

**PROBLEM OF THE
MONTH**

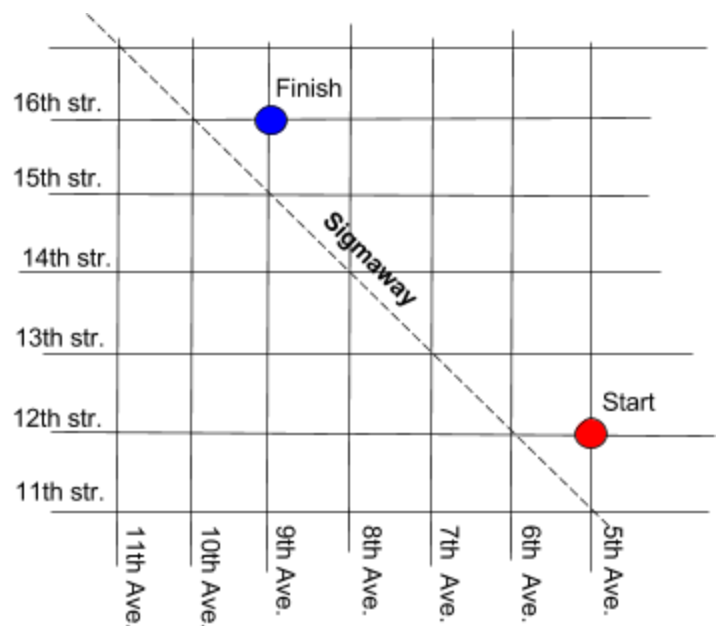


September, 2017

MATHEMATICS

5 points: A tourist arrives in SigmaCity. The city is somewhat similar to Manhattan: Streets follow East-West direction, and their numbers are increasing from South to North. Avenues are perpendicular to the Streets and their numbers are increasing from East to West (see the Figure. Please ignore the Sigmaway - it is needed for 10pt problem only). The tourist starts his journey near the train station located at the corner of 12th Str. and 5th Ave. He knows that there is a Sigma Museum somewhere northwest of him, so at each intersection, depending on the traffic light, he would go North or West, with equal probability. What is the probability that the tourist will eventually reach the Museum located at the intersection of 16th Str. and 9th Ave.?

Hint: try to calculate how many ways are there to get to the museum.



10 points: A tourist arrives in SigmaCity. The city is somewhat similar to Manhattan: Streets follow East-West direction, and their numbers are increasing from South to North. Avenues are perpendicular to the Streets and their numbers are increasing from East to West (see the Figure). The tourist starts his journey near the train station located at the corner of 12th Str. and 5th Ave. He knows that there is a Sigma Museum somewhere northwest of him, so at each intersection, depending on the traffic light, he would go North or West, with equal probability.

However, there is one more street in the city called Sigmaway that runs diagonally as shown in the Figure. The tourist never crosses it, knowing that the museum is located at the same side of the Sigmaway as the train station. At the intersection with the Sigmaway he always turns North. What is the probability that the tourist will eventually reach the Museum located at the intersection of 16th Str. and 9th Ave.?

Hint: Let us sketch the map of streets and avenues in such a way that they form a square grid (the scales are not relevant in the context of this problem). Now let us fold this paper map along the Sigmaway diagonal, and imagine that the tourist walks on the paper map as he did in 5pt. problem...

PHYSICS

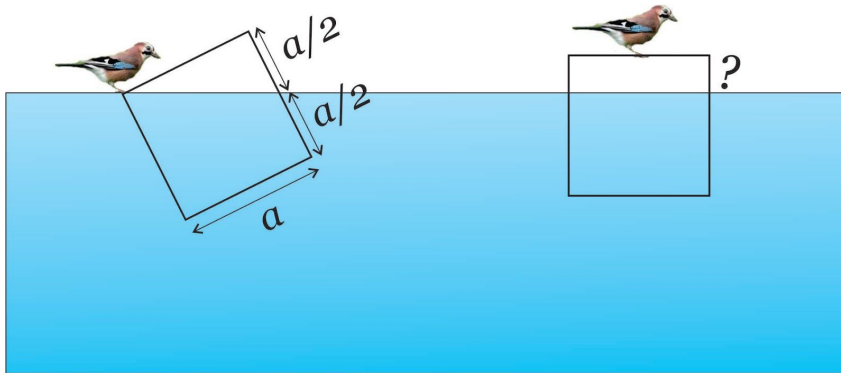
This month problems are on the Archimedes' principle and on the law of the lever. You might find the following links useful.

Buoyancy: <http://hyperphysics.phy-astr.gsu.edu/hbase/pbuoy.html#arch3>
https://en.wikipedia.org/wiki/Archimedes%27_principle

Center of mass: <http://hyperphysics.phy-astr.gsu.edu/hbase/cm.html>

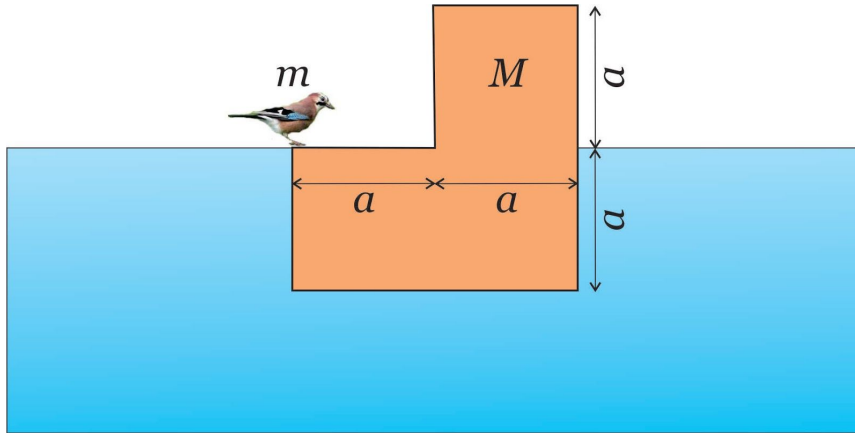
Torque and equilibrium: <http://hyperphysics.phy-astr.gsu.edu/hbase/torq.html>

5 points: A bird of mass m stands at the corner of the wooden log of square cross-section and mass M floating in the water, such that the corner at which the bird stands is level with water, while the level of water on the opposite side of the square log is at the middle of that side (see picture below). Find the level of water (how far the log will be immersed in water) when bird moves to the center of the log.



Hint: What volume of the log is immersed in water? Use Archimedes' principle.

10 points: A bird of mass m stands at the corner of the wooden log with an L-shaped cross-section with dimensions shown in the Figure. The log is floating in the water, so that the corner at which the bird stands is level with water and the bottom of the log is horizontal. Find the mass of the log M .



Hint: Use the law of the lever for the bird and the part of the log above the water relative to the center of the log.

NO HINTS AVAILABLE FOR CHEMISTRY, BIOLOGY, OR CS PROBLEMS