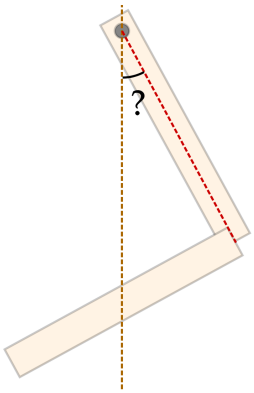
Physics



5 points:

During a workshop on double pendulums[1], everyone attaches one end of a ruler to a rod so that the ruler can rotate freely and a second ruler to the first ruler. The second ruler is also supposed to rotate freely, but by mistake Andrey glues it to the first ruler at a 90° angle. What will the angle be between the first ruler and the vertical when the rulers are allowed to hang freely?

Assume that both rulers are of the same length and mass, the rod is a negligible length from the start of the first ruler, and the mass of the glue is negligible. Note that for full points, justification must be provided in addition to the answer.



Hint:

Find the center of mass.

Solution:

The key realization is that the two rulers will hang so that their center of mass is on the vertical. The center of mass of each ruler is at its center. Since the mass of the rulers is the same, the total center of mass is halfway along the line connecting the centers of the rulers. There is a right triangle formed between the rod, the center of mass, and the projection of the center of mass onto the first ruler. The length of the legs are $L/4$ and $3L/4$, where L is the length of the rulers. Thus, the unknown angle is $\tan^{-1}(\frac{L/4}{3L/4}) = \tan^{-1}(1/3) \approx 18.4°$.

Answer: $\tan^{-1}(1/3) \approx 18.4°$

10 points:

A pitcher throws a baseball forward from a moving train while leaning out the window (very unsafe!). A physicist sits in the locomotive next to the spectacle and notices that the ball travels at a speed V relative to the train. Without much thought, she computes the kinetic energy of the ball and assumes that the pitcher just exerted the amount of energy equal to $m\,V^2/2$, where m is the mass of the baseball. At the same time, a second physicist happens to be on the ground and sees that the baseball is traveling in the same direction as the train with a speed of $2\,V$. He also, without much hesitation, computes the kinetic

[1] Sigma Camp actually did have a workshop on double pendulums as part of JIC. Consider applying to teach your own workshop next year!

energy of the ball and assumes that the pitcher spent an amount of energy equal to $2\,m\,V^2$. Thus, we arrive at a paradox. Assuming that the mass of the train with the pitcher, the first physicist, and everyone else on it is M, compute the exact amount of energy that the pitcher spent throwing the ball in two different inertial reference frames: one moving with speed of V in the same direction as the train and another one at rest. Are these amounts different or the same?

Hint:

No hint this month.

Solution:

First, we consider the inertial reference frame, which moves at the same speed as the train before the throw. The total initial momentum before the throw is

$$P_i = (M+m)\times 0 = 0.$$

After the throw, the ball and the train are moving separately with different speeds and different momenta[2]:

$$P_{\rm ball} = m\,V, \quad P_{\rm train} = M\,V_{\rm train}.$$

Assuming that the friction between the wheels of the train and the rails is negligible, we can apply the conservation of the total momentum:

$$P_{\rm ball} + P_{\rm train} = P_i = 0.$$

Thus, the momentum of the train after the throw is

$$P_{\rm train} = -m\,V.$$

The change in the total kinetic energy of the system in this reference frame is due to the pitcher's efforts and is equal to

$$\Delta E_1 = \frac{P_{\rm ball}^2}{2\,m} + \frac{P_{\rm train}^2}{2\,M} - \frac{P_i^2}{2\,(M+m)} = \frac{m\,V^2}{2} + \frac{m^2\,V^2}{2\,M}.$$

Since $M >> m$, we can approximately write

$$\Delta E_1 \approx \frac{m\,V^2}{2}.$$

The second reference frame is at rest, and we have the following initial momentum before the throw:

$$P_i' = (M+m)\,V.$$

After the throw, we have

$$P_{\rm ball}' = 2\,m\,V, \quad P_{\rm train}' = M\,V_{\rm train}'.$$

The conservation of momentum in this reference frame reads

$$P_{\rm ball}' + P_{\rm train}' = P_i',$$

which gives

$$P_{\rm train}' = P_i' - P_{\rm ball}' = (M-m)\,V.$$

The change in the total kinetic energy of the system in the second reference frame is

$$\Delta E_2 = \frac{(P_{\rm ball}')^2}{2\,m} + \frac{(P_{\rm train}')^2}{2\,M} - \frac{(P_i')^2}{2\,(M+m)}.$$

[2] A careful reader might observe a small mismatch between what the first physicist will see as the speed of the baseball and V. In particular, the first physicist will actually observe that the baseball is moving with the speed $(V - V_{\rm train})$ relative to the train. However, as we will see below, this mismatch is proportional to m/M and thus is negligibly small.

Computing each contribution separately, we get

$$\frac{(P'_{\text{ball}})^2}{2\,m} = 2\,m\,V^2, \quad \frac{(P'_{\text{train}})^2}{2\,M} = \frac{(M-m)^2\,V^2}{2\,M}, \quad \frac{(P'_i)^2}{2\,(M+m)} = \frac{(M+m)\,V^2}{2}.$$

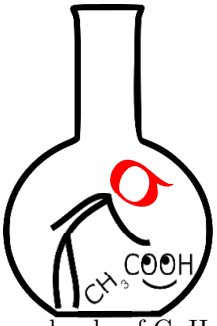
The first contribution coincides with the conclusion of the second physicist, but there is more! Carefully bringing all the terms back together, we derive

$$\Delta E_2 = 2\,m\,V^2 + \frac{M\,V^2}{2} - m\,V^2 + \frac{m^2\,V^2}{2\,M} - \frac{(M+m)\,V^2}{2} = \frac{m\,V^2}{2} + \frac{m^2\,V^2}{2\,M},$$

which resolves the paradox:

$$\Delta E_1 = \Delta E_2 \approx \frac{m\,V^2}{2}.$$

Chemistry



5 points:

When one organic compound can be obtained from another one by addition of one or several CH_2 fragments, these compounds are *holomogous*. Thus, ethane (CH_3CH_3) and propane ($CH_3CH_2CH_3$) are homologous.[3]

Two compounds A and B are homologous that differ by a single CH_2, and they are composed only of C, H, and probably O. The amount of oxygen required for the complete combustion of compound B is twice that for compound A. Calculate the formulas for A and B.

Hint:

How much oxygen is needed for the combustion of CH_2?

Solution:

Answer:
The first answer that comes to mind is CH_2 and C_2H_4. However, that answer is incorrect for two reasons.
First, CH_2, called "carbene", has a very short lifetime, so it cannot be considered a chemical compound.
Second, in organic chemistry, two compounds are considered homologous when two conditions are met: they differ by one or several CH_2 *and* they belong to the same class of compounds. While CH_2 and C_2H_4 meet the first condition, the second condition is not met: CH_2 is an unstable particle, but C_2H_4 is a quite stable chemical compound, which is the *first* member of the class of compounds called "unsaturated hydrocarbons", or "alkenes".
Therefore, the correct answer is different. To find it, let's calculate the amount of oxygen needed for combustion of CH_2:

$$2\,CH_2 + 3\,O_2 \longrightarrow 2\,CO_2 + 2\,H_2O.$$

From the above equation, the compound **A** is a compound that requires 1.5 molecules of oxygen for combustion. This compound is methanol:

$$2\,CH_3OH + 3\,O_2 \longrightarrow 2\,CO_2 + 4\,H_2O.$$

Obviously, the next member of this homology series, ethanol, is the compound **B**:

$$2\,CH_3CH_2OH + 6\,O_2 \longrightarrow 4\,CO_2 + 6\,H_2O,$$

or

$$CH_3CH_2OH + 3\,O_2 \longrightarrow 2\,CO_2 + 3\,H_2O.$$

10 points:

By mixing the acid A with the base B, followed by evaporation and crystallization of the mixture, Alice prepared a salt C1. She asked Bob to reproduce her results, but she didn't explain all details of her experiment. Bob took the same acid and the same base, mixed them, but his experiment yielded another salt, C2. Subsequent analysis demonstrated that C1 and C2 were pure compounds (not mixtures), and their chemical composition was somewhat different. To verify Alice and Bob's results, Cynthia took the same acid A and the same base B, mixed them according to her own procedure, and obtained a new compound: the salt C3. All three compounds, C1, C2, and C3, were pure compounds with different physical properties. All

[3]Usually, addition of CH_2 cannot be done in one step. What is important, this transformation is theoretically possible.

of them were salts, they were soluble in water, and the C1 solution was moderately acidic, C2 solution was slightly basic, and C3 solution was strongly basic. Can you explain how that can be possible? Provide at least one example of C1, C2, and C3.

Hint:

Some acids, such as H_2SO_4, may have more than one active hydrogen.

Solution:

Answer:
Some acids, for example, sulphuric acid, have more than one active hydrogen. When one mole of H_2SO_4 is mixed with one mole of NaOH, the reaction is as follows:

$$H_2SO_4 + NaOH \longrightarrow NaHSO_4 + H_2O.$$

The product of this reaction, $NaHSO_4$, is called "an acid salt": it is simultaneously a salt and an acid: in water, it dissociates on Na^+ and $HSO_4{}^-$, and the latter is acidic, because it can dissociate further:

$$HSO_4{}^- \longrightarrow H^+ + SO_4{}^{2+}$$

However, $HSO_4{}^-$ dissociates less easily than H_2SO_4, so the solution of $NaHSO_4$ is less acidic than a solution of sulphuric acid. Therefore, when we mix solutions of NaOH and H_2SO_4, two outcomes are possible. First, if we take one mole of NaOH per one mole of H_2SO_4, we get sodium bisulphate ($NaHSO_4$), which is moderately acidic, but if we mix *two* moles of NaOH per one mole of H_2SO_4, we get sodium sulphate (Na_2SO_4), which is neutral. In our case, the situation is even more interesting. It seems that the acid **A** has not two, but three active hydrogens. Most likely, it is phosphoric acid (H_3PO_4). The three reactions Alice, Bob and Cynthia observed were as follows. First,

$$H_3PO_4 + NaOH \longrightarrow NaH_2PO_4 + H_2O$$

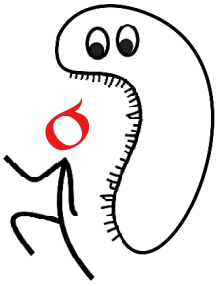
The compound obtained by Alice is NaH_2PO_4, or sodium dihydrophosphate. This salt is slightly acidic, because it has two active hydrogens. Second,

$$H_3PO_4 + 2\,NaOH \longrightarrow Na_2HPO_4 + 2\,H_2O$$

The compound obtained by Bob is Na_2HPO_4, or sodium hydrophosphate. This salt is slightly nearly neutral. To understand why it is not acidic, it is necessary to keep in mind that phosphoric acid is not a strong acid, and NaOH is a strong base. When a weak acid and a strong base form a salt, that salt is not neutral, but basic (an example is laundry soda, Na_2CO_3, which is a strong base). Third,

$$H_3PO_4 + 3\,NaOH \longrightarrow Na_3PO_4 + 3\,H_2O$$

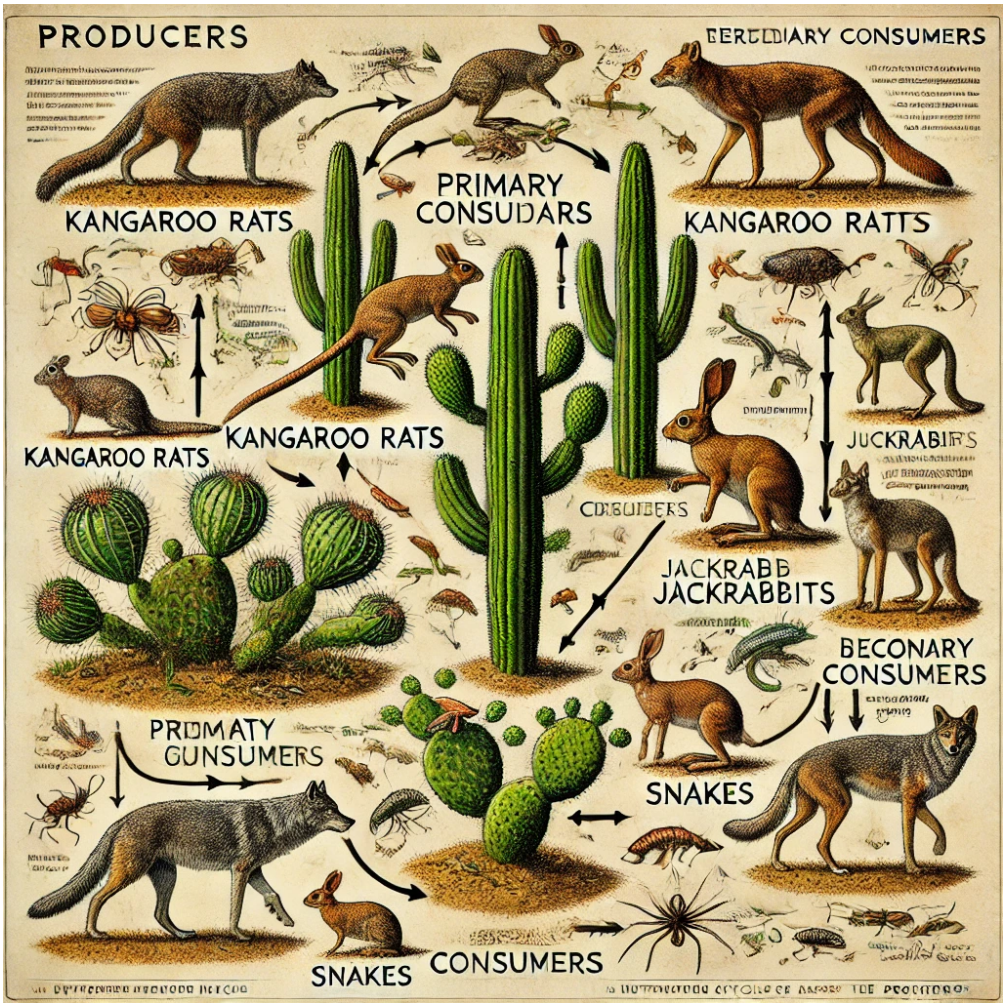
The compound obtained by Cynthia is Na_3PO_4, or trisodium phosphate. Like Na_2CO_3, this salt is strongly basic.
Other answers may be proposed, but this is the most obvious.



Biology

5 points:

While preparing a biology homework assignment, a student asked ChatGPT to draw a diagram showing the food web of the ecosystem of some desert populated by cacti, kangaroo rats, coyotes, grasses, jackrabbits, snakes, bacteria, lizards, birds of prey, shrubs, and insects. The AI created the picture shown below.



There are a few errors in the drawing, but some segments of the web seem reasonable. Please find as many errors as you can and explain what is correct in the diagram. Try to focus on general and fundamental errors, not just on minor typos. Explain your answer.

Draw a more realistic food web that includes these species.

Hint:

Keep in mind that the species in a food web are grouped in several "layers", and that material transfer is always unidirectional.

Solution:

Answer:
There are many errors in the ChatGPT-generated diagram. Some of them include:

1. Spelling/Image not matching description
2. Kangaroo rats should only be shown as primary consumers
3. Coyotes should be tertiary consumers, not primary or secondary
4. Snakes should be secondary/tertiary consumers

The fundamental errors in the diagram are that in many instances, the flow of energy is directed in the wrong direction or simply does not make sense. Additionally, decomposers are not shown in the diagram and are not given a clear role.

The fundamental errors in the diagram are that in many instances, the flow of energy is directed in the wrong direction or simply does not make sense. Additionally, decomposers are not shown in the diagram and are not given a clear role.

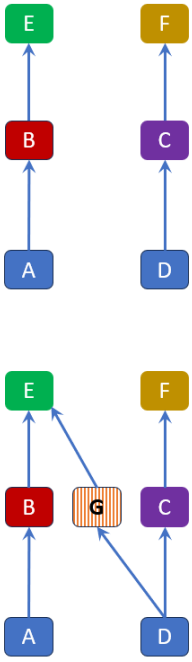
Some correct information includes:

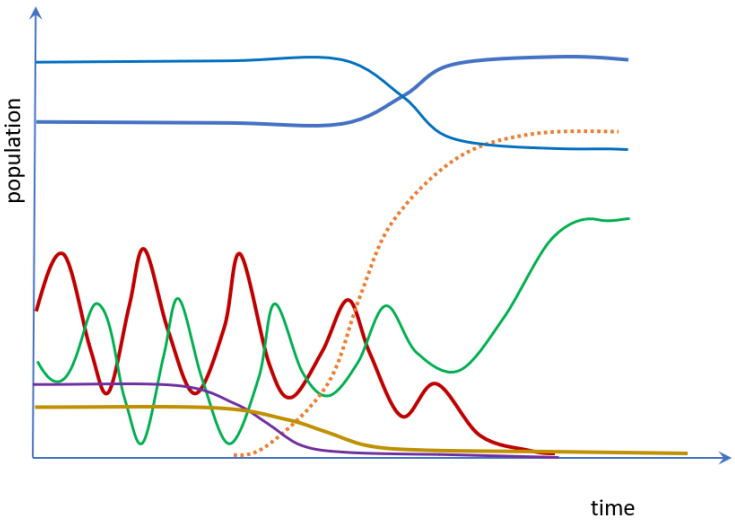
1. Kangaroo rat/jackrabbit is a primary consumer
2. Some of the arrows are correct (for example, the arrow pointing from the jackrabbit to the coyote)

Your revised diagram should clearly illustrate the flow of energy, starting from producers and moving through primary, secondary, and tertiary consumers, ultimately ending with decomposers. The producers are cacti, shrubs, and grasses. The primary consumers are kangaroo rats, jackrabbits, and insects. The secondary consumers are lizards and snakes. The tertiary consumers are coyotes and birds of prey.

10 points:

Some stable ecosystem had the food web shown in the top left panel of the figure below. After an invasive species was introduced, the new food web was as shown in the bottom left panel.





The introduction of the invasive species G led to a significant change of the population dynamics (right panel). Explain the observed changes and propose possible candidates for the species A - G.

Hint:

The population dynamics of two species demonstrated an oscillating behavior, which disappears after the invasive species is introduced. Can you explain why?

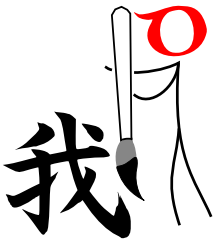
Solution:

Answer: The most interesting event here is the complete extinction of species **B**. It is clear from the food web diagram that **B** and **G** are not competing for food, so the reason that **G**'s population growth negatively impacts **B** is not obvious. Species **B** and **E** represent a classic example of oscillatory population dynamics in a predator-prey system. These types of systems are characterized by negative feedback, where an increase in the population of **B** leads to an increase in the population of **E**, thereby negatively impacting the population of **B**, which begins to decline. When **B** declines, this leads to a decline in **E**, so the population of **B** begins to grow, and the cycle repeats.

This oscillatory behavior is stable, and the main reason for this is the fact that **B** is the only or main food source for **E**, and the population of the latter depends only on the former. However, if **E** gets a new food source (in our case, it is **G**), even a significant reduction in the population of **B** is not able to undermine the food source for **E**: the population of **E** continues to grow even when **B** declines. As a result, a new equilibrium is established, where **G** becomes the main food source for **E**. Since **E** becomes more numerous, its population no longer depends on the population of **B**. Moreover, **G** reproduces faster than **B**, and/or **E** is able to catch **B** more easily than **G**, so **B** disappears completely. As for **G**, its growth rate and life cycle differ from those of **B**, so the populations of **G** and **E** do not exhibit oscillatory behavior.

The roles of the other species in the ecosystem are much easier to explain: **A** and **D** are primary producers (e.g. plants), **B**, **C**, and **G** are primary consumers (e.g. ungulates or rodents), and **E** and **F** are predators. The almost complete disappearance of **C** after **G**'s invasion is explained by competition with **G**, the disappearance of **F** is caused by the disappearance of **C**, the growth of the population of species **A** is caused by the disappearance of **B** (then **A** stabilized, apparently because it had an additional consumer not included in the scheme), and the decline of species **D** is caused by the increase in the number of **G**. But these changes are not so interesting, because they are obvious.

Linguistics & Applied Sciences



5 points:

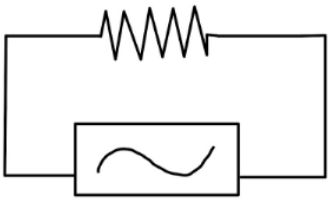


Figure 1

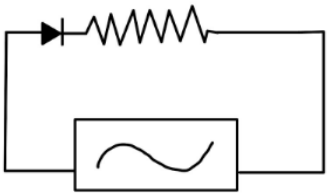


Figure 2

Consider an alternating voltage source which produces a sinusoidal voltage connected to a resistor, as shown on the circuit in **Figure 1**:

1. Plot the current through the resistor as a function of time.

2. A diode is a device that only allows current to flow in one direction (indicated by the arrow). Consider adding a single diode to the circuit as shown in **Figure 2** (left). Plot the current through the resistor as a function of time.

3. Now, imagine that you are given some electric wires, a battery, a *direct current* (DC) motor and some diodes. Unfortunately, you can't tell which side of the battery is positive and which side is negative. First, design a circuit that uses diodes to make sure that the DC motor will always rotate in the same direction, regardless of the battery's polarity. Then, plot two diagrams: (a) a circuit diagram that indicates how current would flow through the circuit for both battery positions, and (b) a graph that plots current through the motor as a function of time.

Hint:

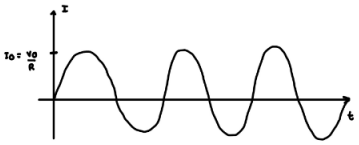
Part 1: Recall that according to Ohm's law, $I = V/R$.

Part 2: Imagine how a diode affects the current coming from an alternating-voltage source.

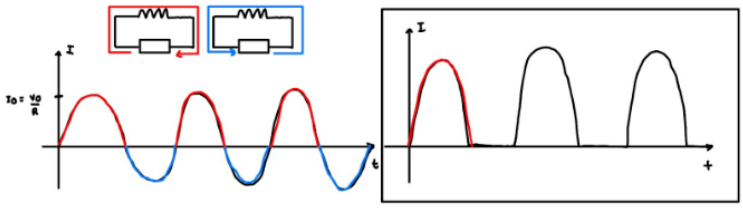
Part 3: No hints.

Solution:

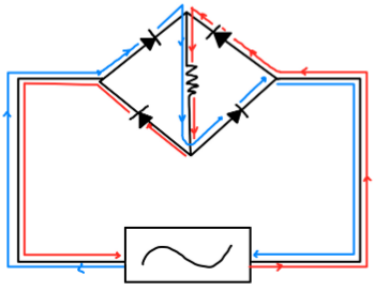
Part 1: In the problem statement, it is given that the voltage source produces a sinusoidal voltage. Therefore, the voltage as a function of time looks like a sine wave. **Ohm's Law states that I = V/R**. Thus, the graph of the current through the resistor as a function of time is also represented by a sine wave:



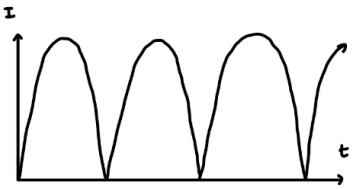
Part 2: Let's look at the original graph we created in Part 1. Circuits with an alternating voltage source have currents that are negative half of the time and positive the other half. Unlike our original circuit, the circuit in Part 2 has a current that is constrained to only flow in one direction instead of two, due to the diode that we added. Depending on the convention for direction (left or right as positive) that the student considers, there are two solutions for Part 2. We present one below:



Part 3: Here, students have to get creative about how to use the diodes to "steer" the current, such that it always goes in the same direction through the resistor. This is an example of a basic **bridge rectifier** which allows the motor (represented here as a resistor) to rotate in the same direction because current will always flow the same direction. The arrow diagram should look like something below:



A plot showing **constant current** is important for the student to show their ability to distinguish AC from DC. If the student plots a *pulsating DC current* like the one below, it is *more likely* (but not 100% certain) that the student **used ChatGPT to construct their rectifier** — be careful.



10 points:

The list of English sentences on the left was translated into an unknown Oceanian language and scrambled to produce the list on the right. The letter 'ə' (which looks like an upside-down <e>, called 'schwa') represents a neutral sounding, mid-central vowel such as 'o' in 'awesome' or in 'lemon'.

Match the English sentences to the Oceanian sentences. Make sure to **show your work** by explaining the meaning of words and grammatical features you find. Feel free to explain parts of a word in your explanation if you find it necessary. The more you show your work or reasoning, the more partial credit you can get for this problem. Even if you think you matched each sentence - explain how you did it!

1. I shall kill deer with this magic I saw
2. after much thought, they won't kill the witch
3. a witch who's doing magic won't eat deer
4. they came after seeing you
5. what are you doing?
6. they shall come to see you
7. a witch will get an axe after seeing a deer
8. I am thinking
9. get this axe, then come
10. he ate the deer after doing good magic

A. koyb kuj gəsp, kəmən maningəngabo
B. kuj tep gi, kəmən ningəpak
C. etəp gəspan?
D. yad ak kuj nəngi, kəmən pakəng gəyin
E. gos nəng tep gi, koyb mapakəngabay
F. koyb kəmən nəngi, tuw dəngabo
G. nəp nəngi, opay
H. ak tuw di, owan
I. nəp nəngəng ospay
J. yad gos nəngəspin

Hint:

See if you can track how often certain bits and pieces repeat in each list of sentences. For English, count the frequency of certain nouns, verbs, and grammatical features (such as: prepositions, prefixes, uses of past or future tense, etc.). For your unknown language, see if any words or prefixes, suffixes — even if you don't know what they mean. Chances are that if something appears a certain number of times in English, it may appear the same number of times in the unknown language.

However, this problem may not always benefit from this comparative method for words *that do not translate directly* or can have several meanings in one of the languages.

Solution:

I am think ing	*yad* gos nəngə sp **in**
I shall kill deer with *this* magic *I* saw	*yad ak* kuj nəng i, kəmən pak əng gə (y)**in**
what are **you** do ing ?	*etəp* gə sp **an**?
get *this* axe, then come	*ak* tuw d i, o (w)**an**
he a te the deer after do ing *good* magic	kuj *tep* g i, kəmən ningə p **ak**
a witch who's do ing magic wo n't eat *deer*	koyb kuj gə sp, *kəmən* ma ningə ngab **o**
a witch will get an axe after see ing *a deer*	koyb *kəmən* nəng i, tuw də ngab **o**
they c a me after see ing *you*	*nəp* nəng i, o p **ay**
they shall (be) com (ing) to see *you*	*nəp* nəng əng o sp **ay**
after *much* thought , **they** wo n't kill the witch	gos nəng *tep* g i, koyb ma pakə ngab **ay**

Affixes:
-əng : prospective tense (translated as 'shall')
-i : dependent clause, indicates 'prior' event (after 'verb A'-i (ing), B happens)
ma- : negative (not, will not)
-ngab- : future tense (for 3rd person: he, she, they, etc)
-p- : past perfect (for 3rd person: he, she, they, etc)
-sp- : present, progressive (I am doing something right now)

Nouns: *gos*: thought *kəmən*: deer *koyb*: witch *kuj*: magic *tuw*: axe

Verbs: də- : to get (something) gə- : to do (often added at the end of another action verb)
nəng- : to think (gos nəng), to learn, to see, to perceive ning- : to eat o- : to come pak- : to kill

Other words: *ak*: this *etəp*: what *nəp*: you (object/accusative) *tep*: good, well *yad*: I

In this Oceanic language, sentences or clauses usually have a subject-object-verb order. If the subject is a pronoun, it can be dropped as long as the **last verb** in the sentence is conjugated to reflect the subject's POV. This final, conjugated verb usually belongs to the **independent clause** (an independent clause doesn't have conjunctions or prepositions like 'after', 'with', etc). Only the last verb reflects the subject person, but the language sometimes chains several verbs in a row (such as in sentence D). **Dependent clauses** are often marked with an -i ending on the verb, without conjugating for POV. This means that the first part of the sentence (before the comma) usually indicates an event that comes first, while the conjugated verb refers to an action that takes place at a later point in time.

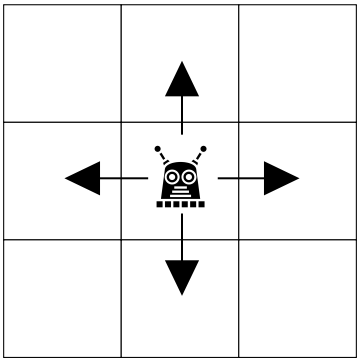
Some other notes: The auxiliary verb gə- is often added after other verbs, similar to how English '*get* going' works. **The sound 'ə'** is often added to combine grammatical affixes together.

Answer:
A 3 B 10 C 5 D 1 E 2 F 7 G 4 H 9 I 6 J 8
1 D 2 E 3 A 4 G 5 C 6 I 7 F 8 J 9 H 10 B

Computer Science

- Your program should be written in Java or Python-3.
- No GUI should be used in your program (e.g. `easygui` in Python).
- All the input and output should be done through files named as specified in the problem statement.
- Java programs should be submitted in a file with extension `.java`; Python-3 programs should be submitted in a file with extension `.py`. **No `.txt`, `.dat`, `.pdf`, etc. Programs submitted in the incorrect format will not receive any points!**

Shurik is running a robotics semilab, where he is designing a moving robot with pathfinding capabilities. The robot moves on an infinite board, where, on each square of the board, it can decide to move in four directions: left, right, up, or down.



5 points:

To test the robot's basic movement capabilities, Shurik wants to ensure that the robot can get from point A to point B. Shurik gives the robot its starting position and the desired position to which it should move, and wants the robot to come up with a sequence of moves, and verify that its moves will get itself to the desired position before making them.

Write a program that receives the robot's starting position, desired position, and sequence of moves, and determines whether the robot will reach its desired position after the move sequence.

Your program should read the input file `input.txt`, which contains four lines:

- The first line contains a single nonnegative integer n, denoting the number of moves the robot makes.
- The second line contains two space separated integers denoting the x- and y-coordinates of the robot's **starting** position on the infinite board.
- The third line contains two space separated integers denoting the x- and y-coordinates of the **desired** position on the infinite board.
- The fourth line contains a list of n space separated characters "`L`", "`R`", "`U`", or "`D`", indicating that the robot moves left, right, up, or down, respectively.

Your program should produce the file `output.txt`, which contains either "`YES`" if the robot ended at its desired position after following the sequence of moves from its starting position, or "`NO`" otherwise.

Sample Input 1:

```
6
0 -1
5 -2
R R R R R D
```

Sample Output 1:

```
YES
```

Sample Input 2:

```
7
-1 5
2 3
R D R R D L U
```

Sample Output 2:

```
NO
```

Sample Explanation 2:

Although the robot reached the desired position $(2, 3)$, it did not end at that position.

Hint:

No hint this month.

Solution:

The 5pt solution is available on the SigmaCamp GitHub repository here:

https://github.com/SigmaCode/POM-QQ/tree/main/CS/2024-2025/September

10 points:

Next, Shurik programs the robot to solve a maze by finding the shortest path from the start to the finish, and record its movements along the way. The maze consists of obstructions that are placed on squares of the board. The robot will always avoid any square that has an obstruction on it. One day, the campers designed a maze and the robot navigated it. However, the next day, a camper discovers a bug in Shurik's code: the robot may not have taken the shortest route in the maze! Unfortunately, the maze was disassembled and no one remembers what it looked like.

Write a program that takes as input the movements of the robot, and determines whether there exists at least one maze such that the path the robot took in that maze was the shortest one.

Your program should read the input file `input.txt`, which contains two lines:

- The first line contains a single nonnegative integer n, denoting the number of moves the robot made.
- The second line contains a list of n space separated characters "`L`", "`R`", "`U`", or "`D`", indicating that the robot went left, right, up, or down, respectively.

Your program should produce the file `output.txt`, which contains either "`YES`" if there exists at least one maze arrangement such that the path the robot took was the shortest one to get from the robot's starting position to its ending position, or "`NO`" otherwise.

Sample Input 1:

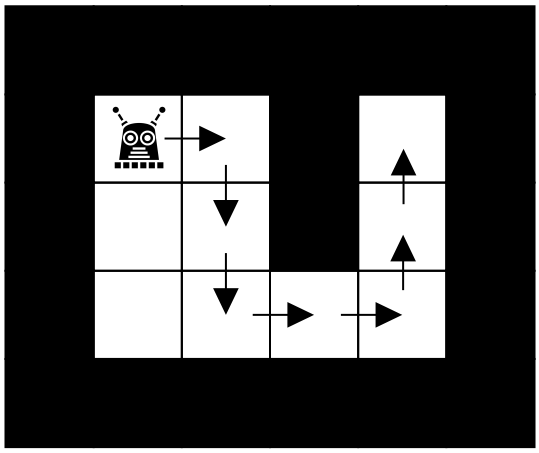
```
7
R D D R R U U
```

Sample Output 1:

```
YES
```

Sample Explanation 1:

Below is one possible maze that the robot could have navigated where its path is a shortest one (black squares indicate obstructions):



Sample Input 2:

```
14
R D L D D R R D R R U U U L
```

Sample Output 2:

```
NO
```

Hint:

Suppose that the path taken by your robot so far *is* the shortest, it takes one more step, and now the path is no longer the shortest. What can you say about the square on which the robot is now on?

Solution:

The 10pt solution is available on the SigmaCamp GitHub repository here:

https://github.com/SigmaCode/POM-QQ/tree/main/CS/2024-2025/September