PROBLEM OF THE MONTH

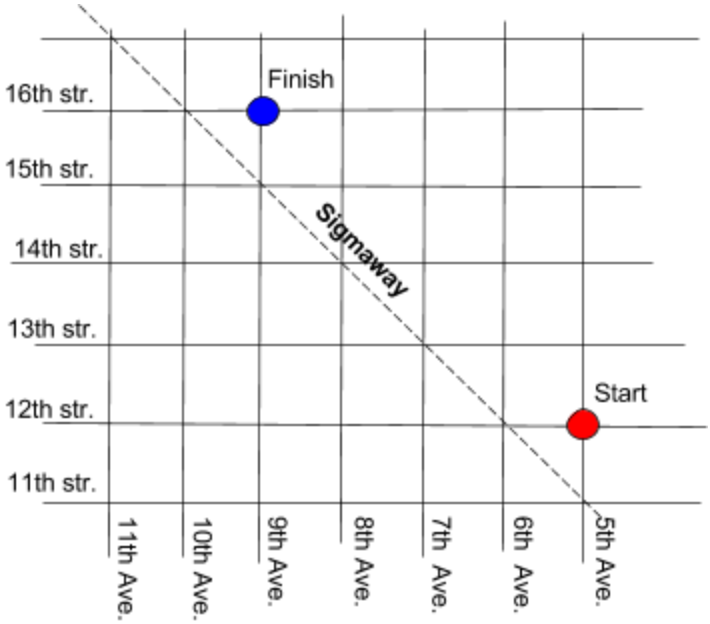
POM

2017-2018

September, 2017

MATHEMATICS

5 points: A tourist arrives in SigmaCity. The city is somewhat similar to Manhattan: Streets follow East-West direction, and their numbers are increasing from South to North. Avenues are perpendicular to the Streets and their numbers are increasing from East to West (see the Figure. Please ignore the Sigmaway - it is needed for 10pt problem only). The tourist starts his journey near the train station located at the corner of 12th Str. and 5th Ave. He knows that there is a Sigma Museum somewhere northwest of him, so at each intersection, depending on the traffic light, he would go North or West, with equal probability. What is the probability that the tourist will eventually reach the Museum located at the intersection of 16th Str. and 9th Ave.?



Hint: try to calculate how many ways are there to get to the museum.

Answer: 35/128

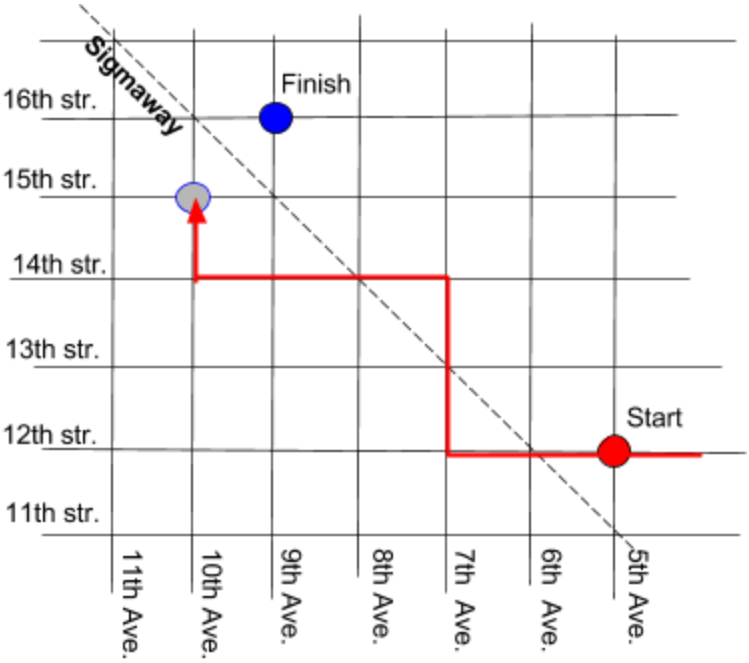
Solution: No matter how the tourist walks, he needs to walk exactly 8 blocks to get to the Museum: 4 to the North, and 4 to the East. The total number of routes of length 8 is $2^8 = 256$, all of the having the same probability. Out of them, we need to select only those that have exactly 4 segments heading North (there rest 4 will be heading east). This number is ${}_8C_4 = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4!} = 70$. The probability to ending up near the museum after walking 8 blocks is therefore 70/256=35/128. Which way the tourist walks after that is irrelevant.

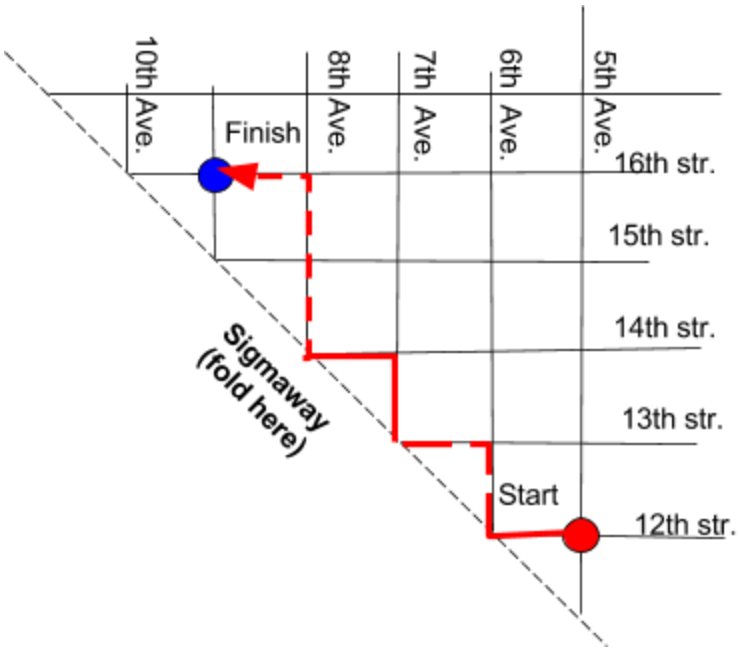
10 points: A tourist arrives in SigmaCity. The city is somewhat similar to Manhattan: Streets follow East-West direction, and their numbers are increasing from South to North. Avenues are perpendicular to the Streets and their numbers are increasing from East to West (see the Figure). The tourist starts his journey near the train station located at the corner of 12th Str. and 5th Ave. He knows that there is a Sigma Museum somewhere northwest of him, so at each intersection, depending on the traffic light, he would go North or West, with equal probability.
However, there is one more street in the city called Sigmaway that runs diagonally as shown in the Figure. The tourist never crosses it, knowing that the museum is located at the same side of the Sigmaway as the train station. At the intersection with the Sigmaway he always turns North. What is the probability that the tourist will eventually reach the Museum located at the intersection of 16th Str. and 9th Ave.?

Hint: Let us sketch the map of streets and avenues in such a way that they form a square grid (the scales are not relevant in the context of this problem). Now let us fold this paper map along the Sigmaway diagonal, and imagine that the tourist walks on the paper map as he did in 5pt. problem...

Answer: 63/128

Solution: Let us sketch the map of streets and avenues in such a way that they form a square grid (the scales are not relevant in the context of this problem). Now let us fold this paper map along the Sigmaway diagonal (see Figures below). When we do so, the left part of the map will go under the right one. Let the tourist walk on the paper map as he did in 5pt. problem (ignoring the Sigmaway). If we look at the tourist's walk at the folded map, regardless whether his is locate on the front or on the back, it will look as if he walks only at the "allowed" part of the city. Furthermore, he will still be going either north or east, with equal probability at each intersection, except at the diagonal (Sigmaway). At the diagonal he would always go North (the tourist from 5pt problem will think that he made a choice between East and North, but on the folded map they are the same). This means that we can solve the 10 pt problem by first solving 5pt. The probability of getting near Museum is equal to probability of getting there in the previous problem PLUS the probability of getting to point X which is a mirror image of the museum location with respect to the Sigmaway. That point is located at the intersection of 10th Ave. and 15 Str. of the original unfolded map. One needs to walk 3 blocks north and 5 to the east, to get there, so the probability of ever getting to point X is ${}_8C_4/256 = 56/256$. The overall result is (56+70)/256=63/128.


Sigmaway
Finish
16th str.
15th str.
14th str.
13th str.
12th str.
11th str.
Start
11th Ave.
10th Ave.
9th Ave.
8th Ave.
7th Ave.
6th Ave.
5th Ave.


10th Ave.
8th Ave.
7th Ave.
6th Ave.
5th Ave.
Finish
16th str.
15th str.
14th str.
13th str.
12th str.
Start
Sigmaway
(fold here)

PHYSICS

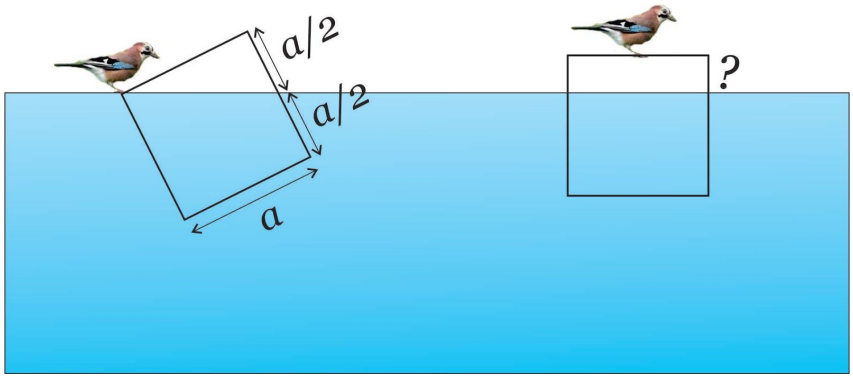
This month problems are on the Archimedes' principle and on the law of the lever. You might find the following links useful.

Buoyancy: http://hyperphysics.phy-astr.gsu.edu/hbase/pbuoy.html#arch3
https://en.wikipedia.org/wiki/Archimedes%27_principle

Center of mass: http://hyperphysics.phy-astr.gsu.edu/hbase/cm.html

Torque and equilibrium: http://hyperphysics.phy-astr.gsu.edu/hbase/torq.html

5 points: A bird of mass m stands at the corner of the wooden log of square cross-section and mass M floating in the water, such that the corner at which the bird stands is level with water, while the level of water on the opposite side of the square log is at the middle of that side (see picture below). Find the level of water (how far the log will be immersed in water) when bird moves to the center of the log.

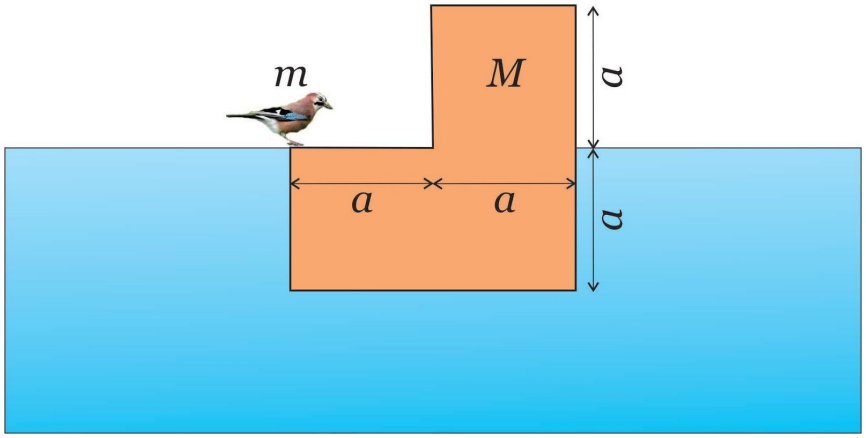


Hint: What volume of the log is immersed in water? Use Archimedes' principle.

Answer: $3/4\ a$

Solution: By Archimedes' principle $(M+m)g = \rho V g$, where m is the mass of the bird and V is the volume immersed into water (ρ is the density of water and g is the free fall acceleration). It is clear that the volume V immersed into water is the same in both cases. In the first picture from simple geometry we find that the log is immersed by ¾ of its volume. Therefore, in the second picture the log is immersed by $3/4\ a$.

10 points: A bird of mass m stands at the corner of the wooden log with an L-shaped cross-section with dimensions shown in the Figure. The log is floating in the water, so that the corner at which the bird stands is level with water and the bottom of the log is horizontal. Find the mass of the log M.



Hint: Use the law of the lever for the bird and the part of the log above the water relative to the center of the log.

Answer: $M = 6m.$

Solution: Let us consider all torques acting on the log relative to the center of the log (geometric center of the large square). Both buoyancy force and the weight of the bottom (immersed) part of the log have zero torque relative to the center. This is clear because one can think of both of these forces as being applied to the center of mass of the bottom part and the lever arm of these forces with respect to the center is zero. From the balance of the torques of the weight of the bird and of the weight of the upper part of the log we have $mga = (M/3)g(a/2)$ again assuming that the weight of the upper part of the log is applied to its center of mass. We immediately obtain $M = 6m.$

CHEMISTRY

5 points:

You are organizing the Escape-the-Lab game. In this game, players have to obtain a four digit number to open the last lock. To solve this puzzle, players use the following set of objects: two sets of bottles with some colorless aqueous solutions inside (the bottles are labeled as 1, 2, 3, 4, and A, B, C, D, accordingly), and a 4x4 rack with empty test tubes. The test tubes are labeled as shown on the scheme below:

1a 2a 3a 4a
1b 2b 3b 4b
1c 2c 3c 4c
1d 2d 3d 4d

These labels are a clue: players are supposed to mix a solution from the bottle 1 and a solution from the bottle **'a'** in the test tube 1a, a solution from the bottle 2 and the bottle **'a'** in the test tube 2a, etc. When the solutions are added to all test tubes, precipitation occurs in some of them, so they will look like this:

o o x o
o o o o
o o o x
o x x x

where 'o' means a clear solution, and 'x' means a precipitation.
That corresponds to the binary numbers 0010 (2), 0000 (0), 0001 (1), and 0111 (7), which is the code (2017) that opens the lock.

Question: which chemicals should you use to prepare solutions 1, 2, 3, 4, A, B, C, and D?

Solution:

There are many solutions to this problem, a possible solution can be:

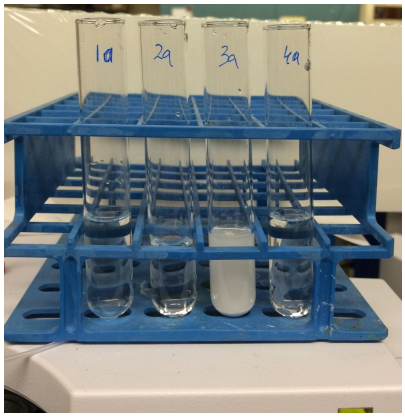
5pt: **1** water, **2** barium nitrate, **3** silver nitrate, **4** zinc sulfate; **a** NaCl, **b** water, **c** barium nitrate, **d** sodium carbonate.

The products of these reactions are as follows (only precipitated solids are shown):

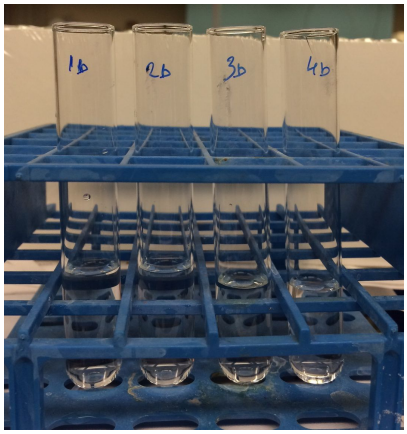
	H_2O	$Ba(NO_3)_2$	$AgNO_3$	$ZnSO_4$
NaCl:	1a	2a	3a AgCl	4a
H2O:	1b	2b	3b	4b
$Ba(NO_3)_2$:	1c	2c	3c	4c $BaSO_4$
Na_2CO_3:	1d	2d $BaCO_3$	3d Ag_2CO_3	4d $ZnCO_3$

Below are the photos of each row taken separately (after the chemicals have been mixed):

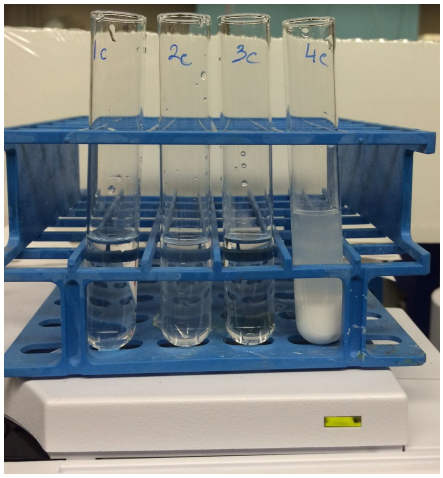
Row A:



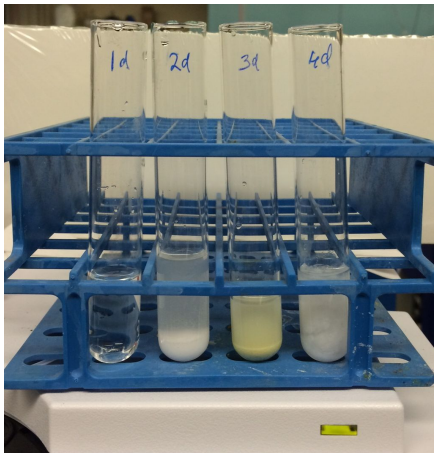
Row B:



Row C:



Row D:



If you provide a different solution, you will get a full score if your solution works.

10 points:

You are organizing the Escape-the-Lab game. In this game, players have to obtain a four digit number to open the last lock. To solve this puzzle, players use the following set of objects: two sets of bottles with some colorless aqueous solutions inside (the bottles are labeled as 1, 2, 3, 4, and A, B, C, D, accordingly), and a 4x4 rack with empty test tubes. The test tubes are labeled as shown on the scheme below:

1a 2a 3a 4a
1b 2b 3b 4b
1c 2c 3c 4c
1d 2d 3d 4d

These labels are a clue: players are supposed to mix a solution from the bottle 1 and a solution from the bottle **'a'** in the test tube 1a, a solution from the bottle 2 and the bottle **'a'** in the test tube 2a, etc. When the solutions are added to all test tubes, precipitation occurs in some of them, so when the secret number is, for example, 2018, the array of test tubes will look like this:

o o x o (i.e. 0010)
o o o o (i.e. 0000)
o o o x (i.e. 0001)
x o o o (i.e. 1000)

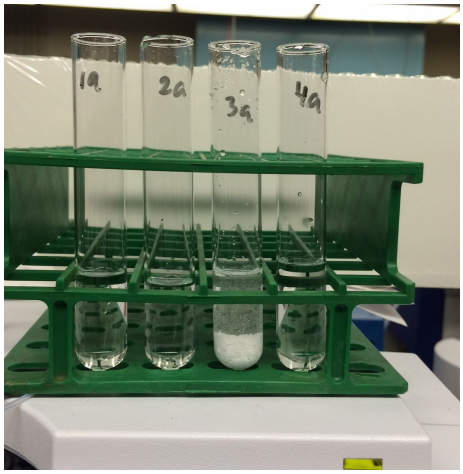
where 'o' means a clear solution, and 'x' means a precipitation; in that case it corresponds to the binary numbers 0010 (2), 0000 (0), 0001 (1), and 1000 (8).

Question: which chemicals should you use for solutions 1, 2, 3, 4, A, B, C, and D if you need to encode the number 2018?

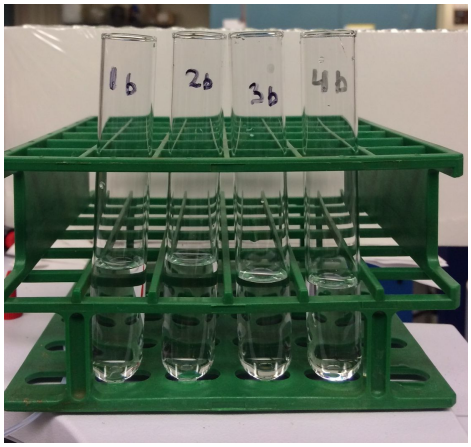
Solution: There are many solutions to this problem, a possible solution can be:
1 sodium sulfate, **2** water, **3** silver nitrate, **4** NaOH; **a** NaCl, **b** water, **c** magnesium acetate (or chlorate), **d** barium nitrate.
The products of these reactions are as follows (only precipitated solids are shown):

	Na_2SO_4	H_2O	$AgNO_3$	NaOH
NaCl:	1a	2a	3a AgCl	4a
H2O:	1b	2b	3b	4b
$Mg(CH_3COO)_2$:	1c	2c	3c	4c $Mg(OH)_2$
$Ba(NO_3)_2$:	1d $BaSO_4$	2d	3d	4d

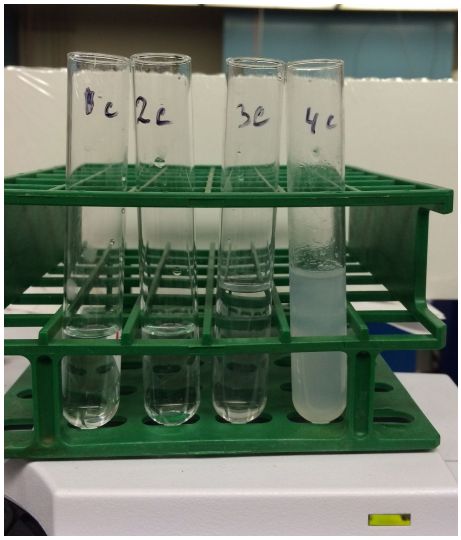
Below are the photos of each row taken separately (after the chemicals have been mixed):
Row A:



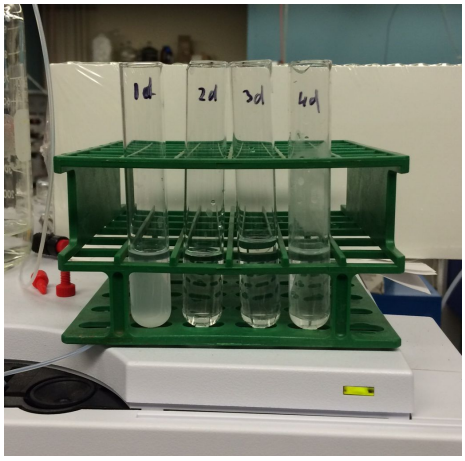
Row B:



Row C:



Row D:



If you provide a different solution, you will get a full score if your solution works.

BIOLOGY

5 points:

- *How did they happen to be so little?* – asked Dorothy. – *I never saw such small pigs before.*
- *They are from the Island of Teenty-Weent,* - said the Wizard, - *where everything is small, because it's a small island.*

This is the explanation that the Wizard of Oz gave Dorothy in L. Frank Baum's book. Is this explanation for the smaller pig size valid only in the author's made-up universe, or could an animal's size be different depending on whether or not that animal inhabits an island in the real world? If so, do you think that the Wizard's smaller island = smaller animals statement would always be true? Explain your answer and give examples.

Solution:

The phenomenon described in the "Wizard of Oz" really takes place, and it is called "insular dwarfism": many cases have been described when the body size of large mammals or birds decreased when their population was living in isolation of small islands. However, an opposite phenomenon, "insular gigantism" was also described: the latter occurs predominantly with small animals.

Several explanation have been proposed for these phenomenae, and the most plausible is as follows. Small island ecosystems are significantly different from the ecosystems of the mainland where the insular species originally formed. As a result, insular environment provides ecological niches that are different from the mainland's niches, and some of them may be unoccupied. For example, mainland elephants are big because their body size allows them to break and eat trees (while grass is being eaten by numerous antelopes and zebras) and to protect themselves from large predators. On small islands, elephants may encounter a situation when neither antelopes nor lions are present, and the overall amount of food is limited. That means smaller size elephants have an evolutionary advantage over larger animals (they need less food, and they don't need to protect themselves from lions), which leads to dwarfism.

On another hand, some species whose small size provides protection against continental predators and an access to certain resources (a typical example is small rodents) may benefit from the increase of body size when they live on a small island where their typical predators are absent, and the competition for food with other species is limited. In this case insular gigantism is observed.

One example of insular gigantism is Komodo dragon, the world's largest lizard. The most plausible mechanism of this gigantism is that these lizards occupied the ecological niche that is normally occupied by large mammalian carnivores (large cats, bears or wolves). The lack of competition with these predators created a situation when large body size leaded to obvious advantages (better ability to catch deer) with no disadvantages (in the absence of large predators Komodo dragons have no natural enemies).

10 points:

One night Dr. Z, the Chief Physician of the Crystal City Memorial Hospital came home very disappointed.

- *Today I spent all day collecting data from the archives of our hospital about patients who were diagnosed with cancer in the last 50 years.* - he told his wife, Mrs.Z and showed her a table:

Years	Oncology cases
1950s	88
1960s	87
1970s	101
1980s	479
1990s	2658
2000s	3012

- *As you know, up until mid 80's our Crystal City was a small town of hunters, surrounded by forests. But then they found minerals and built factories right next to the city. I am sure that pollution of the environment with industrial wastes is the reason for the growth in the number of cancer patients. I will demand the closure of factories, in order to save lives!* - said Dr. Z

Mrs. Z, who had also graduated from medical school, wasn't that certain of her husband's interpretation of these oncology data, and she gently offered him to discuss it the next day, so that she could do some research of her own beforehand.

What explanation(s) of the oncology data other than environmental pollution could she have in mind?

What additional data do you think Mrs. Z wanted to find on her own before discussing the topic with her husband?

Solution:

Mrs. Z was right in her assertion that the conclusion made by her husband was methodologically flawed. There were several problems with his conclusion, and the most important ones were:

1. Dr. Z operates with *absolute* numbers (the amount of cancer cases), whereas the correct conclusion can be done only when the *relative* numbers (the number of cancer cases per 1000 inhabitants) are considered.
2. Dr. Z discusses the number of oncology *cases*, i.e. the number of cases when cancer was diagnosed. One of the factors responsible for the apparent increase of cancer cases may be just an improvement in early diagnostic technique (e.g. introduction of MRI or PCR tests). In other words, if cancer is being detected at earlier stage, that may look like an apparent increase of cancer cases, although no actual increase occurs.
3. Dr. Z excluded demographic data from the analysis. The onset of cancer is more likely in older age, so even a simple increase of life expectancy means the cancer is diagnosed in larger number of people: in 50s, majority of people simply didn't live long enough to get cancer: they were dying earlier as a result of other diseases.
4. Dr. Z ignores the fact that the oncology statistics he refers to shows that the situation has a tendency to stabilise in the last decade: the increase of oncology cases is slowing down. If this is not connected with the city's depopulation (which is unlikely), that means the problem with environmental pollution has already been resolved (as a rule, environmental pollutants have a long lasting cumulative effect, so if the amount of new oncology cases is decreasing, that means currently the population is not being affected by pollutants).ted above)

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it. We use this site when we grade the solutions. Failure to run on this site will lead to automatic point reduction.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc.
  No .pdf, .doc, .docx, etc! Again, failure to follow these instructions will lead to point reduction.

5 points:

Write a program that given two substrings and a longer string determines whether the longer string contains both substrings in a non-overlapping way. For example, if "abc" and "bce" are the substrings, then "abce" fails, but "abcabce" works. Substrings and the string should be entered from input. You can use substring search function in your selected programming language (such as find() in Python and indexOf() in Java).

Solution:

algorithm:

find all occurrences of substring1 in long_string
for each one:
    split long_string at this location
    find substring2 in the left and right parts

Python:

```
def find_all(long_str, sub):
  start = 0
  while True:
    start = long_str.find(sub, start)
    if start == -1: return
    yield start
```

```
    start += len(sub) # use start += 1 to find overlapping matches

def find_2_sub(long_str, sub1, sub2):
  for start in find_all(long_str, sub1):
    left_part = long_str[:start]
    if sub2 in left_part:
      return True
    right_part = long_str[start+len(sub1):]
    if sub2 in right_part:
      return True
  return False

long_str = input("enter the long string: ")
sub1 = input("enter the first substring: ")
sub2 = input("enter the second substring: ")
res = find_2_sub(long_str, sub1, sub2)
print("'%s' and '%s' are%s found in '%s' (non-overlapping)" % (sub1, sub2, "" if res else " not", long_str))
```

Java

```
import java.util.Scanner;
public class Find2Sub {
  private static boolean find2sub(String strLong, String sub1, String sub2) {
    for(int idx = 0; (idx = strLong.indexOf(sub1, idx)) >= 0; idx++) {
      String leftPart = strLong.substring(0, idx);
      if(leftPart.contains(sub2))
        return true;
      String rightPart = strLong.substring(idx+sub1.length());
      if(rightPart.contains(sub2))
        return true;
    }
    return false;
  }

  public static void main(String[] args) {
    Scanner input = new Scanner(System.in);
    System.out.print("enter the long string: ");
    String strLong = input.nextLine();
    System.out.print("enter the first substring: ");
    String sub1 = input.nextLine();
    System.out.print("enter the second substring: ");
    String sub2 = input.nextLine();

    boolean res = find2sub(strLong, sub1, sub2);
    System.out.printf("'%s' and '%s' are%s found in '%s' (non-overlapping)\n", sub1, sub2, res ? "" : " not",
strLong);
  }
}
```

10 points:

You are given a dictionary (a set of strings, which your program should obtain from input), and then given a "test" string (also entered from input), which is likely a mistype of one of the dictionary strings. Your program should decide which of the dictionary strings was most likely mistyped. In addition to your code, please explain your approach; doing so in program comments is the best way. Points will be assigned for the code as well as for the the explanation.

Solution:

This is an open ended problem, so there is no one perfect solution. One approach to this problem is illustrated here:

`http://norvig.com/spell-correct.html`