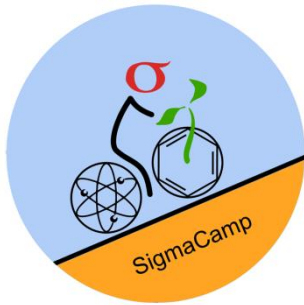




Problem of the Month. Solutions: January 2013

MATHEMATICS

Alice wants to buy a piece of land, exactly level, which would be bounded by four boundary lines, each measuring exactly 100 meters long; two boundary lines running exactly north-south, and two running exactly east-west. Can Alice buy such piece of land in the USA? State your reason.

Solution

This problem is best understood when looking at a geographical globe. Since the Earth's surface is nearly spherical, the conditions set out by Alice could only be satisfied at the Equator. Hence, in the United States there can be no such piece of land.

Namely, the desired piece of land is bounded by two meridians and two parallel circles. Since the arc of a parallel circle intercepted by two fixed meridians is steadily shortened as the circle moves away from the equator, the center of the land Alice wants must lie on the equator; she cannot get it in the United States.

PHYSICS

There are two water fountains in a hallway, side-by-side. They are identical in every way except for the water pressure: The water stream of fountain A reaches 16 cm above the nozzle, and that of fountain B - 36 cm. Would it be faster to fill up a water bottle from fountain A, or B? How much faster?

Solution

The rate at which water flows out of the fountain is proportional to the velocity with which it leaves the nozzle, while the height that the fountain reaches is proportional to the square of this velocity (kinetic energy). Hence, the rate of water flow is proportional to the square root of the fountain's height. Therefore, the taller fountain would fill the bottle 1.5 times faster.

CHEMISTRY

You decided to prepare sodium carbonate from baking soda. From the December POM you know that it can be prepared by heating baking soda in an oven. You placed 168 g of baking soda in a bowl, weighed it and recorded its weight. Then you put the bowl into a hot oven. After 1 hour you took the bowl and weighed it again. The bowl lost 31 g. You correctly concluded that the loss of weight was due to decomposition of the baking soda.

You know that this reaction proceeds in such a way that each molecule of baking soda decomposes independently on others, so the probability of each molecule's decomposing is constant (at given temperature).

Can you tell:

- 1. Is the decomposition complete?*
- 2. If not, how much time the reaction needs to complete (assume precision of your scales is 0.001g)?*

Solution

Sodium bicarbonate decomposes according to the following equation:



(**Note**) That is why baking soda is called sodium *bi*-carbonate: it contains an extra carbonate residue (as compared to sodium *carbonate*), which can be released upon heating. An arrow ($\uparrow$) demonstrates that these compounds are gases at reaction conditions.

From this chemical equation we can calculate a mass change. The mass decreases because of the evaporation of water and carbon dioxide (gas) formed during the reaction (the soda is in an oven, so the water formed evaporates immediately). The mass of one mole of water is 18 g, the mass of carbon dioxide is 44 g, so after full decomposition of *two* moles of sodium bicarbonate the mass of the solid formed decreases by $18+44=62$ g.

Two moles of sodium bicarbonate weigh $2 \cdot (23+1+12+16 \cdot 3) = 84 \cdot 2 = 168$ grams. In other words, it appeared that you took (so serendipitously) exactly two moles of baking soda for your reaction :-). That makes our further calculations easy: since two moles of baking soda produce 62 grams of gases (water + carbon dioxide), the total weight loss of our sample should be 62 grams. That means 31 g weight loss corresponded to exactly 50% decomposition. Therefore, we can easily answer the first question:

► Answer 1: No, the reaction is not complete; only 50% of the baking soda decomposed.

(**Note**) Actually, we deliberately took a whole number of moles to make computation easier, and to save your time and efforts for the second part, which is not so straightforward.

Now, when we established that only 50% of baking soda decomposed for 1 hour, we have to predict the time needed for decomposition of the remaining 50%. One would think the answer is obvious: if 84 grams of baking soda decompose in 1 hour, 84 more grams will decompose in the same time afterwards. However, that is not the case. Remember, each molecule of baking soda decompose *independently* of each other, so if 50% of molecules decompose in one hour, that means that the *probability* of each molecule's decomposing during a one hour period is 50%. The same is true for any amount of soda, be it two moles, two million moles, or two thousandth of a mole: a half of *any* amount of baking soda will decompose in one hour at this temperature, which means that only 50% of the remaining 84 grams of baking soda will decompose during the second hour.

That sounds counter-intuitive, but that is true. To demonstrate that, let's do a thought experiment: 100 students wear white T-shirts, and each of them also has a black T-shirt and a dime. During a first 1 hour period, each student tosses a dime: heads means that he changes his white T-shirt to the black one and stops tossing; tails means that after 1 hour he tosses again. Obviously, since the probability of heads and tails is equal, during the first hour approximately 50 students don the black T-shirts. However, since the results of their tossing have no effect on the tossing during the next hour, the probability for each one of remaining 50 students' tossing heads is still 50%, so approximately 25 of them get heads within the second hour, and so on. As a result, T-shirt changing will proceed as follows (we consider an ideal case when exactly 50% of students get heads each time):

After 1st hour: 50 students wear white T-shirts, 50 students wear black ones;

After 2nd hour: 25 white, 75 black;

After 3rd hour: 12 (or 13) white, 88 (or 87) black;

After 4th hour: 6 (or 7) white, 94 (or 93) black;

And so on.

The analogy between the T-shirt change and the baking soda decomposition is direct and full: when we are heating the soda, each molecule is jiggling (due to thermal vibrations), and this process is similar to tossing a coin: each jiggling can potentially lead to the molecule's falling apart. If the molecule falls apart, it produces sodium carbonate -- water and carbon dioxide, which are stable in these conditions (in other words, it "wears a black T-shirt" and stops tossing), if the molecule doesn't fall apart, it continues jiggling, which may cause falling apart later (in other words, it behaves as a student in a white T-shirt).

Now we are ready to obtain an answer for the second question. We know that precision of our scales is 0.001g, so we will no longer see the weight change when the amount of baking (non-decomposed) soda is less than 0.001g. At this point, we can assume the reaction is complete. To calculate the time needed for that one has to be familiar with logarithms, exponents, derivatives and integrals. However, some crude estimate can be done using just basic mathematical knowledge.

► Solution. Let's compute a mass change after next hours. After 2nd hour a half of the remaining baking soda decomposes, so $\frac{1}{4}$ of the initial amount of the baking soda remains non-decomposed (in other words, the weight loss is $62 \times \frac{1}{4} = 15.5$ grams). Let's continue this series:

After 3rd hour only $\frac{1}{8}$ of the initial amount of baking soda decomposes (and $\frac{1}{8}$ remains), and the weight loss is $62 \times \frac{1}{8} = 7.75$ grams;

4th hour: 3.875 g

5th hour: 1.9375 g

6th hour: 0.96875 g

7th hour: 0.484375 g

8th hour: 0.242188 g

9th hour: 0.121094 g

10th hour: 0.060547 g

11th hour: 0.030273 g

12th hour: 0.015137 g

13th hour: 0.007568 g

14th hour: 0.003784 g

15th hour: 0.001892 g

16th hour: 0.000946 g

In other words, only after 16 hours the mass of the remaining baking soda is so small that our scales are not able to detect it. That means only after 16 hours can we assume the decomposition of soda is complete.

► Answer 2: 16 hours are needed for the baking soda to fully decompose.

(Note) The same considerations can be used to describe a wide range of processes, including the decay of radioactive isotopes.

BIOLOGY

Some animals, for example chameleon, are able to change their skin colors. The color change serves only partly for camouflage, turning chameleons invisible to their predators.

Please, suggest other biological functions of color change in chameleons. Do you know of other animals able to change their skin color? Please, provide at least 4 examples of different animals with an ability to change color and explain why do they do it.

Solution

Myth: Chameleons change color to match their environment.

Chameleons don't change color to match their environment. The color change serves for camouflage only partly. Rather, they change color as a response to mood, temperature, health, communication, and light.

In fact, if a chameleon is attacked by a predator, its color turns reddish with brown

and yellow stripes, as their predators (snakes, mammals) do not distinguish colors well.

Although chameleons at rest tend to assume colors similar to their surroundings, color change is most often used to signify emotional state. Many chameleons are some shade of green or brown at rest, but can become far more brightly colored when frightened, courting, or defending a territory against another chameleon. When startled or threatened, veiled chameleons may darken in color and "play possum." Color also signals changes in light and temperature in the environment. In the morning after a fresh night, the chameleons will warm in the sun, flattening their flanks, which turn black. A sick chameleon will stay pale because they don't have the energy to change color.

Now, not all kinds of chameleons have the ability to change color and some only have limited ability for color changing; for instance, many types of chameleons can only turn green, brown, or gray. Others have the ability to change to a variety of different colors including: pink, blue, red, orange, green, black, brown, blue, yellow, turquoise, and purple. Of the ones that can change color, they'll often change to darker shades, when they are cold. Likewise, if they are hot, they can change to lighter colors which will reflect the sun better, helping to cool themselves off.

Most color changes though are simply due to mood changes or for communication purposes. For instance, the Panther Chameleon will change to red and yellow when they are angry or are getting ready to attack. This serves as a warning to other chameleons and humans to back off. Some types of male chameleons will also change to mixed bright colors when they are trying to attract a nearby female chameleon.

Instead of vocalizing or using pheromones, chameleons communicate visually by changing the colors and patterns of their skin. Different colors and patterns mean different things—similar to how the colors of a traffic light direct drivers.

For example, the brighter colors a male displays, the more dominant he is. So male chameleons can attract a mate or defend their territory by flashing bright colors to each other. To communicate submission or surrender, a male will display drab browns and grays. Females also use a colorful version of signaling to communicate when they want to reject mates or are pregnant.

Bonus Facts:

Those chameleons that can change to a variety of colors can do so thanks to special cells, collectively called chromatophores. These chromatophore cells are grouped in layers underneath the chameleon's transparent outer skin layer. The upper most layers contain yellow and red pigments. The lower layers are comprised of a colorless crystalline substance and melanin, which both are used to modify the visible color of the upper yellow and red pigments, producing a variety of colors. In addition to that, the melanin layer also controls the intensity of the final output color.

Chameleons that can change color, typically can do that in less than 20-30

seconds.

Chameleons are extremely territorial. They also typically like to be by left to themselves.

Some types of chameleons have tongues that are actually longer than their whole bodies. These tongues can be fully extended as fast as 26 body-lengths per second, which is faster than the human eye can follow.

Chameleons have the ability to see ultraviolet light.

People who own particularly colorful types of chameleons can usually quickly learn to discern the mood of their pets and whether the chameleon is sick or not, simply by looking at its coloration.

Read more at <http://www.todayifoundout.com/index.php/2010/11/chameleons-dont-change-color-to-match-their-environment/#dUCiTxCxioSy3ZHs.99>

<http://news.nationalgeographic.com/news/2008/01/080128-chameleon-color.html>

<http://www.answers.com/topic/why-and-how-do-animals-change-color#ixzz2Ju1VBOOY>

As chameleons are somehow related to iguanas, it is plausible to suggest that the changing color ability developed in a remote ancestor during the dinosaur era. Thus, being a chameleon won't hide your real feelings.

There are in fact some other species that DO copy their environment, using chromatophores, like octopuses or many flatfishes, like flounders.

Cephalopods, a group including octopuses and squids, are especially skillful at changing color rapidly; they can turn different colors in less than a second. Several kinds of fish and some amphibians and lizards are also able to change colors, though their transformations take a bit more time than those of the cephalopods. Such animals change their colors for a number of reasons. Those that can rapidly change colors do so to startle or confuse predators or to better blend in with their environments, a technique known as camouflaging. Camouflaging can either be used by an animal that wants to hide from an attacker, or by an animal that doesn't want to be seen by its prey. Color changes can also be used to attract a mate.

Some animals undergo color changes with a change in seasons. Certain mammals and birds that live in cold climates, for example, have white fur and feathers in the winter so they can blend in with the snow and be less noticeable to their predators. Some songbirds will grow brightly colored, attractive feathers for the mating season. Those feathers are replaced by duller colors after mating is over. These color changes are also caused by pigment cells, located beneath the fur or feathers.