Problem of the Month

POM

2022-2023

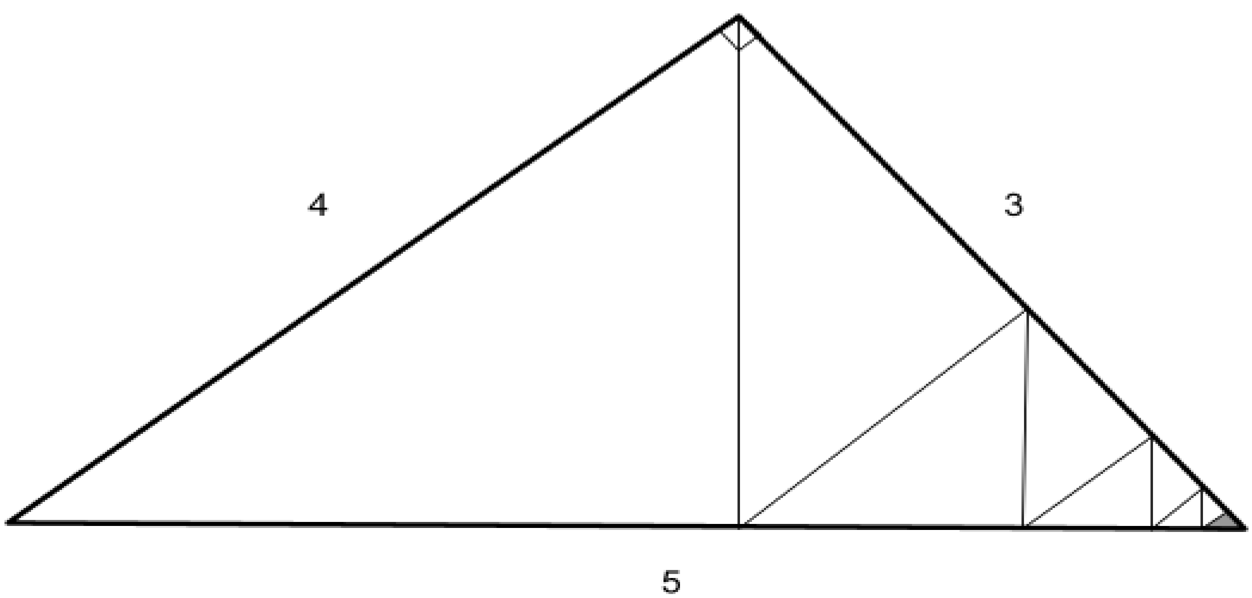
September 2022

# MATHEMATICS

**5 points:**

A right triangle with side lengths 3, 4, 5 is drawn. Then, a line perpendicular to its hypotenuse is drawn through its right angle, resulting in two smaller triangles. This process is repeated seven additional times with one of the smaller triangles, as shown in the picture. What is the ratio of the area of the shaded triangle to the area of the original large triangle?

The image is not to scale.

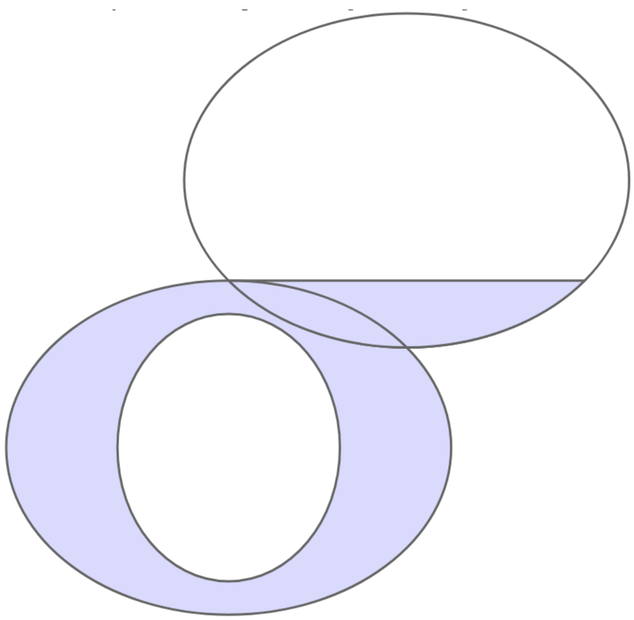



**Hint:** Are some of the triangles in the image related in any special way? Could this be used to solve the problem faster?

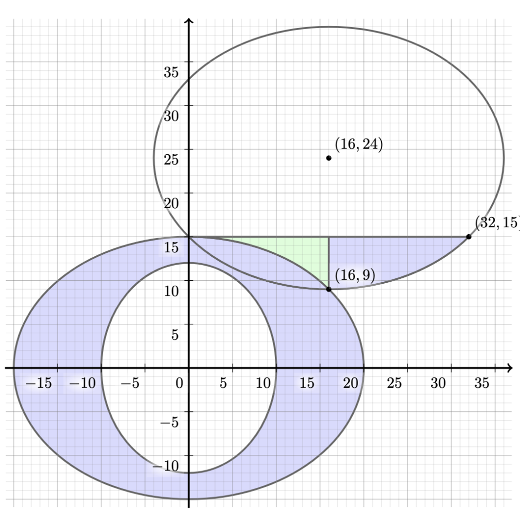
**10 points:**

Alice draws a lowercase sigma (σ) on graph paper using three ellipses and a line. She starts by drawing two ellipses whose semi-major axes, which run parallel to the x-axis, have length 20, and whose semi-minor axes have length 15. The first ellipse is centered at the origin, and the second is centered at (16, 24), and the ellipses have equations $(\frac{x}{20})^2 + (\frac{y}{15})^2 = 1$ and $(\frac{x-16}{20})^2 + (\frac{y-24}{15})^2 = 1$, respectively. She then draws a chord of the second ellipse which goes through (0, 15) and runs parallel to the x-axis. Finally, she draws a third ellipse centered at the origin with a semi-major axis of 12 parallel to the y-axis and a semi-minor axis of 10 parallel to the x-axis, given by the equation $(\frac{x}{10})^2 + (\frac{y}{12})^2 = 1$. Calculate the area of Alice's sigma (the shaded area).

**Don't think you know enough about ellipses? Read up [here](here).**



**Hint:**



# PHYSICS

## Preamble

Hi everyone! Welcome to Physics POM 2022-2023. There are a couple of practical points:

- **It's okay if the Problem of the Month takes a month to solve.** New concepts are hard, and one of our goals this year is to expose our contestants to much more of the vast array of cool ideas, concepts, and techniques we see across all of physics, both within and beyond what you might see at olympiads. This leads to problems that are difficult to solve if you don't know the trick, principle, or conceptual idea behind them, but fall before you quickly once you do. *So don't give up!* Each month we'll link resources and provide hints and ideas to guide you to the correct path. If you read the provided materials carefully, you'll get there!

- **We have a new side contest!** In addition to the 5pt and 10pt problems, we are introducing a third tier of problem designed to explore the material deeper and be more challenging than the main contest, for those into that sort of thing. Because we don't want to fight the other subject POM teams to the death (we *do* still need them after all) over whether Physics gets a larger share of points to hand out to contestants, we can't just create a 20pt category. **So - there will be a separate prize come Sigma 2023 for the contestant that performs best in this separate set of problems.** The prize itself is to be decided, and will be determined by the overall performance of the winner. Depending on interest, we may host an office hour after the deadline to explain the Challenge problem solution and answer any related questions. **Solutions to these problems can be intricate and require multiple lines of calculation. Please present your solution clearly and legibly, or it may be misgraded if the grader can't follow what you are attempting to communicate.**

- **If you have any questions at all about Physics POM logistics this year, please email Alex Frenkel (this year's subject lead) at frenkelalexander1@gmail.com with the subject line `Physics POM Inquiry'.**

# Introduction (Pre-ramble)

The history of the development of physics is long and full of terrors, and we don't plan to bore you by throwing a history textbook at you. That being said, while Newton formalized his laws of motion in *Principia Mathematica* in 1687 the problems presented below could very likely have been solved by ancient Greek philosophers over two thousand years ago (and here we think we've come so far since then! Don't worry too much - next month we'll graduate to at least the 18th century.)

The modern perspective, regardless, does begin with the three basic principles laid out by Newton - principles that you've seen copied and elucidated thousands of times over. Accounts of the original translation differ, and I don't know Latin, so I will trust Wikipedia's phrasing:

1. *Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it.*
2. *The change of motion of an object is proportional to the force impressed; and is made in the direction of the straight line in which the force is impressed.*
3. *To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.*

There are certain folks (you know the type) that will hand these three principles to you, pat you on the back, and send you on your way - "You now have all you need to solve any mechanics problem!" While technically correct, to do this is analogous to handing someone a saw, a hammer, and a bucket of nails, asking them to turn a forest into a neighborhood, then calling them an idiot when they stand around lost and confused. The tools are there, but there is a vast wealth of tricks and techniques to using them that might appear straightforward only once they've been revealed to you (I'll take any conceptual excuse to link a Penn and Teller performance). Ideas like conservation laws, normal force, kinetic and static friction, air resistance, buoyancy force, the idea that tension always points parallel to a freely hanging string, Hooke's principle, Hooke's other principle or any of the hundred or so ideas, facts, and methods presented in this absolutely gorgeous introduction to Mechanics problem solving by Jaan Kalda (an Estonian physicist whose fantastic content we will be pulling from all year) or a similarly wonderful introduction to olympiad problem solving by Kevin Zhou (a brilliant colleague of mine at Stanford).

Even to link a barrage of (pedagogically elucidated) information, while a drastic improvement, is still quite unfair (although we highly, highly, ***highly*** recommend carefully perusing the above resources if and when you have the time). Like a chess tactic becomes infinitely easier when you know it involves a queen sacrifice, olympiad problems become much simpler once you know the idea you're meant to apply. The best olympiad solvers are those with a talent for quickly and accurately assessing which of the hundreds or thousands of tricks from their bag are the ones applicable to any particular problem. This is why for the problems presented below (and future months as well), we will not only provide resources, but *also* do our best to direct you to the needles in the conceptual haystack that are most relevant. This is our compromise for this year's spike in difficulty.

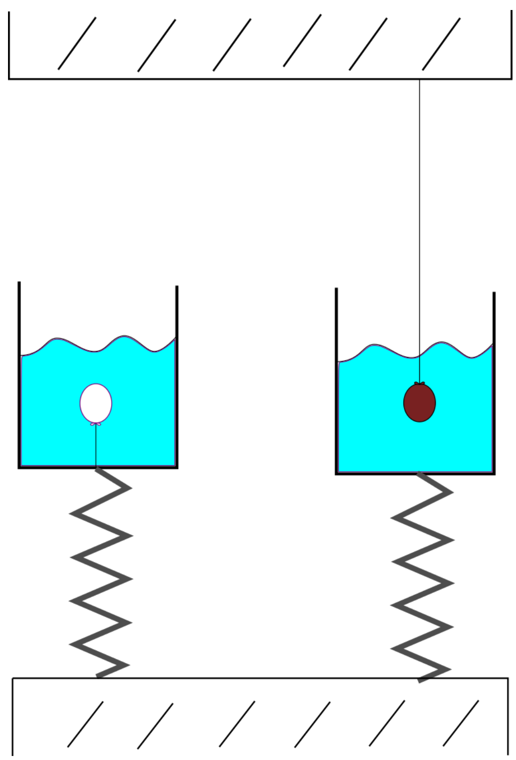
Mechanics can often feel boring due to its familiarity in everyday life. The flipside is that classical mechanics has *precisely* the tools we need to describe, explain, and predict the vast majority of everyday human experience. The curved trajectory of a spinning soccer ball, how much water you can splash at your friend by dragging your hand across a pool's surface, the structural stability of enormous arched cathedrals, or even how far you'd expect an action movie protagonist to *actually* be tossed by an explosion's shock wave - the vast majority of questions you can ask about what happens to objects in everyday life can be turned into an olympiad problem via a cleverly simplified model. Furthermore, the intuition behind energy, momentum, static and elastic collisions, and the mass flow of particles permeates all the way to every corner of cutting edge physics. More pertinently - these ideas will show up in future POMs this year, even if the monthly themes will stray to other areas of physics.

To demonstrate this last point, the mechanics problems this month are for the most part fluid dynamics themed. Fluids may seem intimidating at first, and exhibit a wealth of fascinating properties and phenomena, but one can get very far by considering a fluid as a collection of many elastically colliding particles, especially when the fluid is undergoing laminar flow (as in this month’s challenge problem!). They are a useful playground for developing your physical intuition, and practice thinking about fluids will strengthen your understanding of many phenomena you directly experience in everyday life.

So, without further ado - good luck!

**5 points:**

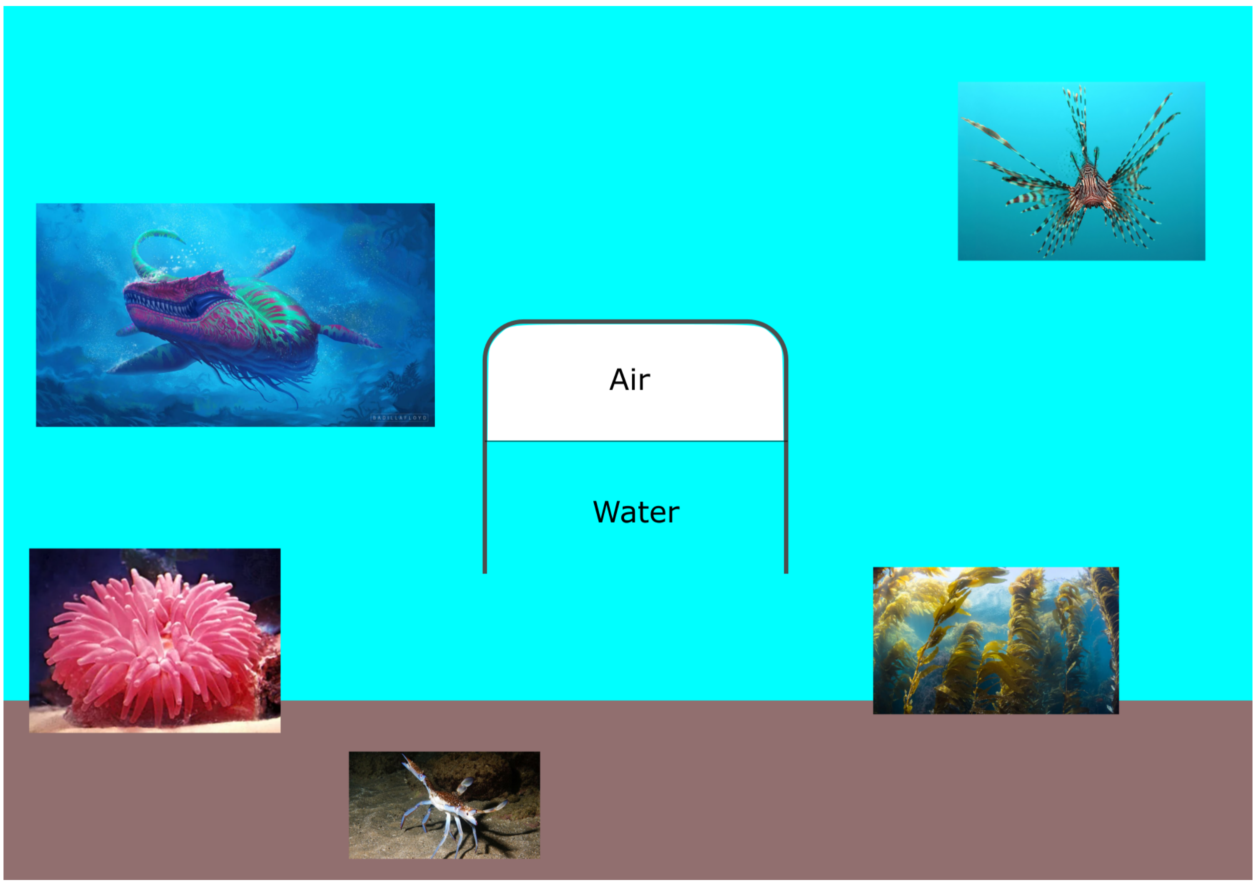
Two springs of identical spring constant support two identical tubs, filled with identical amounts of water (pictured below). The tub on the left has a balloon filled with air tied to its floor. The tub on the right holds an identical balloon, filled with an identical volume of mercury, suspended by a sturdy string from the ceiling. Provide an explanation for whether the spring on the left, the spring on the right, or neither will be compressed more. Assume the rightmost spring does not compress far enough for the mercury balloon to reach the water level in its container. **No points will be awarded for a correct answer with no solution.**



**Hint:** Remember Newton's third law. What counteracts buoyancy force?

**10 points:**

For our second problem, we place a bucket of mass $M$ resides in placid, idyllic water. It floats at depth $D$, supported by an air pocket trapped inside. At this depth, all forces acting on the bucket cancel. A passing megalodon disturbs the bucket, causing it to begin floating upward. At what depth will its acceleration reach $1g$? Assume the initial volume of this air pocket is $V_0$, and the bucket is large enough that at no point does air spill out of the bottom.



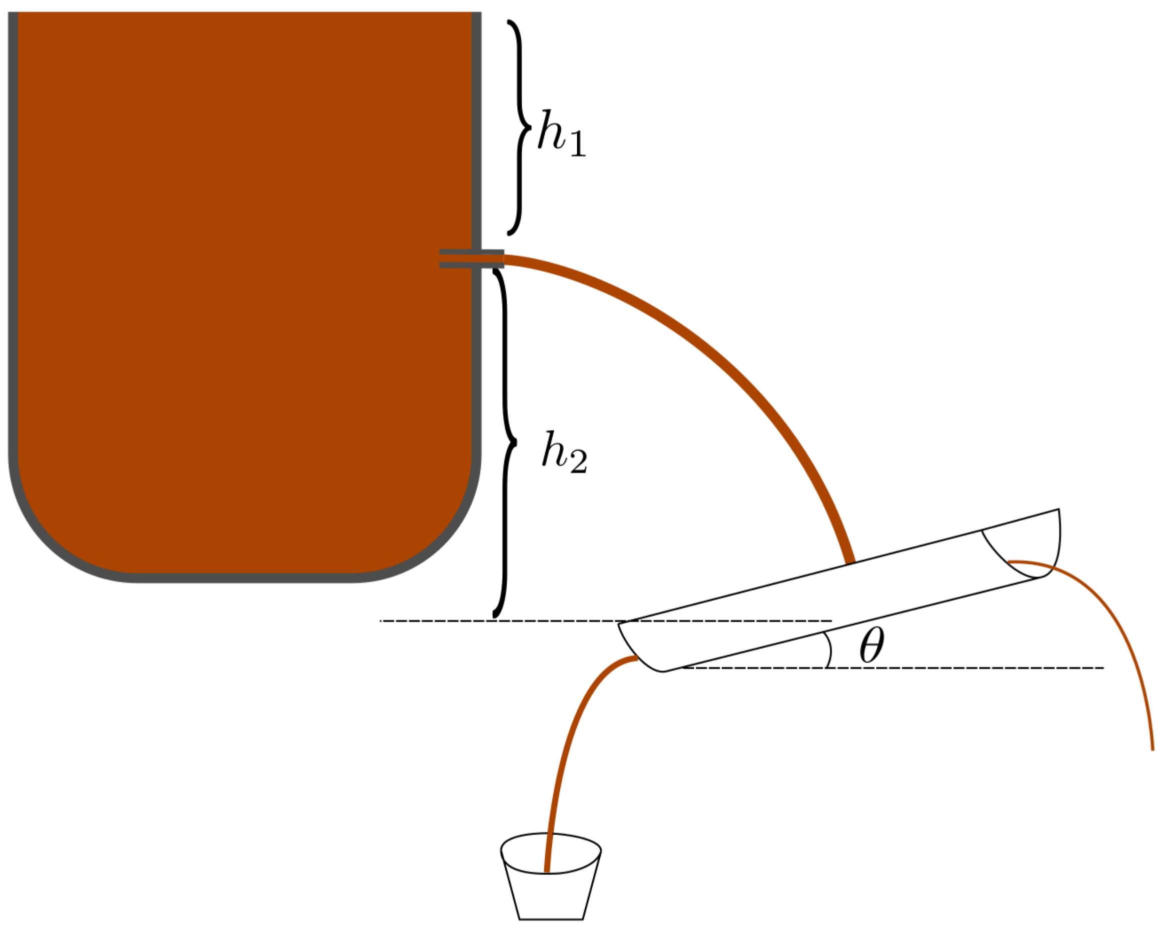

**Hint:** As the bucket rises, the pocket of air expands due to the lowering pressure, and the effective amount of displaced water due to the air changes.

**September Physics Challenge Problem (submit here)**

**Warmup:** Problem 52 of Kalda. Understanding and submitting the solution to the warmup is worth 10% of the total score this month.

**The main event:** A bored Sigma camper has drilled a hole in their grandmother's expensive samovar! The camper proceeds to fill their cup by letting the tea pour into a pipe, which then redirects it into their cup. However - the camper forgot to make the pipe long enough, and some of the tea is spilling over the top. How much tea is entering the camper's cup per second?

Assume the hole is drilled at height $h_1$ below the tea level in the samovar, the tea hits the pipe at height $h_2$ below the hole, the pipe is inclined at angle $\theta$, the tea undergoes laminar flow, you can neglect air resistance, and the pipe is short enough that all the tea that is directed up the pipe after collision spills out the top. Use $\rho$ for the density of the tea.



# CHEMISTRY

**5 points:**

Alice built a mini hydroponic farm in her house, and decided to grow tomatoes there. She prepared a solution containing 10 mM ammonium chloride and 10 mM potassium dihydrophosphate, which was supposed to provide tomatoes with the three most essential elements: potassium, nitrogen, and phosphorus. However, she quickly found that tomatoes were growing very slowly, and she asked Bob for advice. "Check the pH", he said, "It seems your solution is too acidic." Alice quickly realized that he was right, and she decided to fix that problem by adding lime water (a solution of calcium hydroxide) until pH reached the value of 7.4. After that, Alice let the solution obtained sit for one day to let all small particles precipitate, and then she used the clear liquid for growing tomatoes. However, although tomatoes did grow, their leaves were dark, dull, blue-green, and may become pale in severe deficiency. Alice again asked Bob's help, and he immediately found the reason. "Alice, you forgot that ....." What did Bob say?

**Hint:** no hint this month

**10 points:**

Consider this statement: "In some geographical location, the following phenomenae are observed: (i) The water from natural wells has a strong H2S smell; (ii) when you bite a local apple and leave it for several minutes, the surface becomes brown; (iii) teapots become covered with a thick scale when they have been extensively used during several days; (iv) when you are pumping fresh water from local wells, it is initially clear and colourless, but if you leave it in a bucket, it quickly becomes turbid; (v) the surrounding area swampy and wooded; (vi) the location is surrounded with red sand dunes." Identify which items in the list correlate with each other and which are mutually exclusive. Explain your answer from a chemist's point of view.

**Hint:** no hint this month

# BIOLOGY

## *Introduction*

Welcome to the Biology POM 2022-2023! We're very excited to introduce you to the world of scientific literature through the POMs this year, and in that vein, here are some tips to approach tackling a scientific article. When answering questions, we expect you to do research and consult outside sources, but you do need to cite every source you use in a consistent format (MLA 9th Edition). Here are some resources to help you with that! As always, don't hesitate to contact Elena Yakubovskaya (elena.yakub@gmail.com) or Sanjana Rao (sanjanarao@uchicago.edu), this year's Biology subject leads, if you have any questions.

**5 points:** Humans have subjected many animals to artificial selection, creating breeds to better fit humanity needs. Provide several examples of such animals and the purpose for which they are bred. Why do you think there are many more examples of vertebrate animals domesticated in this way, but only few among the invertebrates? What difficulties may humans encounter when trying to domesticate invertebrates? Suggest one invertebrate species that would be useful for humans to create an enhanced breed for and describe your artificial selection strategy.

**Hint:** no hint this month

## 10 points:

Poaching is a serious threat to the wild black rhinoceros in southern Africa. Governments are actively managing the black rhinoceros population across southern Africa, mostly by capturing and subsequent translocation by a helicopter. Rhinoceroses are transported in suspension in two common positions: lying on the side and upside down, hanging by their feet [1]. Which way is safer for a rhinoceros's health and why? How is the safest transportation orientation different for other big mammals? Provide examples. How would you set up an experiment to test the best orientation for transportation?

*(The Pulmonary and Metabolic Effects of Suspension by the Feet Compared with Lateral Recumbency in Immobilized Black Rhinoceroses (Diceros bicornis) Captured by Aerial Darting)*[1]

**Hint:** no hint this month

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[1] *Radcliffe et al. Journal of Wildlife Diseases, 2021.*

# LINGUISTICS

## 5 points:

An anagram is a word or phrase that can be rearranged to create another word or phrase. An aptigram is an anagram that creates a synonym when rearranged, and an antigram creates an antonym. Given these three definitions, find an existing prefix used in English (like apti- or anti-) and create your own type of anagram. Provide a definition and three examples of your new type of anagram. The examples, both before and after rearranging, must use distinct valid English words, and proper nouns are not permitted.

Example: aptigram (apti- from Latin, meaning fitness)

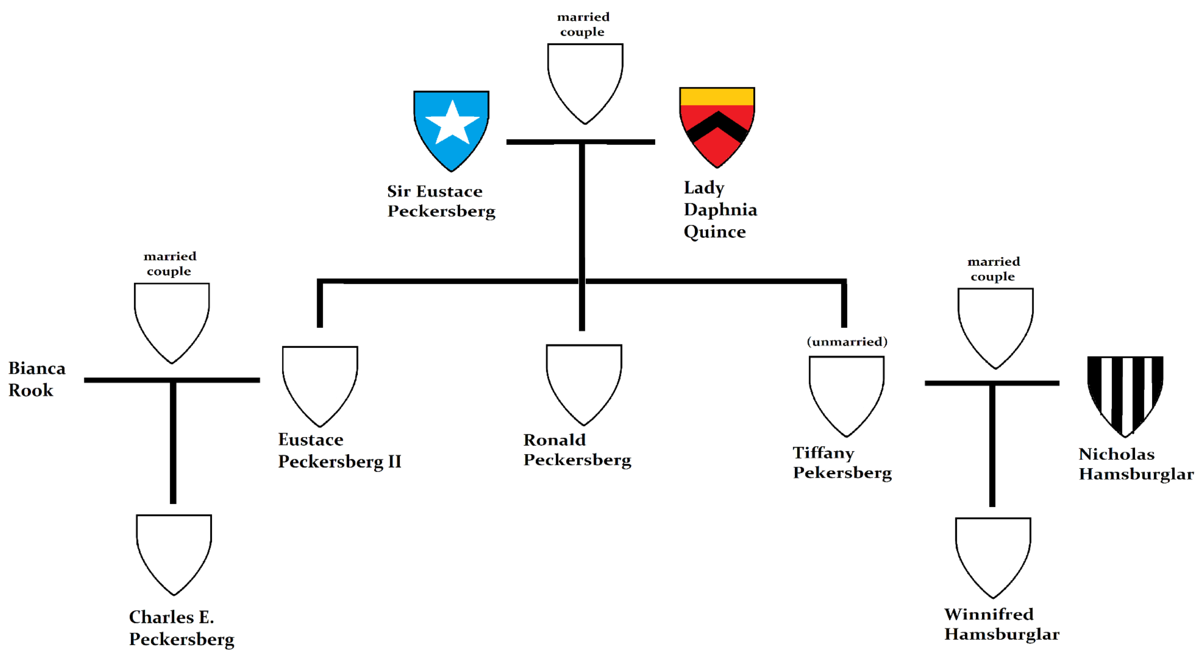
past due = date's up
dormitory = dirty room
a gentleman = elegant man

**Hint:** no hint this month

## 10 points:

Heraldry is the broad term for things related to the study and design of heraldic achievements, most commonly known as coats of arms. Coats of arms are passed down through families, but change as they are passed down to different family members.
Below is a family tree with only three of the coats of arms provided. Please research the rules for passing down coats of arms and add your own illustrations to the blank spaces in the tree.

Note: We understand you might find conflicting sources, so please explain why you choose to draw each coat of arms that way and cite your sources. Please do your best to be clear with your drawings, but if you're worried that they aren't clear you can provide a description next to them. The diagram below can be downloaded here: [https://drive.google.com/file/d/1szQOeKiHpjcvRXiDoGJ-N6vOsv71_zhC/view?usp=sharing](https://drive.google.com/file/d/1szQOeKiHpjcvRXiDoGJ-N6vOsv71_zhC/view?usp=sharing)




**Hint:**
You're looking for rules about:
- combining coats of arms after a marriage
- symbols added to coats of arms based on children's birth order
- women inheriting coats of arms

# COMPUTER SCIENCE

While computer science is a lot of fun (we hope!), it can also be complicated and difficult to learn. We want these problems to be challenging but approachable, so this year's CS POM is introducing some new resources to make learning the skills to succeed more accessible:

- This year, the CS POM team has decided to introduce a lecture for September, which will consist of three 20-minute time blocks, during which three sessions will be offered concurrently. These beginner-friendly sessions will be taught by members of the CS POM team, and will cover a wide range of CS topics, mostly centering around Python. If you haven't coded before, we suggest making an account with [Replit](), a website that can run your code, which we'll use during the sessions. The lecture will be offered on **Sunday, October 2nd at 3PM EST** at https://sigmacamp-org.zoom.us/j/88483055125. Can't make it? We will record all the sessions and post links to the recordings as well as notes from some sessions on the POM website. Participants can attend as many or as few of the time blocks as they want - it's up to you to decide which sessions you'd like to attend! Depending on the lecture's success, it may be offered again in future months with some new topics. The schedule of sessions is below:

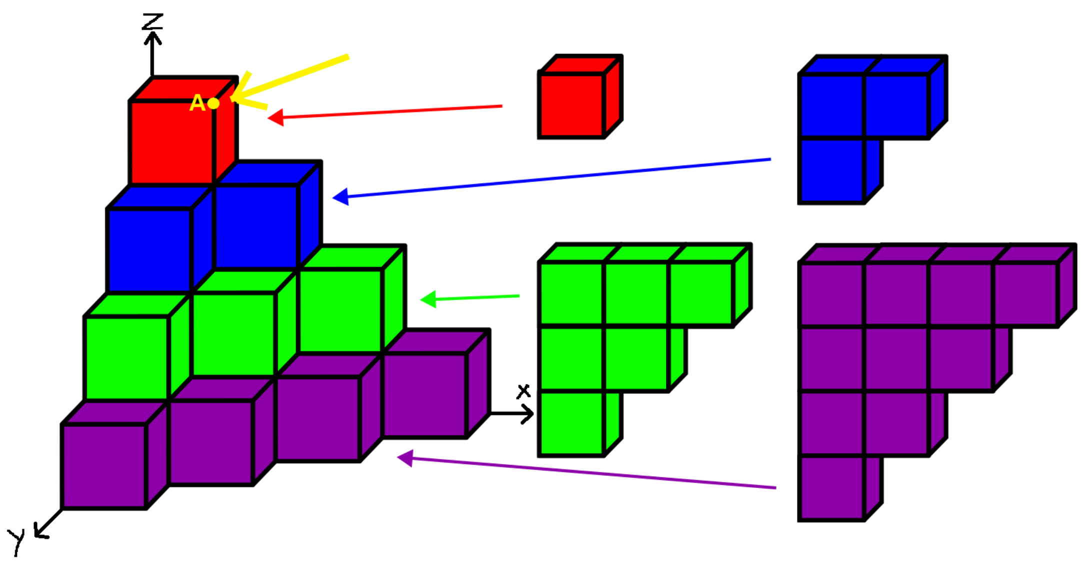
<table>
<tr><td colspan="2">Introduction</td></tr>
<tr><td rowspan="3">3:05 - 3:25 PM EST</td><td>Get Started with Python with Anna Rosner</td></tr>
<tr><td>Read and Write a File in Python with Maya Smith</td></tr>
<tr><td>Python Logic Basics with Anatoly Zavyalov</td></tr>
<tr><td colspan="2">5 minute break</td></tr>
<tr><td rowspan="3">3:30 - 3:50 PM EST</td><td>Data Processing and Validation with Anna Rosner</td></tr>
<tr><td>Read and Write a File in Python with Maya Smith</td></tr>
<tr><td>Python Logic Basics with Anatoly Zavyalov</td></tr>
<tr><td colspan="2">5 minute break</td></tr>
<tr><td rowspan="3">3:55 - 4:15 PM EST</td><td>Data Processing and Validation with Anna Rosner</td></tr>
<tr><td>Python Data Structures with Anar Amgalan</td></tr>
<tr><td>Depth-First Search and Breadth-First Search with Anatoly Zavyalov</td></tr>
</table>

- Additionally, the CS POM team has put together a list of some of their favorite CS resources. Any that we feel are relevant to a problem will be linked within it, but otherwise you can find the whole list here. Without further ado, here are the Computer Science rules and problems:

- Your program should be written either in Java or Python 3
- No GUI should be used in your program
- All the input and output should be done through files named as specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python 3 programs should be submitted in a file with extension .py.
  **No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!**

## 5 points:

The picture below shows a pyramid built out of identical cubes. The base of the pyramid is a right triangle, and the pyramid is built up along the z-axis. The top of the pyramid has 1 cube, the second layer from the top has 3 cubes, the next layer has 6 cubes, etc. For example, the picture below shows a 4-layer pyramid:




Write a program that receives the coordinates of the outermost point of the top cube (A, pointed to by the yellow arrow in the diagram above) and calculates the total number of cubes in the pyramid. Your program should read the input file **input.txt**, which will consist of one line containing the x, y, and z coordinates separated by spaces.

Example input file:

```
1 1 4
```

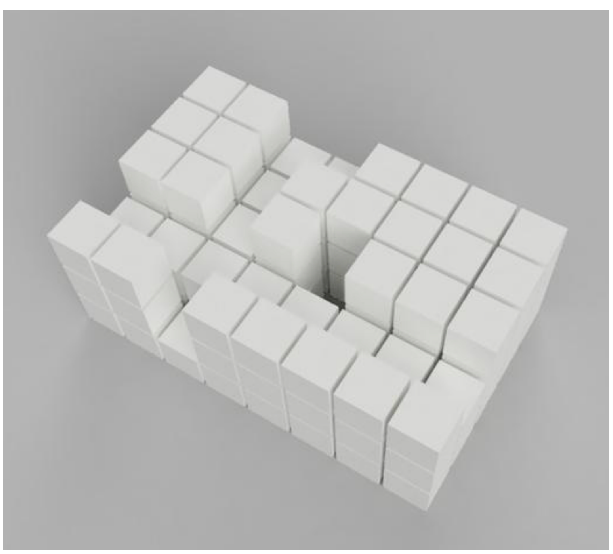
Your program should produce the output file **output.txt**, containing a single integer number representing the volume of the pyramid (i. e., total volume of all the cubes comprising the pyramid). If the given input does not correspond to a possible pyramid, write IMPOSSIBLE to the output file.

Note that the provided coordinates are integer numbers, and that **the size of each cube is not necessarily 1x1x1**.

**Hint:** no hint this month

## 10 points:

A three-dimensional figure is constructed from identical cubes with overall dimensions l x w x h cubes. All the cubes are aligned along the grid lines with no displacement. From this figure, projections are made from three sides: front, right, and top. To make a projection of a figure from a view, look at it straight on. For each cell where any cube is visible, that cell is shaded. If no cube is visible, that cell is empty.
For example, the projections of the below figure would be the following:



```
## #####
######## (from the front)
########

# ###
##### (from the right)
#####

########
########
#### ### (from the top)
########
########
```

Using these projections and given values for l, w, and h, your program will calculate the maximum number of cubes that could be in the figure or identify if such a figure is impossible (eg, projections are inconsistent). Your program should read the input file **input.txt**, of which the first line will include l, w, and h, each separated by spaces, and the following lines will include the projections, each separated by an empty line. In the projections shaded cells are represented by #.

Example input file:

```
3 4 3
####
####
# ##

###
###
###

####
 ###
####
```

Your program should produce the output file **output.txt**, containing a single integer number representing the maximum number of the cubes in the figure to satisfy given projections. If the projections given do not correspond to any possible figure, write IMPOSSIBLE to the output file.

**Hint:** no hint this month