PROBLEM OF THE MONTH

2019 - 2020

January, 2020

MATHEMATICS

5 points:

One day the Little Red Riding Hood decided to take some pies to her grandmother's house. Shortly after leaving her house, however, she realized she was really hungry and ate the three largest pies, and as a result the total weight of the pies decreased by 35%. A little bit later in her journey, she once again wanted a snack and ate the three smallest pies, as a result of which the pies' total weight decreased by another 5/13 compared to the previous weight. How many pies did the Little Red Riding Hood leave her house with?

10 points:

The line l intersects the segment AB at point D so that $AD = a$ and $BD = b$. Construct a circle that goes through points A and B and carves out the smallest possible segment from line l. What is the length of this segment in terms of a and b?

PHYSICS

5 points: Imagine that you don't have a freezer. There is an easy trick that would allow you to achieve a rather low temperature: mix salt and ice. As salt causes the ice to melt, the temperature of the mixture drops. This way, the melting temperature of ice can be brought down to as low as $T_1 = -21^0C$. What fraction of ice will be melted when that temperature is achieved if you started with ice at its regular melting point, $T_0 = 0^0C$? Assume the system to be thermally isolated. Specific heat capacities of liquid water and ice are $C_w = 4.2\frac{kJ}{kg\cdot^0C}$ and $C_i = 2.1\frac{kJ}{kg\cdot^0C}$, respectively. Neglect heat capacity of salt. Latent heat of ice melting is L=334 kJ/kg.

10 points: Commercial heat pads are based on the solution of sodium acetate. Combined with water, this salt may form a crystal with a melting temperature close to $T_m = 58^oC$. This crystal will not typically form spontaneously when the solution is cooled down starting from higher temperatures. In other words, the liquid would remain "supercooled" below the melting point. The crystallization can be triggered by local mechanical stress, leading to heating up of the solution.
Assume that the heat pad of certain mass M is placed into a thermally isolated container that contains the same mass of water, at temperature $T_0 = 20C$. What will be the final temperature inside that container, once the crystallization is triggered? Specific heat capacities of water and the sodium acetate solution are $C_w = 4.2\frac{kJ}{kg\cdot^0C}$ and $C_s = 3\frac{kJ}{kg\cdot^0C}$, respectively. Ignore the mass and heat capacity of all the parts of the head pad except for the sodium acetate solution itself. The composition of the solution is such that it can crystallize completely at T_m, the latent heat of its crystallization is L=270 kJ/kg.

CHEMISTRY

5 points:

One of Sigma workshops is “Fruit electricity”. By using two different metals, electricity is produced from lemons, apples or other juicy fruits. For the first time, a copper wire and iron nails (galvanized common nails obtained from the Home Depot) were used, and the voltage obtained from the single fruit cell was approximately 1V. For the second time, a copper and X-acto knife blades were used, and it produced about 0.7V. How can you explain this significant difference?

10 points:

A coin made from a copper-silver alloy with unknown silver content weighs 1 gram. To this coin, an excess of nitric acid was added, and when the reaction had ceased (all solids “dissolved”), the clear and transparent liquid obtained was evaporated to dryness. The solid remainder was dissolved in 100 ml of water, and 10 grams (excess) of sodium sulfide was added. The precipitate formed was filtered and completely dried. The mass of the residue was 1.38 grams. What is the percentage of silver in the alloy?

BIOLOGY

5 points:

Why do birds and mammals maintain a constant temperature? A small bird consumes substantially more food than an cold-blooded lizard of the same weight, which warms in the sun during the day and cools again at night.

10 points:

Viruses do not only parasitize cells causing their death, but also facilitate horizontal gene transfer between organisms and may even define biogeochemical cycles. Imagine that all the viruses on Earth disappeared at once. What environmental (short-term) and evolutionary (long-term) consequences will this lead to?

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc. **No .pdf, .doc, .docx, etc!**

The Game of Hungry Sigma-Fish

You are given a rectangular field throughout which a number of hungry Sigma-fish is spread out. Each fish is denoted by an integer representing its size. Empty fields are represented by a period "." Here is an example of a simple field:

```
..4....3
.....2..
.2......
.......1
```

Each move goes like that: each Sigma-fish [simultaneously] moves one field (vertically, horizontally or diagonally) towards another Sigma-fish of smaller size (if any). The direction is chosen toward the closest smaller fish, with the following tie-breakers:

- larger target has a priority
- in case of equal size targets or alternative fields to move, the priority is directional, in the following order of decreasing priority: N, NE, E, SE, S, SW, W, NW.

At the end of each move, if two or more fish end up on the same field, they merge into a fish with the combined size (sum of their previous sizes).

5 points:

For this problem the game field is a square NxN.

Write a program that will receive on the input:

- size N of the field (optionally)
- field composition, which consists of periods and integers

Your program should determine whether each of the integers 1..N is found in the set of coordinates of the fish on the board and print YES or NO correspondingly. In case of NO, as a bonus, please print which integers are missing.

Note: coordinates start with 1, thus the coordinates of the top left field of the board is (1, 1), and the coordinates of the bottom right field of the board is (N, N).

10 points:

Write a program that will receive on the input:

- size of the Game of Hungry Sigma-Fish field as integers N and M (optionally)
- field composition, which consists of periods and integers

Your program should print the final field state. You may optionally elect to print all intermediate field states. Finally, one point will be reserved for the quality of presentation.

LINGUISTICS

5 points:

Dr. Sigmund Sigman, a former Sigma camper and current linguist, recently discovered an advanced civilisation of hominids living in ice caves near the Sigmen Glacier in Antarctica. The hominids, who call themselves Sigmamen, use squid ink to write on seal skins. Dr. Sigman had trouble translating the writing system of the natives, especially its numerical calculations, in part because there allegedly were multiple correct representations of the same number. However, after learning to verbally communicate, he could agree with the natives on some common numerical values, which are written below in the Sigmaman number system:

Number of fingers on two hands:

Variant 1: 

Variant 2: 

Number of letters in the English alphabet:

Variant 1: 

Variant 2: 

Number of U.S. states:

Variant 1: 

Variant 2: 

Number of U.N. representatives:

Variant 1: 

Variant 2: 

Number of days in a year:

Variant 1: 

Variant 2: 

Current year:

Variant 1: 

Variant 2: 

Given the following examples, help Dr. Sigman by doing the following:

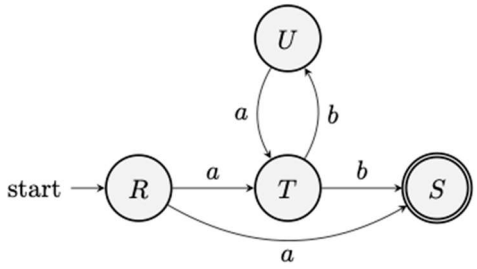
Question 1: Determine the decimal translation of each of the Sigmaman numerals used in the examples. Express your answers in Hindu-Arabic numerals (ex: 0, 1, 2, 3) and/or explain your answer in words.
Question 2: Using the Sigmaman number system to write TWO different variants of:
a. The number of letters in the Russian alphabet
b. The number of representatives in the E.U. parliament

Don't forget to explain how you found your answer!

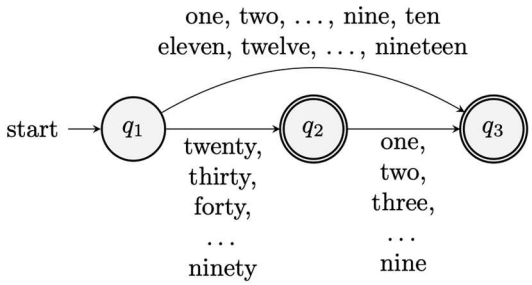
10 points:

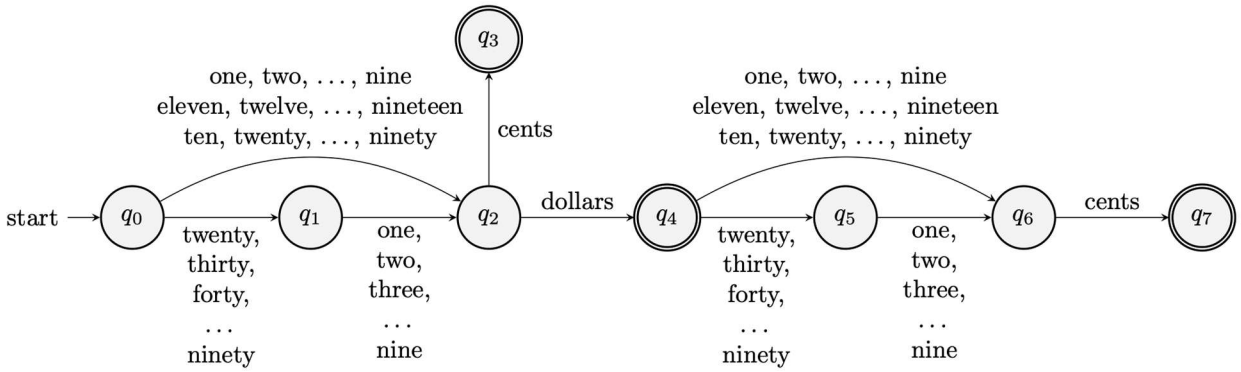
Finite state automata (FSA) are graphs used for recognizing whether some string is valid (*"grammatical"*) in some language. To check whether a string is valid, we start in a vertex of the FSA marked with a "start", and proceed to subsequent vertices based on the elements in the string, reading them one after another. If we finished the string and ended up in a vertex marked with a double circle (such vertices are called *"final states"*), the input string is *grammatical* in a given language. If we exhausted the string but didn't end up in a final state, the string is *ungrammatical*. Similarly, if we encountered an element in the string which does not have a corresponding arrow going out of the vertex we are currently in, the string is also *ungrammatical*.

For example, for the automaton below, the following strings are grammatical: *a*, *ab*, *abab*, *ababab*, etc., and the following strings are ungrammatical: *b*, *aa*, *ba*, *bb*, *aba*, *abb*, *abaa*, etc.



FSAs are often used in natural language processing. The two examples below show FSAs recognizing English phrases for numbers from 1 to 99 and for prices in dollars and cents under $100, respectively.





Problem: Write an FSA for time-of-day expressions like *eleven o'clock*, *twelve-thirty*, *midnight*, *a quarter to ten* and others -- ideally, your FSA should cover all possible time-of-day expressions used in English, and only them, i.e. your FSA should not accept any expression that is ungrammatical in English.