PROBLEM OF THE MONTH

POM

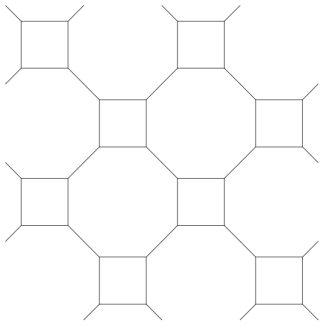
2021 - 2022

September, 2021

MATHEMATICS

5 points: A grasshopper starts at the origin and is jumping along a line. For each jump it can choose either direction. First, it jumps 1cm, then 2cm, then 3cm, and so on. Is it possible that after 2021 jumps, the grasshopper ends up exactly at the origin? What is the minimum number of jumps larger than 2021 the grasshopper can take, starting from the origin and jumping in the manner described, to land on the origin again?

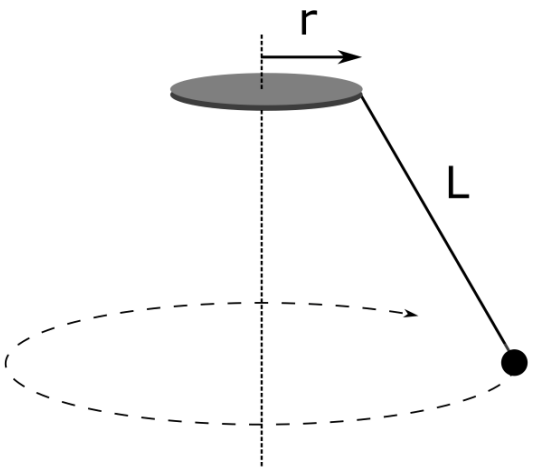
10 points: Consider the following infinite two-dimensional lattice made out of regular octagons and squares. What is the average area per vertex for this lattice if all edges of the lattice have the same length *a*?



PHYSICS

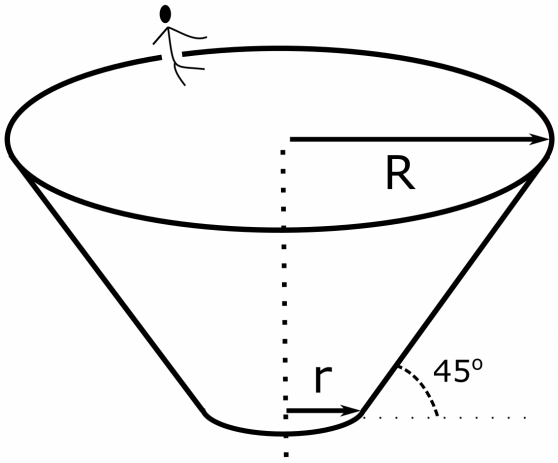
5 points:

A weight of mass 1kg is hanging from the edge of a disk on a thin thread. The breaking force of the thread is 20 N. The disk can spin on its axis. The disk begins stationary, and then slowly starts to accelerate. To what height with respect to its initial position will the mass rise before the string breaks?



10 points:

A water park visitor gets onto a funnel-shaped water slide with a slope of 45 degrees, top opening *R = 5 m*, and bottom opening *r = 1 m*. The rider launches themselves on the water slide horizontally with the speed *v = 5 km/hr*. The slide is wet, so there is no friction. What will be the rider's speed at the exit of the funnel slide?



CHEMISTRY

5 points:

Emelianiya huxleyi is a tiny unicellular marine algae. This organism is unique because it is arguably the only species whose life cycle directly affects the Earth's climate. These algae produce massive seasonal blooms that affect atmospheric carbon (CO_2) due to two planetary scale processes. The first process is the binding of the atmospheric CO_2 due to photosynthesis. The second process is the conversion of hydrocarbonate ion (HCO_3^-) dissolved in water into insoluble calcium carbonate. The latter is used by algae for building the algal carbonate shell, and it precipitates when the algae die. Formation of calcium carbonate can be illustrated using the following (incomplete) scheme:

$$HCO_3^- + Ca^{2+} \rightarrow CaCO_3$$

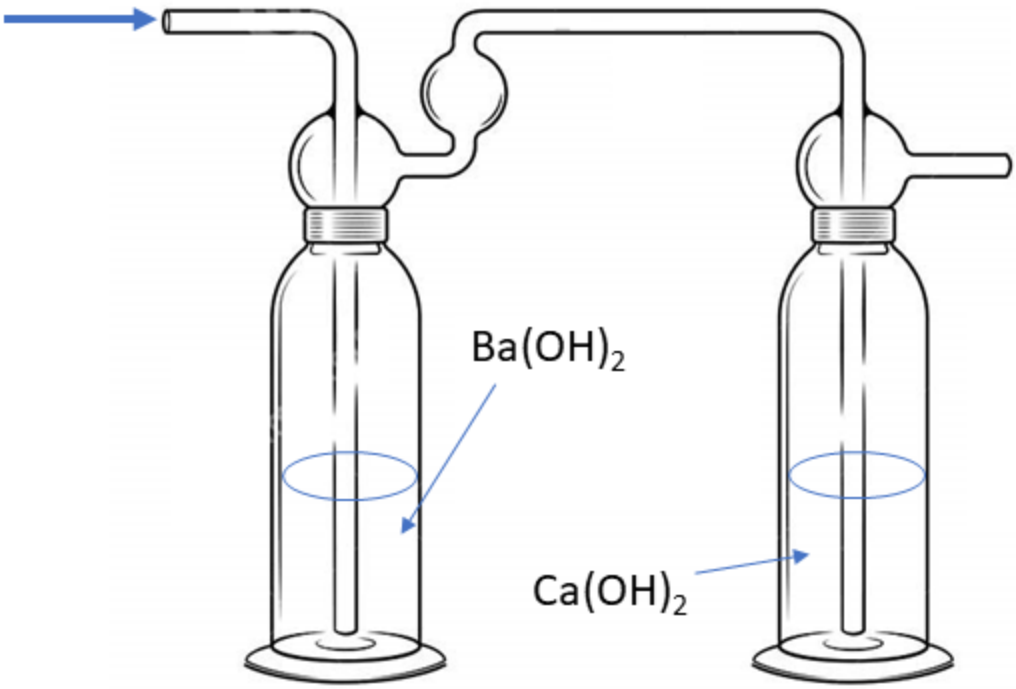
Please, complete the equation and tell how this process affects the overall carbon balance in the atmosphere, i.e. how the amount of atmospheric CO_2 changes per one molecule of calcium carbonate formed.

10 points:

Two Drexel's flasks were connected sequentially (as shown on the figure below), the first one was filled with 40 mL of saturated $Ba(OH)_2$ solution, the second one was filled with 40 mL of saturated $Ca(OH)_2$ solution (the temperature was 20°). Lesha, who was walking by, blew bubbles through them (the direction is shown with a blue arrow), and white precipitation quickly formed in the first flask, whereas the liquid in the second flask remained clear. Zhenya collected the precipitate by filtration, dried it, and its weight appeared to be 197 mg.

Next day, Zhenya prepared fresh solutions, but poured $Ca(OH)_2$ into the first flask, and $Ba(OH)_2$ into the second one. Lesha, who again was walking by, decided to repeat his trick, but precipitation was observed in both flasks. The weight of the two precipitates was 94 and 12 mg, accordingly.

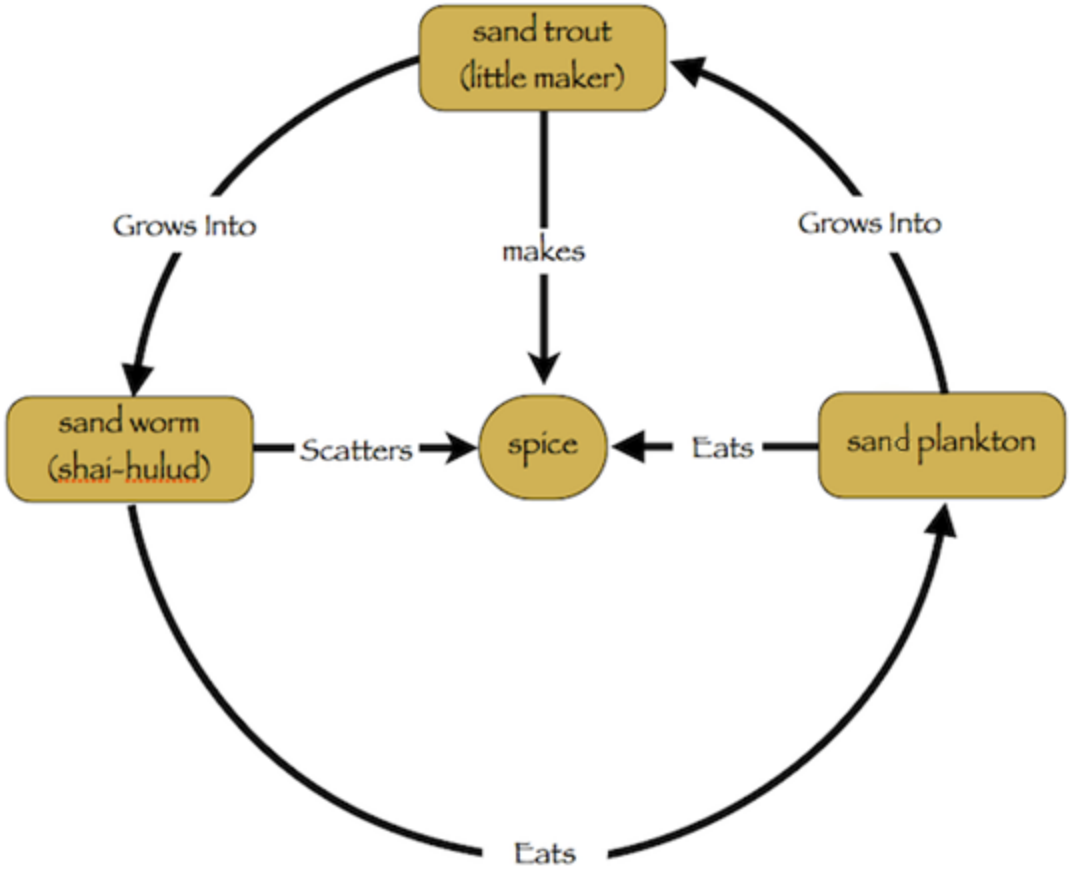
Assuming that the amount of CO_2 exhaled by Lesha was the same both times, how much of CO_2 does he exhale?


Ba(OH)2
Ca(OH)2

BIOLOGY

5 points:

The planet Arrakis described in the *Dune* novel series is populated by giant sandworms, which play a key role in the planet's ecosystem. The life cycle of Dune's sandworms constitutes a complex ecological feedback system, and different sources provide different versions. One of the summaries is as follows:

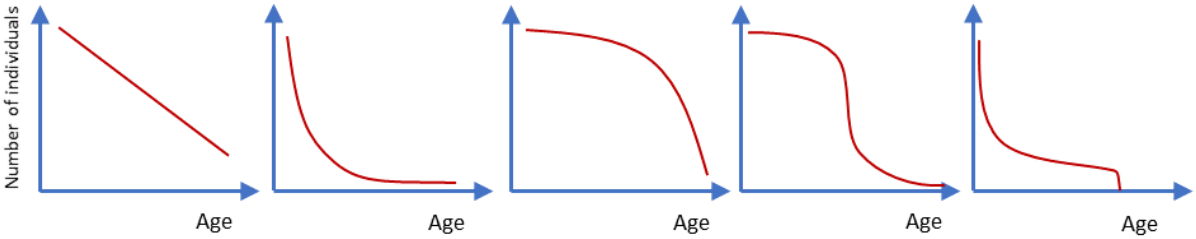


During its development, a sandworm passes several phases (Fig.1). Sandworm larvae develop from "sand plankton", which feed upon spice scattered by sandworms. Sandtrout, which are flat and rhombus shaped creatures, find underground water and bind to each other to form "living cisterns" beneath the surface of Arrakis. By doing that, they make the planet dry enough to make the existence of sandworms possible. In those "living cisterns", sandtrout converts water into a mixture known as pre-spice mass and a large amount of CO_2. This causes gigantic explosions, where a huge amount of pre-spice mass is moved closer to the planet surface and the majority of sandtrout are killed. A small amount of survived sandtrout form cysts, which, after a 6-year hibernation period, yields a pre-sandworm form. The latter may grow into a full-size

sandworm, which, like whales, are travelling across the desert sands and devouring sand plankton.
Please answer if the above described ecological system can exist, at least, theoretically. If you think this description is incomplete, explain what is missing from that description. If you think this life cycle is impossible in principle, explain why.
In your explanations, please ignore biochemistry related aspects, focus on ecology.

10 points:

A group of researchers monitored the population of several species on a large island X. They obtained the following plots for five different species A, B, C, D, and E (Figs A-E), where population density is plotted vs age. For the convenience of comparison, the data are shown in relative figures.



A B C D E

By looking at the plots, answer the following questions and explain your answers:
1. Which of those species are closer to the top of the food chain pyramid?
2. If few representatives of one of those species will be transferred to an island Y with a similar ecosystem, which of those species is more likely to demonstrate fast and explosive growth?
3. Which of the species A-E is more likely to care for their offspring?
4. Which of those species is more likely to be a mammalian and which is an arthropod?
5. Which of those curves are likely to be a result of incorrect measurements?

COMPUTER SCIENCE

- Your program should be written in Java or Python-3
- No GUI should be used in your program: eg., easy gui in Python
- All the input and output should be via files named as specified in the problem statement
- Java programs should be submitted in a file with extension .java; Python-3 programs should be submitted in a file with extension .py.
  No .txt, .dat, .pdf, .doc, .docx, etc. Programs submitted in incorrect format will not receive any points!

According to the famous myth, inhabitants of the Tower of Babel spoke many different languages. Some spoke multiple languages, some spoke just one. For example, Anatu spoke Akkadian and Aramaic, Nergal spoke Aramaic and Moabite, Nuska spoke Moabite and Palmyrene, and Tiamat spoke just Palmyrene. This allowed Anatu, Nergal, Nuska and Tiamat to communicate as there was always a translation path between any of them: indirect communication is possible if there are pairs of people who can use the same language, who can relay information along the path between the individuals. Your task will be to identify the largest groups of people capable of communicating with each other.

Your program will receive an input file named **input.txt,** which contains the data on what languages the inhabitants of the Tower of Babel can speak. Each line of the file contains a name of the inhabitant followed by a list of languages he/she speaks, all in the comma-separated format. For example:

```
Name_1, language_1, language_3
Name_109, language_1, language_31, language_10
Name_2, language_11, language_33, language_10
Name_3, language_11, language_33, language_10
Name_4, language_2, language_3
```

5 points:

Find up to 2 languages that allow the largest group of people to communicate with each other.

Your program should produce an output file named **output.txt** that on the first line lists the 2 languages that allow communication between the largest number of people (in comma-separated format), and on the second line lists the names of all individuals who can

communicate with each other using these 2 languages (also in comma-separated format). Both lists should be sorted in ascending order.

In the example above, the combination of language_1 and language_10 lets 4 people to communicate with each other, making the output:

```
language_1, language_10
Name_1, Name_109, Name_2, Name_3
```

Here, Name_2 and Name_3 both can speak to Name_1 through Name_109.

10 points:

Find up the largest group of people capable of communicating with each other.

Your program should produce an output file named **output.txt** that one the first line lists the set of languages that enables the communication within the group (in comma-separated format), and on the second line lists the names of all individuals who can communicate with each other (also in comma-separated format). Both lists should be sorted in ascending order.

In the example above, the combination of language_1, language_3 and language_10 lets all people communicate with each other, making the output:

```
language_1, language_10, language_3
Name_1, Name_109, Name_2, Name_3, Name_4
```

Here, Name_2 and Name_3 both can speak to Name_1 through Name_109, and Name_4 is connected to the group via Name_1.