PROBLEM OF THE MONTH

December 2013

2013-2014

MATHEMATICS

Two bricks, weighing 6 kg and 12 kg, are made of tin alloys with different percentage tin content. Someone cuts off a piece of mass m from each brick. Each piece is then melted together with the remainder of the other brick, and a new brick is then cast from the resulting material. The percentage content of tin in the new bricks is equal. Find the mass m.

Solution

Let us assume that the content of lead (fraction) in 6 kg and 12 kg bricks are x and y, respectively. Then the content of lead in 12 kg brick after re-casting is $[(12-m)\,y+mx]/12$. Indeed, the amount of lead in the brick after the piece m was cut off is $(12-m)\,y$ and the amount of lead in the joined piece of mass m is mx.

Similarly, the content of lead in 6 kg brick after re-casting is $[(6-m)\,x+my]/6$.

We have equation $[(12-m)\,y+mx]/12\ =[(6-m)\,x+my]/6$ or after simplification
$x-y=m(x-y)/4$.

As according to the problem x and y are different we can cancel $(x-y)$ and obtain $m=4\,kg$ as a final answer.

PHYSICS

Using as little equipment as you can, determine the density of water ice in your freezer relative to the density of water in your tap. Please, try using only items that would be available in the 19th century (e. g. no electronic kitchen scales; freezer is OK, in the 19th century you would do that in winter time).

This is an experimental problem. You have to design the measurement, optimize the conditions for your experiment to guarantee the best accuracy possible, and perform the measurement. Describe your experiment in sufficient detail so that it can be reproduced by a person grading your work and estimate the accuracy of your measurement.

Solution

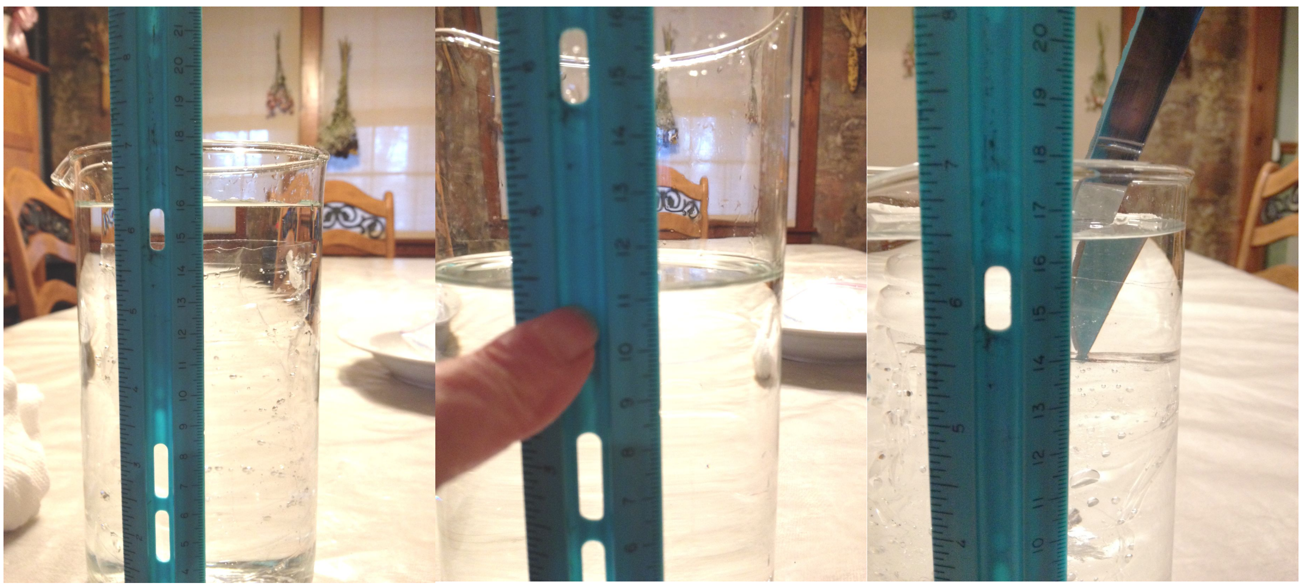
The easiest way to solve this problem is by comparing the volume of an identical mass of ice and water using the Archimedes buoyancy law. The important consideration here is that because this difference has to be measured with good accuracy in order to obtain accurate result, the measured quantity has to be much larger than the accuracy of the measurement device. In the solution proposed below, the volume ratio is obtained by measuring the height of the water level filling the glass vessel using a simple ruler. The smallest scale on the ruler is 1 millimeter, and it is reasonable to believe that the measurement accuracy with such a ruler is 0.5mm. Hence, water levels measured in this experiment must be much larger than ~0.5mm.

Experiment A.

1. Take a glass vessel with vertical walls, such as a glass jar. It need not be cylindrical, and walls need not even be rigorously vertical all the way, they only need to be vertical over a large fraction of vessel's height, so that its cross-sectional area stays constant. This is commonly the case for a glass jar around the middle of its height.
2. Take another vessel, such that it fits in the first one. Here we cut the corner by using a plastic bottle, but we could have used another glass jar, which would require proper correction accounting for the thickness of its walls.
3. Insert smaller vessel in the larger one and fill them with water, (a). The water level is $h_1 = 160.0 \pm 0.5$ mm. We need to check that the additional uncertainty introduced by the water meniscus does not degrade the 0.5 mm accuracy of our measurement. This is indeed the case in (a) – (c).
4. Remove the inner vessel filled with water and put it in the freezer. Measure the level of the remaining water, $h_0 = 111.5 \pm 0.5$ mm, (b). The volume of the smaller vessel together with the water removed is , $V_1 = S*(h_1-h_0)$, where S is the cross-section of the larger glass vessel. The volume of the walls of the smaller vessel itself, which are of a finite material thickness, is measured in a similar way, by submerging it in the larger jar and measuring the change in the water level. In our case this volume is negligible – the level difference is smaller than our measurement error of 0.5 mm.
5. When water in the inner vessel (plastic bottle in our case) freezes, submerge it back in the

larger vessel and measure the water level height, $h_2 = 164.5 \pm 0.5$ mm, (c).

6. The Volume of the obtained ice, $V_2 = S^*(h_2\text{-}h_o)$. Again, this includes negligible volume of the walls of the smaller vessel (plastic bottle).
7. Neglecting the water evaporation/sublimation in the freezer, the mass of the removed water equals to the mass of the obtained ice. Therefore, the ice density relative to the density of water equals to the volume of water relative to that of ice, $\rho_{ice}/\rho_{water} = V_1 / V_2 = (h_1\text{-}h_o)/(h_2\text{-}h_o) = 0.915 \pm 0.014$. We truncated the result at the digits determined by the accuracy of our measurement, which we have estimated from the relative errors of each of our height-difference measurements, $\approx 0.5/(160.\text{-}111.5) \approx 0.010$, and $\approx 0.5/(164.5\text{-}111.5) \approx 0.009$. Adding these in squares, as appropriate for the independent Gaussian deviations, we obtain the overall relative accuracy estimate, ≈ 0.014, or $\approx 1.4\%$. Note that 1.4% accuracy for the ice density itself corresponds to $\approx 16.4\%$ accuracy in the measured difference between the density of water and the density of ice, $\rho_{water} - \rho_{ice} = 0.085 \pm 0.014$.



Answer: $\rho_{ice}/\rho_{water} = 0.915 \pm 0.006.$

CHEMISTRY

A 250 mL conical flask (so called “Erlenmeyer flask”) containing 10 grams of magnesium carbonate has been placed on the right scale of the pendulum balance scales. An identical flask, containing 10 grams of *calcium* carbonate, has been placed on the left scale. In addition to the flasks, there are two identical glass beakers with 80 grams of 30% nitric acid[1] on each scale. The scales are at equilibrium, as shown on the photograph.

Question 1. What happens to the scales after we pour the acid from each beaker into the corresponding flask, and place each beaker back to the original position? Will the scales remain at equilibrium? If not, which scale will go up?

Question 2. Will the result be different if we use only a half of the acid? Three quarters of the acid? (Of course, the rest of the acids remain in the beakers, and the beakers remain on the scales).

The SchoolNova web site contains some information needed to solve this problem http://schoolnova.org/nova/classinfo?class_id=chemistry101&sem_id=f2013

[1] 30 grams of nitric acid per 100 grams of the *final solution.*

Solution

Both calcium and magnesium carbonates are the carbonic acid salts, so addition of nitric acid leads to formation of free carbonic acid in both cases:

$$CaCO_3 + 2\,HNO_3 = Ca(NO_3)_2 + H_2CO_3 \tag{1}$$

and

$$MgCO_3 + 2\,HNO_3 = Mg(NO_3)_2 + H_2CO_3 \tag{2}$$

Carbonic acid, being unstable, decomposes immediately onto water and carbon dioxide.

$$H_2CO_3 = H_2O + CO_2 \tag{3}$$

The latter compound is a gas, it escapes immediately, so the mass of the content of each flask decreases. Therefore, to answer the questions 1 and 2, we need to calculate the amount of carbon dioxide formed in (and escaped from) each flask.

First of all, let's calculate the maximal weight loss of 10 gram samples of $CaCO_3$ or $MgCO_3$ due to CO_2 formation. It is clear from the equations 1, 2, and 3 that one molecule of calcium or magnesium carbonate produce exactly one carbon dioxide molecule. That means we can set two proportions:

Proportion 1:

$(40 + 12 + 16 * 3)$ grams of $CaCO_3$ $->$ yield 44 grams of CO_2
10 grams of $CaCO_3$ $->$ yield X grams of CO_2

and

Proportion 2:

$(24 + 12 + 16 * 3)$ grams of $MgCO_3$ $->$ yield 44 grams of CO_2
10 grams of $MgCO_3$ $->$ yield Y grams of CO_2

Even without solving these proportions, one can see that Y is greater than X (because the numerator in both proportions is the same, whereas the denominator in smaller in the second case). That means the final mass of the material will be smaller in the flask containing magnesium carbonate, so the scale with $CaCO_3$ will go down. (It is easy to calculate the mass difference.

It equals to
$44 * 10/(24 + 12 + 16 * 3) - 44 * 10/(100 + 12 + 16 * 3) = 44 * (100 - 84)/840 = 0.84\ g)$

Unfortunately, that is not a final answer: the above considerations work only when we take a sufficient amount of the nitric acid. In connection to that, to answer the Questions ##1,2, we need to calculate the amount of HNO_3 needed for full conversion of $CaCO_3$ and $MgCO_3$. From the equations 1 and 2, we can conclude that two molecules of nitric acid are needed to react with one molecule of $CaCO_3$ and $MgCO_3$. Therefore, we need to solve the following proportions:

$2 * (1 + 14 + 16 * 3)$ grams of HNO_3 are needed to react with $(40 + 12 + 16 * 3)$ grams of $CaCO_3$

X grams of HNO_3 are needed to react with 10 grams of $CaCO_3$

and

$2 * (1 + 14 + 16 * 3)$ grams of HNO_3 are needed to react with $(24 + 12 + 16 * 3)$ grams of $MgCO_3$

Y grams of HNO3 are needed to react with 10 grams of $MgCO_3$

It is easy to calculate that X equals to $126 * 10/100 = 12.6\ g$, and Y equals to $126 * 10/84 = 15\ g$.

That means three situations are possible:

1. The amount of nitric acid is smaller than 12.6 grams. In that case, the mass of CO_2 formed in both flasks will depend only on the amount of HNO_3 taken (because there is an excess of calcium or magnesium carbonate in each flask). In other words, the amount of gas formed in each flask will be the same, so the scales will remain at equilibrium.

2. The amount of the acid is greater than 12.6 g, but smaller than 15 g. In that case, calcium carbonate will react completely, but magnesium carbonate will not. The mass of the $MgCO_3$ flask will be smaller (more CO_2 forms there), but the weight loss will depend on the amount of HNO_3 taken.

3. The amount of HNO_3 is greater than 15 g. In that case, both calcium and magnesium carbonates react completely, and the mass of the flask on the left scale is greater by 0.84 g (see above).

Now we are ready to answer Questions 1 and 2.

1.
80 grams of our HNO_3 solution contain $0.3 * 80 = 24$ grams of the acid. That is greater than 15 grams, so we have an excess of HNO_3 (case #3), **so the left scale will go down** (see the photo below).

2.

If only a half of the acid solution is taken, the amount of HNO_3 is $0.3 * 40 = 12$ grams. That is less than 12.6 grams (case #1), so **the scales will remain at equilibrium.**

BIOLOGY

We all have been taught by our parents that to stay healthy one should keep his or her body clean. Nature must have taught the same lesson to other animals as well because almost all creatures have developed some ways of cleaning and tidying themselves. Moreover, some animals continue doing it even when there is no apparent need for that; in animal life, cleaning plays a more important role than merely hygiene maintenance.

1. Man-made aids such as bathtubs, soap, plastic combs, and brushes are not available to animals, so they must manage with whatever equipment nature provides. Please list at least 5 different ways in which animals clean themselves (try to provide examples covering different animal taxa, e.g. insects, birds, mammals, etc.).

2. Please provide at least 3 reasons other than simple hygiene why different animals clean and groom themselves and each other.

Solution

For animals, staying clean is as necessary as eating. If birds don't have clean feathers, they can't fly fast enough to catch a meal or avoid becoming one. Cleaning themselves and putting their fur, feathers or other skin coverings in good order is known as personal grooming, a form of hygiene. Extracting foreign objects such as insects, leaves, dirt, twigs and parasites, are all forms of grooming.

I. How do animals stay clean?
That's depend on the animal.
Some animals clean themselves. Monkeys, baboons, chimpanzees, and gorillas rely on their friends to help them stay clean. But the most interesting cleaning techniques involve one species helping another stay clean (cleaning symbiosis).

How do animals clean themselves?

Cleaning themselves*: Tongues* are important in the cleaning activities of many animals. You a probably familiar with the tongue baths taken by the family cat, but did you know that wild members of the cat family also lick themselves clean? The bobcat licks every reachable part of its body; then it licks its paw and uses the wet foot to wash its face and behind its ears. Young bobcats are kept clean by their mothers just as kittens are bathed by the family cat.
Bats spend at least an hour each day washing themselves in catlike fashion. Those parts of the bat's body, which cannot be reached with its tongue, such as the inside of its ear, are cleaned with its *saliva-moistened wing thumbs*. Special attention is given to cleaning the wings to keep them soft, and the bat massages each joint of its wing "hands" with its thin, red tongue every day.
Ants are among the cleanest of all insects. They lick themselves every few minutes with oily saliva that cleans and oils their bodies. They also lick each other as a part of their grooming activities.

Since their antennae are quite sensitive, they also must be kept very clean. To clean them, the ant lifts its *front leg* over one antenna and pulls the antenna through a special brush of hairs, which grows at its “wrist.” The brush is then pulled through the ant’s mouth to remove any dirt. This action may be repeated several times on each antenna until both are cleaned to the ant’s satisfaction.
Members of the dog family, such as coyotes and wolves, use their tongues to clean themselves and their young, but they also use other methods of grooming. *Scratching and biting*, for instance, help rid their bodies of fleas and ticks, tangled hair, and burrs. *Shaking* removes dirt, loose hair, dead fleas and ticks, and anything else that may be loosely caught in the hair. Shaking also removes excess water when the animals are wet. *Rubbing and rolling* in grass and other vegetation help clean the animal’s coat, dry it, and, on occasion, improve its odor. Wild members of the dog family seldom take *water baths*, and this is probably just as well since too much water can dry their skins.
Beavers, which spend so much of their time in the water, groom their fur coats whenever they waddle ashore. The beaver sits upright and uses its *forepaws* to shake water out of its ears. It may then scratch the hair on its head, rub its eyes, comb its whiskers, and scratch its belly. A double-claw toenail on the second toe of each hind foot is used to comb and groom the fur.
A porcupine spends very little time combing its hair, and it is impossible for the creature to use its teeth or claws to groom the fur and hair on its back and tail. It occasionally licks its paws and scratches, but to shed loose quills the porcupine usually shakes itself like a dog. To groom its coat this prickly animal simply erects the hair on its body and then relaxes it. This smooth and straightens its appearance.
In addition to tongue baths, some animals, such as rabbits and squirrels, take *dust baths*. The animal looks for a spot where the soil is either sandy or powdery dry. Then it lies down, rolls about, kicks out its legs, pulls itself in circles, and finishes the bath with a leap and a shake. This dust bath helps to remove parasites from the animal.
Many birds, such as chickens, pheasants, and quail, also take dust baths.
House sparrows enjoy both water and dust baths. Vultures and owls enjoy taking *sunbaths*. The vulture perches on a high branch and spreads its wings so the sunshine can reach every feather. The sunshine makes it feel good and also helps kill many of the germs picked up from the dead animals the vulture eats. The owl may lie in a sandy area and spread first one wing and then the other as it sunbathes, or it may stand on a stump, let its wings droop down as far as they will reach, and turn its face to the sun.
Fish are constantly in the water, and they couldn’t possibly need a bath, but they still get parasites on their bodies. To try to remove them the fish may roll to the side and *fling itself against a plant or rock*. This action rubs the side of the fish, usually the gill section, against the plant or rock and may remove some of the irritating parasites from its body.
http://www.tpwd.state.tx.us/publications/nonpwdpubs/young_naturalist/animals/natures_bath_time/
http://query.nytimes.com/mem/archive-free/pdf?res=F20B12FA385B13738DDDA00994D8415B838DF1D3

How do animals help each other to stay clean?

Mutual grooming:

1. Social grooming:
Social grooming is defined as a grooming activity between individuals. Social grooming tends to be a reciprocal interaction and it involves more than one individual.
The partnerships formed during social grooming are long-lasting, much like the relationship you have with your best friend. It might be unsurprising, then, that social grooming in primates serves primarily a social purpose, allowing animals to bond and build relationships.

Social grooming involves gentle touches as well as stroking, scratching, and massaging.
Studies have shown that allogrooming (social grooming) is not at all unique to primates but is also observed in other mammals, insects, reptiles and birds. Impalas groom each other; in particular, it is very common for impala fawns to groom their mothers. Honeybees have been observed grooming each other to remove pollen grains from their wing bases European green lizards also groom each other routinely.
http://academic.reed.edu/biology/professors/srenn/pages/teaching/web_2008/dklj_site_final/adaptive.html
http://knowingneurons.com/2013/02/20/social-grooming-its-not-just-for-monkeys-and-prairie-voles/

2. Cleaning symbiosis is a mutually beneficial behavior involving the removal of ectoparasites, diseased and necrotic tissue between cooperating species.
Cleaning symbiosis is well known among marine fish, where some small species of cleaner fish, notably wrasses, but also species in other genera, are specialized to feed almost exclusively by cleaning larger fish and other marine animals. Other cleaning symbioses exist between birds and mammals, and in other groups.

There are many unlikely animal pairings in the world, but some of the most uncanny ones involve birds. They have been known to climb in and clean the teeth of a crocodile, lounge on the backs of buffalo, moose, hippos, elephants and zebras and even to share a residence with ants – their natural prey. The first set of symbiotic wonders focused mainly on underwater matches – this set focuses on fliers of the friendly skies:
Plovers and Crocodiles, Honeyguides and Ratels, Egrets and … All kind of large animals, Oxpeckers and Zebras, Ostriches and Zebras, Woodpecker and Tree Ants, Meat Ants and Leafhoppers.
http://webecoist.momtastic.com/2009/03/01/symbiotic-bird-animal-relationships/
http://www.wetwebmedia.com/clngsymbfshs.htm

II. Other reasons for grooming:
In addition to removing parasites and lowering the risk of disease, grooming can also serve to *strengthen social bonds, reduce tension, calm, soothe, appease, reassure, reconcile after an aggressive interaction, gain favor with more dominant individuals or indicate sexual receptivity.*
Among primates, social grooming plays an important role in *establishing and maintaining alliances and dominance hierarchies, for building coalitions, and for reconciliation after conflicts; it is also a resource that is exchanged for other resources, such as food and sex.*
Primates groom socially in moments of boredom as well, and the act has been shown to reduce tension and stress. It is often associated with observed periods of relaxed behavior, and primates have been known to fall asleep while receiving grooming.
Domesticated animals, especially cats and dogs, will groom trusted humans as a *sign of affection.*

COMPUTER SCIENCE

Write a Java program that reads 2 integers from the stdin (not from local variables) and returns true if only one of the integers is in the range [13,19].

For example:
onlyOneTeen(15,5) -> True
onlyOneTeen(1,19)->True
onlyOneTeen(13,14)->False

The program should reject improper input (such as strings, floats, etc) and use boolean expressions.

2 points for handling improper input
7 points for the correct onlyOneTeen function (I will be using my input for testing)
1 point for a short, easily readable program.

Some resources:
http://www.youtube.com/watch?v=PWez5mVXACc
http://www.youtube.com/watch?v=YeLkiLq14Qo
http://theoryapp.com/conditionals-in-java/
http://www.erpgreat.com/java/java-boolean-logical-operators.htm
http://www.dummies.com/how-to/content/how-to-use-boolean-expressions-in-java.html

Solution

SOLUTION 1 (using do while loop for input validation):

```
import java.util.Scanner;

public class Ideone{

  public static void main (String[] args) {

       Scanner input = new Scanner(System.in);

       int int1 = 0, int2 = 0;
       boolean isNumber1 = false, isNumber2 = false;

       System.out.println("Please enter an integer");
```

```
        // Keep asking for user input until we get an integer
        do{
          if(input.hasNextInt()){
                int1 = input.nextInt();
                isNumber1 = true;
          }
          else{
                System.out.println("Your input is not an integer ( or outside of [-2147483648,
2147483647] range. Try again:");
                // Take the non-integer user input and ignore it ( do not assign to any variable )
                input.next();
          }
        } while(!isNumber1);

        // Repeat the above for the second number
        System.out.println("Please enter an integer");
        do{

          if(input.hasNextInt()){
                int2 = input.nextInt();
                isNumber2 = true;
          }
          else{
                System.out.println("Your input is not an integer ( or outside of [-2147483648,
2147483647] range. Try again:");
                // Take the non-integer user input and ignore it ( do not assign to any variable )
                input.next();
          }
        } while(!isNumber2);


        boolean int1_is_Teen = (int1 >= 13 && int1 <= 19);
        boolean int2_is_Teen = (int2 >= 13 && int2 <= 19);

        System.out.println("There is only one teen number in " + int1 + ", " + int2 + " : " + (
(int1_is_Teen && !int2_is_Teen) || (!int1_is_Teen && int2_is_Teen)) );

   }

}
```

SOLUTION 2 (using try catch for input validation) :

```
import java.util.Scanner;


public class Ideone{
```

```
  public static void main (String[] args) throws java.lang.Exception
  {

        Scanner myScanner = new Scanner(System.in);
        boolean loop = true;
        int a = 0, b = 0;

        do{
          try {
                System.out.print("Please enter an integer: ");
                a = myScanner.nextInt();
                loop = false;
          }
          catch(Exception e)
                {
                  System.out.println("Not an integer, or outside of [-2147483648, 2147483647] range
:( ");
                  myScanner.nextLine();
                }
        } while (loop);

        loop = true;
        do{
          try {
                System.out.print("Please enter another integer: ");
                b = myScanner.nextInt();
                loop = false;
          }
          catch(Exception e)
                {
                  System.out.println("Not an integer, or outside of [-2147483648, 2147483647] range
:( ");
                  myScanner.nextLine();
                }
        } while (loop);

        boolean a_is_Teen = (a >= 13 && a <= 19);
        boolean b_is_Teen = (b >= 13 && b <= 19);

        System.out.println("There is only one teen number in " + a + ", " + b + " : " + ( (a_is_Teen &&
!b_is_Teen) || (!a_is_Teen && b_is_Teen)) ) ;

  }
}
```