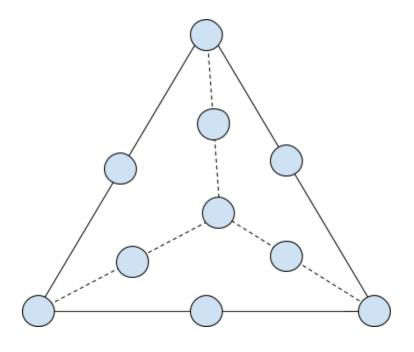# MATHEMATICS

5 points:

There are 10 small spheres on a tetrahedron, as shown below. Is it possible to label each sphere with a different digit from 0 to 9 such that the sum of the three digits on each of the six edges is the same?

Hint: Try to compare the total sum of the digits and the total sum of the edges.

10 points

Alex cut a rectangular sheet of paper along a straight line. Then, he cut one of the two resulting pieces along a straight line. Then, he did the same to one of the resulting three pieces, and so on. Prove that after enough such cuts it will be possible to find, among the resulting pieces of paper, 100 polygons with the same number of vertices (for example, 100 triangles, or 100 quadrilaterals, etc).

Hint: In order to make a polygon with one more side than the original, it is necessary to produce a triangle. For example, to create a pentagon by cutting a quadrilateral in two pieces, the second piece must be a triangle.

PHYSICS

5 points:

The United States one-cent coins aka pennies, minted before and after 1982 are made of substantially different materials. Design and carry out an experiment in which you measure one physical property of the material from which pennies of each type are made to establish that the materials are indeed different. A physical property of the material does not include the variation in the shape of coins and their appearance due to wear and tear. Examples are density, heat capacity, thermal expansion, electrical conductivity.

Hint: The easiest experiment is to measure density. You may use several coins of the same type to improve accuracy.

10 points:

A ball is dropped with no initial speed from the height h above the plate. The plate is moving upward with constant speed u. Find the time interval between subsequent collisions of the ball with the plate. Neglect air resistance and assume that collisions are absolutely elastic. Acceleration of gravity is g.

Hint: Switch to the reference frame of the plate.

CHEMISTRY

5 points

During Sigma2019, in Fire Chemistry semilab (http://sigmacamp.org/2019/semilabs/fire) , Mark decided to prepare an aqueous solution of silver nitrate ($AgNO_3$). To this end, he asked Luigia, his assistant, to bring some water. He poured water into a test tube and added few crystals of silver nitrate. He expected to obtain a clear solution, but to his surprize, he got the cloudy solution shown in this photo:

https://photos.sigmacamp.org/2019/Day-5-August-15/i-wQLgCNm/A

(the tube on the right, the one Tatiana is holding). He immediately realized his mistake, and asked Luigia: "Please, bring me XYZ water". After Luigia did what Mark asked, he prepared another solution, and it was clear and colorless, as he expected (this solution is shown in the test tube on the left, the one Sophia is holding).
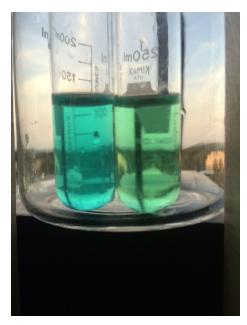
What was the word "XYZ", and why did Mark initially get a cloudy solution?

Hint:

Tap water, especially in Connecticut area, has some admixtures.

10 points:

Aqueous solutions of both cupric chloride ($CuCl_2$) and nickel chloride ($NiCl_2$) are green, and it is not easy to discriminate them based on their color (see the photo below):

Suppose you have some green solution, and you are not completely sure if it is copper or nickel chloride. How can you figure out what it is if, besides that unknown solution and distilled water, you have just test tubes, and you can spend for chemicals:
- just $7?
- just $11?
- just $23?
(shipping cost is not included)
Explain your answer bearing in mind that you are only allowed to use chemicals that are sold by regular suppliers to ordinary customers.

Hint:

There are two possibilities to check that: to add some chemical that may produce different color with nickel and copper ions, and to capitalise of the fact that copper and nickel have different electrochemical properties (see metal activity series)

BIOLOGY

5 points

Recent comparative studies of wolves and dogs demonstrated that dogs have a special muscle raising the inner eyebrow, whereas this muscle is almost absent in wolves. Is that difference just a collateral effect of domestication, or this muscle plays some important role? Explain your answer.

Hint:

Some dogs look more cute than others. Try to think why.

10 points:

Bacteria demonstrate extreme genetic plasticity. One of the important mechanisms responsible for such plasticity is horizontal gene transfer (HGT): a bacterium can transfer a piece of its genome, in the form of a relatively short DNA, to another bacterium (-a). For example, a typical *E. coli* genome contains approximately 5,000 protein-coding genes, whereas the *E. coli*
 pan-genome (all the genes in all the strains) is estimated to contain about 16,000 genes. In other words, a bacterial strain that has no gene, which might be needed in a specific situation, can borrow such a gene from another strain (and lose it later, when not needed). This mechanism has numerous evolutionary advantages, allowing, for example, a quick acquisition of antibiotic resistance. Nevertheless, it is virtually absent (or very rare) in eukaryotes. Please explain why eukaryotes do not use HGT for survival as extensively as bacteria do?

Hint:

In contrast to us, bacteria are single-cellular organisms; in addition, their genome has a single set of genes, which, as a rule are combined in a single chromosome.

COMPUTER SCIENCE

- You can write and compile your code here:
  http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- **Please make sure that the code compiles and runs on** http://www.tutorialspoint.com/codingground.htm **before submitting it.**
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc.
  No .pdf, .doc, .docx, etc!

5 points:

Your program will determine whether a set of given positive integers are consecutive elements of the Fibonacci sequence (see https://en.wikipedia.org/wiki/Fibonacci_number).
The program should receive a list of positive integers on input. The numbers will not necessarily be presented in a sorted order. The program should print whether they are consecutive Fibonacci numbers or not, and, if yes, print them in sequential order.

Example 1:
```
Input:  5 13 8 21
Output: YES 5 8 13 21
```

Example 2:
```
Input:  5 13 8 22
Output: NO
```

Hint:

Don't forget that Fibonacci sequence starts with 0, 1, 1…

10 points:

Your program will determine whether a given positive integer can be written as a sum of consecutive elements of the Fibonacci sequence (see https://en.wikipedia.org/wiki/Fibonacci_number).

The program should receive a single positive integer on input. It should print whether the number can be written as a sum of consecutive Fibonacci numbers, and, if yes, print the sequence.

Example 1:
```
Input:   47
Output: YES 5 8 13 21
```

Example 2:
```
Input:   48
Output: NO
```

Hint:

If a positive integer is a sum of some number of sequential elements of the Fibonacci sequence, the largest number in that sequence can not be larger than the positive integer itself (and can only be equal in one trivial case of 0+1=1).

LINGUISTICS

5 points:
Consider the following singular and plural forms from an Iranian language:

singular	translation	plural
tapus	'hawk'	tapusān
kres	'crack'	kresehār
pastuwer	'rain'	pastuweruna
tabak	'dish'	tabakuna
tingun	'tenacity'	tingununa
kašak	'guard'	kašakān
xer	'snore'	xerehār
lāmbuzen	'swimmer'	lāmbuzenān
uš	'camel'	ušān
tox	'cough'	toxehār
zum	'son-in-law'	zumān
čāl	'swing'	čāluna
šrak	'whistle'	šrakehār
rap	'second'	rapuna
grab	'clomping'	grabehār
meč	'fly'	mečān

Explain how plural is formed and form the plurals of the following nouns:

dār	'gallows'
dzaz	'hissing'
až	'bear'
māmuriat	'trip'
totkamār	'wizard'

10 points:

Consider the following numbers in one of the Uralic languages (spoken in Siberia). Note that the order of translations does not match the order of original phrases.

sompylasar ɛj šitty, muktyssar ɛj ukkyr, sompylasar ɛj sompyla, šittysar, ukkyr ca muktyssar, šitty ca tɛ¯sar, sompylasar ɛj sel'cy, ukkyr ca tōn

20, 38, 52, 55, 57, 59, 61, 99

Translate into this language the following numbers:
41, 48, 77, 98