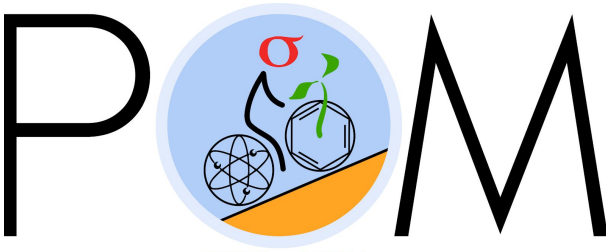
PROBLEM OF THE MONTH



September, 2020

2020 - 2021

MATHEMATICS

5 points:

Consider an infinite 3-dimensional cubic lattice with black nodes at the vertices of the cubes and in the center of every face and white nodes on the midpoint of every edge. What is the ratio of the number of black nodes to white nodes?

10 points:

A cube with an integer side *n* is divided using barriers into 1x1x1 cubes. How many barriers do you need to break down in order to be able to get from any 1x1x1 cube to the edge?
(A single barrier is one face of a 1x1x1 cube)

PHYSICS

5 points: An SUV weighing **2000 kg** and moving with a speed of **15 m/s** fails to stop at a stop sign on an intersection. It hits a sedan weighing **1000 kg** and moving in a perpendicular direction at a speed of **21 m/s**. The collision is completely ***inelastic***: cars stick and continue moving together. How far from the collision point will they come to rest? The coefficient of kinetic friction between the rubber tires and the asphalt is $\mu = 0.7$ (assume that both drivers were applying brakes at the moment of the collision).

10 points: A sedan of mass ***m*** collides with an SUV of mass ***M*** that stopped at an intersection. The collision between the cars is ***neither purely elastic nor inelastic***. The SUV is pushed forward by distance ***D***, while the sedan comes to a stop after moving by distance ***d<D*** from the collision point, in the same direction. What was the speed of the sedan prior to the collision? The coefficient of kinetic friction between the rubber tires and the asphalt is μ (both drivers were applying brakes at the moment of the collision).

CHEMISTRY

This month, the topic is: Dynamic equilibrium in Chemistry.

IMPORTANT! In this PoM season, we do an experiment: each month, an online lecture will be given. This lecture may be helpful for those who want to solve Chemistry PoMs, although it is not supposed to provide direct hints.
This month's lecture takes place at 10:30 am on Sunday, September 27. The Zoom invitation is below:

https://us02web.zoom.us/j/4817690592?pwd=T2djSjRETEpDSHFZdWJpYlBTYzdjQT09
Meeting ID: 481 769 0592
Passcode: 879615

During and after the lecture, you will have an opportunity to ask general questions on the topic. The lecture will be recorded, and it can be found at our youtube channel.

In addition, the solutions will be announced via Zoom too, so you will be able to ask questions if you want. The exact date will be announced later. Check our website for that.

5 points:

When you add a powdered soluble substance, e.g. table salt, to water, the solid dissolves, but if you add too much of it, a saturated solution forms, and the rest of the solid does not dissolve anymore. If you leave that mixture in a closed vessel for several weeks, you will see that, although the total amount of the solid hasn't changed, there is a significant change in the particles' appearance: instead of a large number of small grains, one or several large crystals form. Why does it happen?

10 points:

Hardness of fresh or tap water is a property that has no relation to real hardness. It indicates the amount of dissolved salts, usually hydrocarbons and sulfates of calcium and magnesium. When tap water is boiled or heated, calcium salts may precipitate in a form of calcium carbonate ($CaCO_3$) or, more rarely, calcium sulfate ($CaSO_4$). That results in formation of solid deposits on the inner surface of teapots, boilers, and other equipment. For removal of carbonate deposits, a treatment with some acid, such as acetic acid, dilute hydrochloric acid, or citric acid, can be helpful, because solid calcium carbonate participates in an exchange reaction:

$CaCO_3 + 2HCl \rightleftarrows CaCl_2 + H_2CO_3$

One of the products, carbonic acid, is unstable, it decomposes into water and carbon dioxide (a gas), which escapes to the atmosphere, so the equilibrium shifts to the right side, and all insoluble calcium carbonate converts to the soluble calcium chloride ($CaCl_2$).
That approach does not work for calcium sulfate deposits. Sulfuric acid is not volatile, so if you add HCl to calcium sulfate:

$CaSO_4 + 2HCl \rightleftarrows CaCl_2 + H_2SO_4$

sulfuric acid stays in the solution, and the equilibrium remains shifted to the left side, so no conversion of low soluble calcium sulfate into soluble calcium chloride occurs.

Propose the process that would allow conversion of calcium sulfate deposits into something soluble. How many steps are needed for that?

BIOLOGY

5 points:

Groups of organisms often possess what is known as “collective intelligence,” in which flocks and swarms are able to make optimal “decisions” as a group that far exceed the capabilities of any one of its members. Assuming that ants are identical, how do they manage collective transport of objects several times their sizes--so large, in fact, that most ants are incapable of seeing where they’re going? How would behavior change as a function of the size of the swarm?

10 points:

In May 2020, the Environmental Protection Agency (EPA) approved release of 750 million genetically engineered mosquitoes in Florida Keys. The mutant mosquito are expected to mate with local, non-mutant mosquitoes to produce nonviable female offspring, which is expected to cause nearly complete eradication of the mosquito population.

- What was the reason for introducing the mutation that kills only females, but not males?

- In which chromosome such lethal mutation should be placed to produce a desired effect, and how all of that works?

- What potential effects (positive and negative) eradication of mosquitoes will have on the ecosystem? Which species of the Florida Keys ecosystem will be most endangered ?

LINGUISTICS

5 points:

Some of the following words from a language spoken in Siberia are translated below (in a random order).

vörny, vörz'yny, vörz'ödny, vörödyštny, vörödny, padmyny, padmödny, lebz'yny, lebny, gazhödyštny, gazhodny, seiny, seiyštny

to move oneself, to make someone late, to eat a little bit, to move someone, to be late, to move someone a little, to have fun, to start moving oneself, to start flying

Determine which translation corresponds to which word and translate the rest of the words. Explain how you reached your conclusion.

10 points:

The phrases below from a language spoken in southeast Asia have the given translations in random order.

sampeyan murid adhi aku
murid adhi kanca sampeyan
adhi aku kanca murid sampeyan
aku murid kanca sampeyan
adhi aku murid sampeyan

My younger brother is a friend of your student.
You are the student of my younger brother.
the student of the younger brother of your friend
My younger brother is your student.
I'm the student of your friend.

a. Establish the translations of the words and describe the structure of such sentences from this language. Explain how you reached your answer.
b. According to what you discover, what is another possible translation of the 2nd phrase?

COMPUTER SCIENCE

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files with specified in the problem names
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.
  No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!

Introduction:

Mark decided to digitize his library. He has ***n*** books on his shelf. ***k*** counselors volunteered to help him scan all the books. (You can presume that k <= n). Mark is very particular in the order the books appear on the shelf at any given moment, and therefore counselors are instructed not to mix or move the books, and each counselor can only scan a set of consecutive books.

Your program should read the input file **input.txt**, which consists of 2 rows. The first row contains space-separated values of *n* and *k*. The second row contains n space-separated integers representing the size of each of the books (i.e., number of pages). Example input file:

```
10 3
50 100 75 66 350 254 39 111 205 321
```

You can assume that all books have a positive integer number of pages.

5 points:

Your program should calculate the minimum and maximum number of pages a counselor may need to scan considering all possible divisions of labor.

The program should produce output file **output.txt**, which consists of 2 space-separated numbers - minimum and maximum. For example above, the file will contain:

```
39 1421
```

10 points:

Your program should find the ***fairest*** division of labor (partition of the shelf into k sets of consecutive books). Fairest is defined as a division with the minimum of the total number of pages the busiest counselor (i.e. the counselor who will have to scan the most of the pages) will have to scan.

The program should produce output file **output.txt**, which consists of k lines. Each line *i* will start with the number of pages the *i*-th counselor has to scan, followed by a space, dash, space and then space-separated list of book sizes comprising that total.
For example, for an input file:

```
6 2
10 20 30 40 50 60
```

the output file should contain:

```
100 - 10 20 30 40
110 - 50 60
```