PROBLEM OF THE MONTH

POM

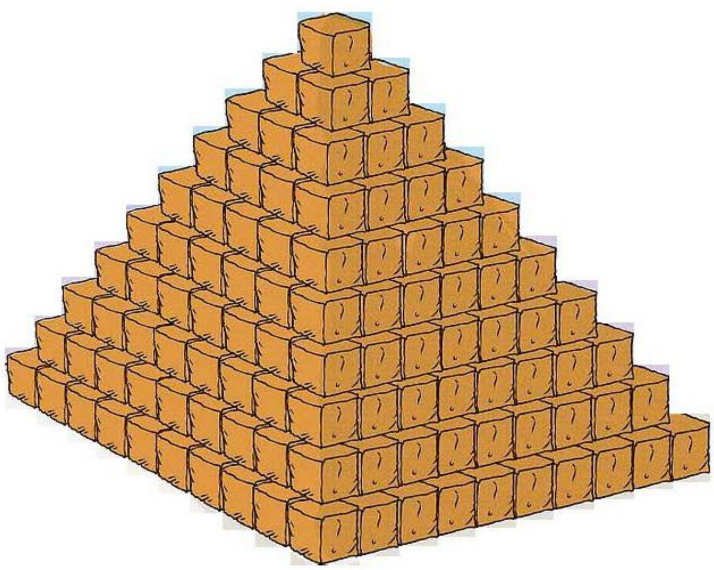
2018 - 2019

September, 2018

MATHEMATICS

5 points:

An ancient ruler has decided to build a Geometry Temple in the form of a square-base pyramid made up of cubes with a side of 1 meter (as illustrated below). The Temple has 100 layers, where the bottom layer is 100x100 meters and the top layer is a single cube. The surface of the Temple is supposed to be covered with gold. What is the total area of gold foil that is needed in order to accomplish this?



Hint: Consider projections of the pyramid from the top and from the sides.

10 points:

While designing the Geometry Temple, a square pyramid made of identical cubes (such as shown in Figure above), the ancient architect had to count the total number of cubes that were needed. In trying to do so for the large, multi-layer pyramid, (s)he discovered the following remarkable relation, which simplified the task:

$$1^2 + 2^2 + 3^2 + ... + n^2 = 1 \cdot n + 3 \cdot (n-1) + 5 \cdot (n-2) + 7 \cdot (n-3) + ... + (2n-1) \cdot 1$$

Prove this equality.

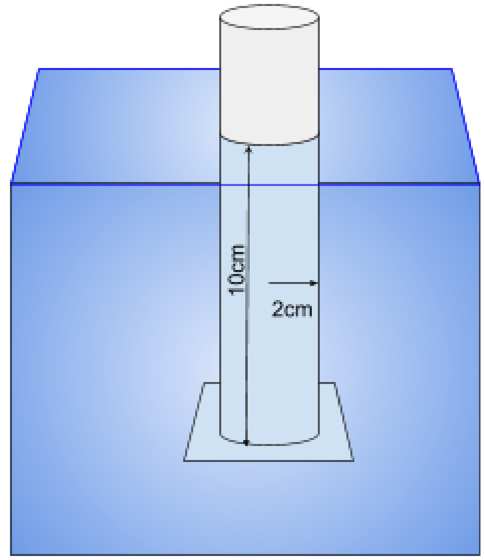
Hint:

As you probably noticed, the problem is about counting the number of cubes in the pyramid. The left part of the equality counts the number of cubes in the pyramid, layer-by-layer along the vertical direction, starting from the smallest 1-cube layer at the top, to the widest, 100-cube layer at the bottom. Obviously, widest-to-narrowest layer is not the only way to group the cubes for counting. Try to figure out whether a grouping exists that would yield the right-hand part.

PHYSICS

5 points:

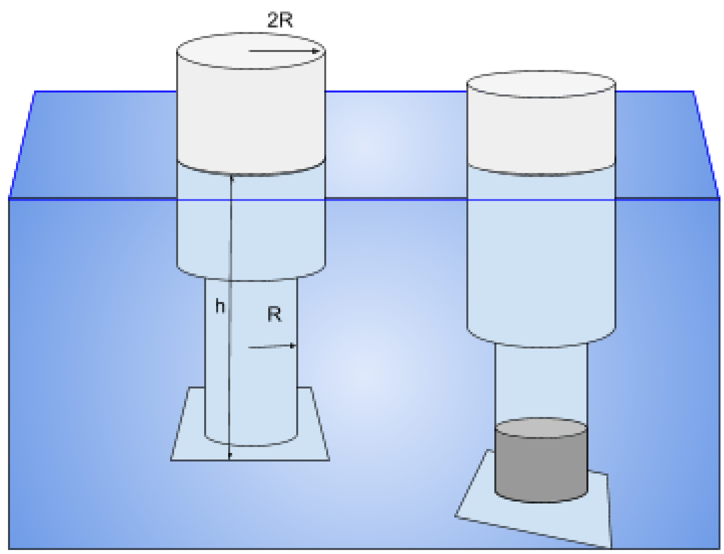
The bottom of a cylindrical vessel submerged in water is not attached (see the figure). Find the maximum mass of sand that can be poured into the vessel until the bottom falls off. Radius of cylinder is ***R=2 cm***. The vessel's position is fixed, with its bottom located at depth ***h=10 cm***. Neglect the mass of the bottom.



Hint: The bottom is kept in place because of the hydrostatic pressure. You can google (or derive) a formula for this pressure at depth ***h*.**

10 points:

A vessel is made of two connected cylindrical parts of radii ***R*** (lower) and ***2R*** (upper), as shown in figure. It freely floats in water, but the bottom of the lower cylinder is not attached. Initially, the bottom is at depth ***h***. The lower cylinder is being filled with sand until the bottom falls off. Find how deep in water the bottom of the vessel was right before this happened. Neglect the mass of the bottom.



Hint: There are two parts here. First, you can find how much deeper the vessel will move when sand of mass ***M*** is added (use Archimedes principle). Second, similarly to 5 pt. Problem, you can find the hydrostatic pressure near the bottom of the vessel and relate it to the mass (***M*** at the moment when the bottom falls off, the weight of the send becomes slightly bigger that the hydrostatic force which pushes the bottom upward).

CHEMISTRY

5 points:

Alice, a college faculty, asked Bob, her technician, to prepare 1 M solution of CsCl for tomorrow experiments. Next day, when Alice started to use the solution prepared by Bob, she noticed something is wrong with it. “Bob, how did you prepare this solution?” - she asked. “Alice, there was no cesium chloride in the lab, so I decided to prepare the solution from available chemicals. I took one kilogram of 1 M HCl solution and one kilogram of 1 M CsOH solution and mixed them together.” “Oh, now I see”, - Alice said. “We definitely cannot use this solution and should probably make another solution.”
Can you explain why the solution prepared by Bob is not good, which mistakes had he made, and how could did Bob fix them?

Hint:

Bob made at least three mistakes. The least obvious is that some water forms in a reaction between CsOH and HCl. Another mistake is that he mixed solutions by weight, whereas 1M means “one mole of a substance per 1 *liter* of a solution”. Try to figure out the last mistake. If you were Alice, and you decided to fix the Bob’s error by adding some chemicals to the solution he prepared, what additional information would you have to know?

10 points:

"*We had two bags of copper chloride, seventy-five ounces of aluminum pellets, five kilograms of high purity citric acid, a saltshaker half-full of mercuric chloride, and a whole galaxy of multi-colored pH papers, rubber balloons, strings etc... Also, a quart of isopropanol, a quart of acetone, a case of Poland Spring water, a pint of raw ether, and two dozen grams of isoamyl alcohol. Not that we needed all that for an air trip, but once you get locked into a serious chemicals collection*, *the tendency is to push it as far as you can. The only thing that really worried me was the ether. There is nothing in the world more helpless and irresponsible and depraved than a man in the depths of an ether binge, and I knew we'd get into that rotten stuff pretty soon.*"
Using the stuff described in this quote, can you launch your iPhone to the sky? Which items listed there are needed for that, and how will you do that?

Hint:

Obviously, the only way to do that using the items from this set is to prepare hydrogen. Usually, if you have some metal and some acid, the idea that comes first is to mix them. However, that will not work in that case: citric acid is too weak to react with aluminium. In addition, aluminium is covered with a thin but very stable film of aluminium oxide that makes it very stable. However, if you find a way to peel this film off the aluminium’s surface, it will react even with water, and a lot of hydrogen will form. The only thing you need to do is to figure out how can this aluminium oxide film be removed.

BIOLOGY

5 points:

During a study of some exotic ecosystem, a group of biologists identified three species (A, B, and C) that normally coexist in this ecosystem. All three species are essential components of the ecosystem. To identify their role, the researchers created an artificial ecosystem that was composed of these three species, and made the following observations:

- Light is essential for a normal growth of this ecosystem;

- Removal of 90% of A from the ecosystem leads to a sharp decrease of the population of B and an increase of the population of C;

- Removal of 90% of B from the ecosystem leads to a sharp increase of the population of A and decrease the population of C;

- Removal of 90% of C from the ecosystem leads to an increase in the population of A and an increase of the population of B.

Only short-term effects were measured in these experiments.

Based on these data, propose a possible architecture of the food chain in this ecosystem and guess what type of organisms the species A, B, and C are.

Hint:

Assume that the only ways of interaction between these three species in this ecosystem are competition and predator-prey relationship.

10 points:

A research ship *HMS Beagle-3* arrived at a tropical archipelago *Larva-y-Escarabajo*, where professor Gaze discovered two new amphibia species. They looked like a newt or salamander, both of them were of the pretty much the same size, but one species had a pale yellow color, whereas the second one was dark-brown. Prof. Gaze found that both species prefer to lay eggs in the small pond, and, although the pale-yellow spice preferred to spend more time in the water than the brown one, he suggested these two amphibia may compete with each other. "If this is so," - argued the ecologist, - "the reduction in the number of pale-yellow newts will improve the living conditions for the brown one, and their population will increase."

Gaze arranged many traps along the island. He released the all brown newts that were caught, collected the pale-yellow ones and released them on another island of the archipelago. Thus, he managed to reduce the population of pale yellow newts by 90 percent.

When a year later, Gaze returned to the island, he was surprised to find that the number of brown newts did not increase, but it decreased significantly.

How can you explain this apparent contradiction of Prof. Gaze's hypothesis and his subsequent observation?

Hint:

What if these two newts are not two different species? Obviously, the idea about a sexual dimorphism is too straightforward, and prof. Gaze checked this hypothesis first, and ruled it out. Do you know other examples (especially in amphibia) when animals belonging to the same species look very differently?

COMPUTER SCIENCE

- **Your program should be written in Java or Python**
- You can write and compile your code here:
  http://www.tutorialspoint.com/codingground.htm
  Please note that *codingground* site modified its structure and now all the input for the program run is entered on a separate tab. This is convenient as the same input can be used across multiple runs without re-entry
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc.
  No .pdf, .doc, .docx, etc!

Common introduction:

Sigma Kingdom has N cities connected by some roads. Your program will receive a map of Sigma Kingdom on input. There, first a number of lines in the map will be provided, followed by that number of lines, containing the following characters:

- Letters A to Z indicate locations of cities (Sigma is a small kingdom, and it can not have more than 26 cities)

- Roads in Sigma Kingdom go strictly horizontally, indicated by character **-**; vertically, indicated by character **|**; or diagonally, indicated by characters **/** and ****. All the roads in Sigma Kingdom connect exactly two cities. There are no roads going from or to nowhere: there is always a city at each end of the road.

- Some roads may change the direction by going through a junction. Junctions, which never overlap, are indicated by character +.

Here are some valid roads:

```
A--B
| / \
```

```
  |/   \
  C     +--D
```

In this example A is connected to B and C, C is connected to A and B, and B is connected to A, C and D.

Roads can cross, one going over the other, but they cannot go over or under a city. This is a valid map:

```
    B
 A  |
  \ |
   \|C
    \|
H---|\----D
    ||\
    || \
    G|  \
     |   E
     F
```

Junctions cannot be located at a point of intersection of 2 or more roads.

This is also valid – A and B are connected:

```
    D  F  H  J
  A-|\-|\-|\-|-B
    | \| \| \|
    C  E  G  I
```

Junctions cannot touch anything but two road segment they join: they cannot touch cities or other roads. Thus, this is not valid:

```
    BC
A   ||
  \ ||
   +||--D
    ||
    FE
```

but this is fine:

```
     BC
A    ||
  \  ||
   +-||--D
     ||
     FE
```

5 points:

Based on the map received by your program from input, figure out and output which of the cities in Sigma Kingdom is the most connected one (has most of the roads going to/from it).

Hint:

Count the number of roads (-, |, /, \) immediately surrounding a town.

10 points:

Given the map of Sigma Kingdom, which your program will get on input, print out connectivity table for Sigma Kingdom. This square table will have all Sigma cities as the names of rows and columns. Then each (i,j) cell would contain 1 if *i*-th city is connected to *j*-th city and 0 otherwise. For example, the correct output for the first example above would be:

```
  ABCD
A 0110
B 1011
C 1100
D 0100
```

Hint:

Starting from each town, follow each road emanating from it by keeping the direction until you reach another town or a junction. If you reach a junction, find a road coming out of it distinct from the road you arrived from, then keep moving in the new direction until you reach a town or another junction. When you reach a town, record that there is a connection between your starting town and the destination town in your connectivity table.